

Steady State Dispersion of Non-buoyant Air Pollutants with Variable Wind Profile and Removal Mechanism

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Abstract:

In this paper, a mathematical model is proposed to study the steady state dispersion of non-buoyant air pollutants emitted from a continuous point source. The effect of wind velocity is taken in the form of a power function that changes with downwind and crosswind distances. The concept of removal mechanism is also incorporated in the model. The concentration profile of non-buoyant air pollutants is computed and variations of the downwind and crosswind distances for a range of parametric values are analyzed. The results are explained graphically. It is found that the curves drawn demonstrate the consistent behavior with different vertical distances for $0 \leq x \leq 1$, while keeping the crosswind distance constant. Also, the downwind distance is increased with a consistent decrease in the concentration profile. The curves with different crosswind distance values and a constant vertical distance also show uniform behavior for $0 \leq x \leq 1$. Concentration profiles have been found to be symmetric for $-1 \leq y \leq 1$ with varying downwind distance values while maintaining a constant vertical distance value.

Keywords: Mathematical model, Air pollutants, Removal mechanism, Variable wind profile.

1. Introduction

The pollution caused by air pollutants is known as air-pollution [20]. Air pollution started from the moment when primitive man knew to make fire, since then, it has increased and is still increasing every moment due to population explosions, rapid industrialization, and urbanization. Thus, there is a growing concern about the pollution of the atmospheric air by man made activities. Many of the gaseous pollutants emitted by natural activities adversely affect the quality of the air, particularly near dense urban areas and near large emission sources. The gaseous air pollutants include carbon monoxide, chlorine, halogenated solvents, hydrocarbons, hydrogen sulphide, nitrous oxide and sulphur dioxide.

Air pollution is normally supposed as the dis-equilibrium condition of air caused due to the introduction of foreign elements from natural and man made sources to the air so that it becomes injurious to biological communities. Therefore, air pollution may be described as the imbalance in quality of air so as to cause adverse effects on the living organisms existing on the earth.

Now-a-days air pollution is one of the biggest issues which affect both public and individual health by increasing morbidity and mortality rates in addition to contributing to climate change. Human health is adversely affected by a wide range of air pollutants. Air pollutants are harmful airborne

substances that threaten the human welfare, and harm environment. Particulate matter (PM) is one of the air pollutants which are tiny particles with a range of diameters that are inhaled into the human respiratory system and can cause respiratory diseases, other diseases such as cancer, and problems with the reproductive, cardiovascular, and central nervous systems are also taking their exposure in human life. Short-term exposure to air pollution is strongly linked to the conditions such as asthma, respiratory disorders, wheezing, coughing, and high hospitalization rates [11].

Particulate and gaseous pollutants are two categories of environmental pollutants. Agricultural materials, chemicals, lead, SPM, RSPM, and other materials are examples of particulate air pollutants. The six primary air pollutants that the World Health Organization (WHO) tracks are lead, sulfur oxides, nitrogen oxides, carbon monoxide, particle pollution, and ground-level ozone. The effects of air pollution on the groundwater, soil, and other environmental components can be disastrous. It poses a serious risk to living things as well. Acid rain, global warming, greenhouse effect, and climate change are realized due to adverse ecological effects of air pollution [19].

A lot of research is done on the intricate connection between emission sources and air quality using air dispersion models. The analytical solutions have multiple benefits since each of the influencing parameters is stated clearly in closed-form mathematics. It is simple to look into how each parameter affects the models individually. Analytical solutions are also helpful in evaluating performance and accuracy of the numerical models [10]. Analysis of the analytical solutions enables important insights into how a system behaves to be obtained. When predicting the impact of a specific source (stack) emission on air quality, dispersion modeling is crucial for investigators.

Understanding the types of air pollutants is crucial for creating an accurate air pollution model. Consequently, when presenting mathematical models, consideration should be given to both the passive nature of the boundaries and the structure and characteristics of the pollutants.

Within the scientific community, mathematical modelling is widely acknowledged for its significance and necessity. In the past, a variety of modeling techniques have been successfully applied to address the dispersion of air pollution. Additionally, consistent efforts are being made to increase prediction accuracy by utilizing the most recent developments in computing technology as well as enhancements to the observational and modeling frameworks [12].

Sharan et al. [12, 13] have attempted to review the significant research on dispersion modelling. In a different work, Sharan et al. developed a mathematical model for low wind conditions by accounting for diffusion downwind.

Essa [7] and Essa et al. [8] have studied the effect of eddy diffusivity on the solution of diffusion equation. In another paper, Essa et al. [9] have provided an analytical solution to the three-dimensional advection-diffusion equations, taking the vertical eddy diffusivity and the wind speed as the dependent variables for the vertical height z , with the assumption that the concentration distribution of pollutants in the crosswind direction has a Gaussian shape.

Using a power law profile, Srivastava et al. [14] have presented a three-dimensional atmospheric diffusion model with a variable removal rate and variable wind velocity.

Bhandari and Verma [4] have presented an analytical model for the dispersion of air pollutants in a finite atmospheric boundary layer using variable separable method. Bhandari et al. [5] have presented a mathematical study on dispersion of non-buoyant air pollutants emitted from point source having variable wind velocity.

An analytical approach to a problem on the dispersion of air pollutants is investigated by Verma et al. [15].

Verma [18] has also provided an analytical approach to the problem of dispersion of air pollutant with constant wind velocity and constant removal rate by taking constant eddy diffusivities into consideration.

By assuming that the wind velocity varies with downwind distance in the form of a wave function and the removal rate as a constant, Agarwal et al. [1] have studied an unsteady state three-dimensional atmospheric diffusion equation for air pollutants emitting from a point source.

Various results have been found for the advection-diffusion model by taking various conditions on wind velocity and eddy diffusivity [16, 17, 18].

Particulates have a property to settle on the atmosphere's surface, therefore, it is important to take into account that they are non- buoyant [6, 2].

Alam and Seinfeld [3] have given a solution of steady state diffusion equation for sulphur dioxide and sulphate dispersion from point source. Lin and Hildemann [10] have given analytical solutions of the atmospheric diffusion equation with multiple sources.

Manisalidis et al. [11] have reviewed environmental and health impacts of air pollution.

In view of the above mentioned researches done on how changing wind velocity affects the dispersion of non-buoyant pollutants from a point source that is located at the ground's origin ($z=h_s$), it is recommended to use a non-positive sink velocity in the vertical direction to take the effects of buoyancy on the pollutant into consideration. Therefore, we have considered the Dirichlet-type boundary condition, which implies that contaminants are removed as soon as they come into contact with the boundaries.

2. Mathematical Model

Consider the steady state dispersion of non-buoyant air pollutants emitted from a point source of strength Q located at height h_s from the ground, where removal mechanism exists. The partial differential equation governing the concentration $C(x, y, z)$ of the non-buoyant air pollutants with boundary conditions can be written as follows:

$$U(z) \frac{\partial C}{\partial x} - w_s \frac{\partial C}{\partial z} = K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} - \alpha C \quad (1)$$

where the downwind diffusion is negligible in comparison to advection. The effect of buoyancy on the problem is modelled by prescribing a negative sink velocity ' $-w_s$ ' in the z - direction where $w_s = |w_s|$, C is the concentration of air pollutants, $U(z)$ is the variable wind velocity and α is the constant removal rate, K_y and K_z are eddy diffusivities in y - and z -directions respectively. The initial and boundary conditions for equation (1) are taken as follows:

$$C(x, y, z) = \frac{Q\delta(y)\delta(z-h_s)}{U(z)}, x = 0 \tag{2}$$

$$C(x, y, z) = 0, \quad y \rightarrow \pm\infty \tag{3}$$

$$C(x, y, z) = 0, \quad z=0 \tag{4a}$$

$$C(x, y, z) = 0, \quad z = H \tag{4b}$$

where δ is the Dirac-delta function. The conditions (4a) and (4b) represent the Dirichlet type boundary conditions.

It is also assumed that the wind velocity $U(z)$ varies with height, which follows the power law and is taken in the following form:

$$U(z) = \frac{U_H}{H^p} z^p$$

where U_H is the wind velocity at reference height H . Hence, equation (1) can be re-written as:

$$\frac{U_H}{H^p} Z^p \frac{\partial C}{\partial x} - w_s \frac{\partial C}{\partial z} = K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} - \alpha C \tag{5}$$

Using the following dimensionless quantities $\bar{x} = \frac{K_z x}{U_H H^2}, \bar{y} = \frac{y}{H}, \bar{z} = \frac{z}{H}, \bar{h}_s = \frac{h_s}{H}, \bar{C} = \frac{U_H H^2 C}{Q}$,

$\bar{\delta}(\bar{z} - \bar{h}_s) = H\delta(z - h_s), \bar{\alpha} = \frac{\alpha H^2}{K_z}, \bar{w} = \frac{w_s H}{K_z}$, equation (5) (on dropping the bars for convenience) becomes

$$Z^p \frac{\partial C}{\partial x} - w \frac{\partial C}{\partial z} = \beta \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} - \alpha C, \quad \beta = \frac{K_y}{K_z} \tag{6}$$

and the given boundary conditions reduce to

$$C(x, y, z) = \frac{Q\delta(y)\delta(z-h_s)}{z^p}, x = 0 \tag{7}$$

$$C(x, y, z) = 0, \quad y \rightarrow \pm\infty \tag{8}$$

$$C(x, y, z) = 0, \quad z = 0 \tag{9a}$$

$$C(x, y, z) = 0, \quad z = 1 \tag{9b}$$

3. Method of solution

For the simplicity of problem, we consider $p=1/2$, and the method of separation of variables has been used to solve equation (6). Thus, we take

$$C(x, y, z) = L(x)M(y)N(z) \tag{10}$$

as trial solution, where $L(x)$, $M(y)$ and $N(z)$ are lonely functions of x , y and z respectively.

Then, equation (6) is reduced to

$$\frac{1}{L} \frac{dL}{dx} = \left(\frac{\beta}{M} \frac{\partial^2 M}{\partial y^2} + \frac{1}{N} \frac{\partial^2 N}{\partial z^2} + \frac{w}{N} \frac{\partial M}{\partial z} - \alpha \right) z^{-\frac{1}{2}} = -\lambda^2 \tag{11}$$

where λ^2 is a separation constant.

To solve equation (11), we write the following two differential equations:

$$\frac{1}{L} \frac{dL}{dx} = -\lambda^2 \tag{12}$$

$$\left(\frac{\beta}{M} \frac{\partial^2 M}{\partial y^2} + \frac{1}{N} \frac{\partial^2 N}{\partial z^2} + \frac{w}{N} \frac{\partial M}{\partial z} - \alpha \right) z^{-\frac{1}{2}} = -\lambda^2 \tag{13}$$

The solution of equation (12) is given by

$$L(x) = C_1 \exp(-\lambda^2 x) \tag{14}$$

where C_1 is an arbitrary constant of integration.

In order to find a solution of equation (13), the method of separation of variables is applied once again, and it is reduced into two ordinary differential equations, which are given by

$$\frac{d^2 M}{dy^2} = \frac{(\alpha + \eta^2)M}{\beta} \tag{15}$$

$$\frac{d^2 N}{dz^2} + w \frac{dN}{dz} + \left(\lambda^2 z^{\frac{1}{2}} + \eta^2 \right) N = 0 \tag{16}$$

where η^2 is another separation constant.

The solution of equation (15) is given by

$$M(y) = C_2 e^{-\sqrt{\frac{\alpha + \eta^2}{\beta}} y} + C_3 e^{\sqrt{\frac{\alpha + \eta^2}{\beta}} y} \tag{17}$$

where C_2 and C_3 are arbitrary constants of integration.

To find a solution of equation (16), we simplify the equation by using $z = \frac{s^2}{4}$ i.e., $s = 2z^{\frac{1}{2}}$

so that $\frac{dN}{dz} = \frac{2}{s} \frac{dN}{ds}$, and $\frac{d^2 N}{dz^2} = \frac{4}{s^2} \frac{d^2 N}{ds^2} - \frac{4}{s^3} \frac{dN}{ds}$

Putting these values of $\frac{dN}{dz}$ and $\frac{d^2 N}{dz^2}$ in equation (16), we have

$$\frac{4}{s^2} \frac{d^2 N}{ds^2} - \frac{4}{s^3} \frac{dN}{ds} + 2 \frac{w}{s} \frac{dN}{ds} + \left(\lambda^2 \frac{s}{2} + \eta^2 \right) N = 0 \tag{18}$$

For solving equation (18), we apply Frobenius series method, so let us take

$$N(s) = \sum_{n=0}^{\infty} C_n s^{k+n}, C_0 \neq 0 \quad \text{i.e., } N(s) = C_0 s^k + C_1 s^{k+1} + C_2 s^{k+2} + \dots \tag{19}$$

Putting the values of $N, \frac{dN}{ds}, \frac{d^2 N}{ds^2}$ in (18), it is deduced that

$$\sum_{n=0}^{\infty} C_n \left[(k+n)(k+n-1) s^{k+n-2} - (k+n) s^{k+n-2} + \frac{w}{2} (k+n) s^{k+n} + \frac{\lambda^2}{8} s^{k+n+3} + \frac{\eta^2}{4} s^{k+n+2} \right] = 0$$

The indicial equation for this is given by

$k(k - 2) = 0 \Rightarrow k = 0$ and 2 which gives the indicial roots as $k = 0$ and 2 .

The recurrence relation is given by

$$C_n = \frac{-\frac{\lambda^2}{8}C_{n-5} - \frac{\eta^2}{4}C_{n-4} - \frac{w}{2}C_{n-2}(k+n-2)}{(k+n)(k+n-2)} \tag{20}$$

Values of constants can be obtained by using equation (20) as follows:

For $n = 1$, we get $C_1 = 0$

For $n = 2$, we get $C_2 = -\frac{wC_0}{2(k+2)}$

For $n = 3$, we get $C_3 = 0$

For $n = 4$, we get $C_4 = -\frac{\frac{\eta^2}{4}C_0}{(k+4)(k+2)} - \frac{\frac{w}{2}C_2}{(k+4)}$

For $n = 5$, we get $C_5 = \frac{-\frac{\lambda^2}{8}C_0}{(k+5)(k+3)}$

For $n = 6$, we get $C_6 = \frac{-\frac{\eta^2}{4}C_2}{(k+6)(k+4)} - \frac{\frac{w}{2}C_4}{(k+6)}$

For $n = 7$, we get $C_7 = \frac{-\frac{\lambda^2}{8}C_2}{(k+7)(k+5)} - \frac{\frac{w}{2}C_5}{(k+7)}$, etc.

Thus, for $k=0$, we have $C_2 = \frac{-wC_0}{2.2}$, $C_4 = \frac{-\frac{\eta^2}{4}C_0}{4.2} - \frac{\frac{w}{2}C_2}{4}$, $C_5 = \frac{-\frac{\lambda^2}{8}C_0}{5.3}$, $C_6 = \frac{-\frac{\eta^2}{4}C_2}{6.4} - \frac{\frac{w}{2}C_4}{6}$, $C_7 = \frac{-\frac{\lambda^2}{8}C_2}{7.5} - \frac{\frac{w}{2}C_5}{7}$, etc.

and for $k=2$, we have $C_2 = \frac{-wC_0}{2.4}$, $C_4 = \frac{-\frac{\eta^2}{4}C_0}{6.4} - \frac{\frac{w}{2}C_2}{6}$, $C_5 = \frac{-\frac{\lambda^2}{8}C_0}{7.5}$, $C_6 = \frac{-\frac{\eta^2}{4}C_2}{8.6} - \frac{\frac{w}{2}C_4}{8}$, $C_7 = \frac{-\frac{\lambda^2}{8}C_2}{9.7} - \frac{\frac{w}{2}C_5}{9}$, etc.

Putting these values of constants in (19), we can find the solution of (16) in the following form:

$$N(z) = k_2 \left[4z - 2wz^2 - 0.665z^3(-w^2 + \eta^2) - 0.45696z^{\frac{7}{2}}\lambda^2 - 0.16665z^4w^3 + 0.333\eta^2wz^4 + 0.228z^{9/2}\lambda^2w + \dots \right] \tag{21}$$

The above is obtained since $k_1=0$ after using boundary condition 9(a).

Again from, the boundary condition 9(b), we get the eigen value equation in the following form:

$$4 - 2w - 0.665(-w^2 + \eta_m^2) - 0.45696\lambda_m^2 - 0.16665w^3 + 0.3333w\eta_m^2 + 0.228w\lambda_m^2 + \dots = 0 \tag{22}$$

Since the above equation is uniformly convergent in $[0, 1]$ for $\lambda_m^2 \leq \eta_m^2$, therefore, we take the value of separation constants so as $\lambda_m^2/\eta_m^2 \leq 1$.

Now, applying the boundary condition (8) in (17), the solution $M(y)$ is obtained as

$$M(y) = C_4 \left[H(y)e^{-\sqrt{\frac{\alpha+\eta^2}{\beta}}y} + H(-y)e^{\sqrt{\frac{\alpha+\eta^2}{\beta}}y} \right] \tag{23}$$

where C_4 is an arbitrary constant and $H(y)$ is the unit step function which is given by

$$H(y) = \begin{cases} 0, & y < 0 \\ 1, & y > 0 \end{cases} \tag{24}$$

Substituting the values of $L(x)$, $M(y)$ and $N(z)$ from equations (14),(23) and (21) in (10), the value of concentration $C(x, y, z)$ is given by

$$C(x, y, z) = \sum_{m=1}^{\infty} [k_m \exp(-\lambda^2_m x) \{H(y)e^{-\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y} + H(-y)e^{\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y}\} f_m(z)] \tag{25}$$

where without loss of generality, it is assumed that $C_1=C_4= 1$ and $f_m(z)$ is given by (21).

Now, applying the boundary condition (7) in (25), it is obtained that

$$Q\delta(y)\delta(z - h) = \sum_{m=1}^{\infty} k_m \{H(y)e^{-\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y} + H(-y)e^{\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y}\} f_m(z)z^{\frac{1}{2}} \tag{26}$$

Multiplying (26) by $f_n(z)$ and then integrating w.r.t. z from 0 to 1 and w.r.t. y from $-\infty$ to ∞ , and using the following properties of Dirac-delta function -

$$\int_0^1 \delta(z - h_s) f_m(z) dz = f_m(h_s), \quad \int_{-\infty}^{\infty} \delta(y) dy = 1$$

as well as using the orthogonality condition of eigen functions, i.e. $\int_0^1 z^{\frac{1}{2}} f_m(z) f_n(z) dz = 0$ if $m \neq n$, the constant k_m is obtained as

$$k_m = \sqrt{\frac{\eta_m^2 + \alpha}{\beta}} \frac{Q f_m(h_s)}{2 \int_0^1 f_m^2(z) z^{1/2} dz} \tag{27}$$

Substituting the value of k_m in equation (25), we finally get

$$C(x, y, z) = \sum_{m=1}^{\infty} [Q \exp(-\lambda^2_m x) \{H(y)e^{-\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y} + H(-y)e^{\sqrt{\frac{\alpha+\eta^2_m}{\beta}}y}\} \sqrt{\frac{\alpha+\eta^2_m}{\beta}} \frac{f_m(h_s) f_m(z)}{2 \int_0^1 z^{1/2} f_m^2(z) dz}] \tag{28}$$

4. Results and Discussion

We study the case of dispersion of air pollutants emitted from a source at $(0,0,h_s)$, where the wind velocity is assumed to vary with the power of vertical height z . The dimensionless concentration is calculated by using (28). The dimensionless parametric values used in the analysis are taken as follows:

Parameters	α	β	h_s	λ	H	Q
Values	2.0	10.0	0.2	10.0	1.0	1.0

In order to illustrate the behavior of the concentration profile of non buoyant air pollutants, we have displayed the concentration distribution graphically for variety of conditions:

In figure (1), we have plotted the concentration of non- buoyant air pollutants against the downwind distance ($0 \leq x \leq 1$) for different values of vertical distance ($z = 0.8, 0.5, 0.2$) keeping the crosswind distance fixed at ± 1 . The concentration profiles decrease regularly with increasing downwind distance.

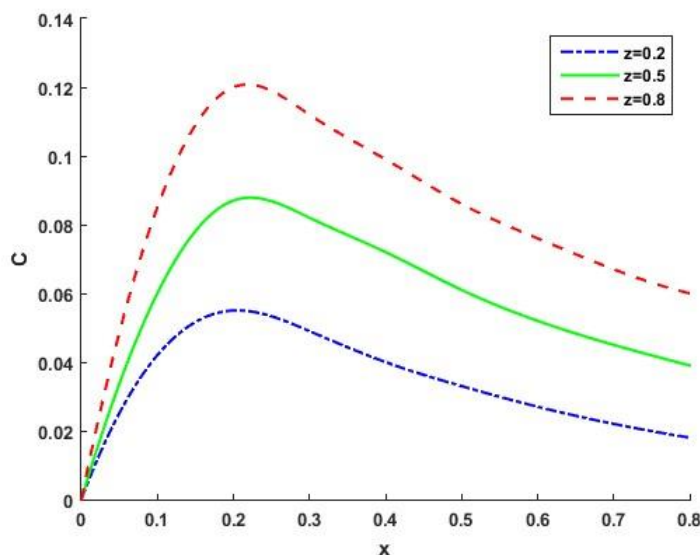


Fig 1: Dimensionless concentration $C(x, \pm 0, z)$ plotted against downwind distance x .

In figure (2), the concentration of non-buoyant air pollutants is plotted with respect to downwind distance ($0 \leq x \leq 1$). For different values of crosswind distance ($y= 0.5, 0.3, 0.1$), the value of vertical distance is fixed at ($z=0.2$).

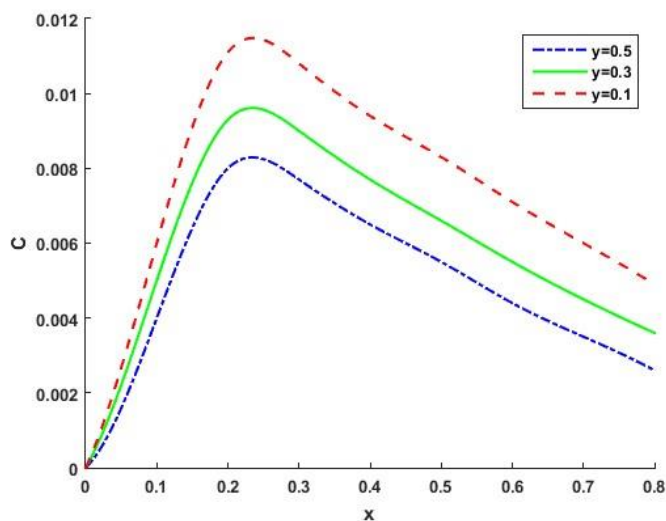


Fig 2: Dimensionless concentration $C(x, y, 0)$ plotted against downwind distance x .

In figure (3), we have plotted the concentration of non-buoyant air pollutants against cross wind distance ($-1 \leq y \leq 1$) for different values of downwind distance ($x=0.1, 0.5, 0.9$) keeping the value of vertical distance fixed at ($z = 0.2$). It is seen that the concentration profiles are symmetric with peak concentrations along the centerline of the plume.

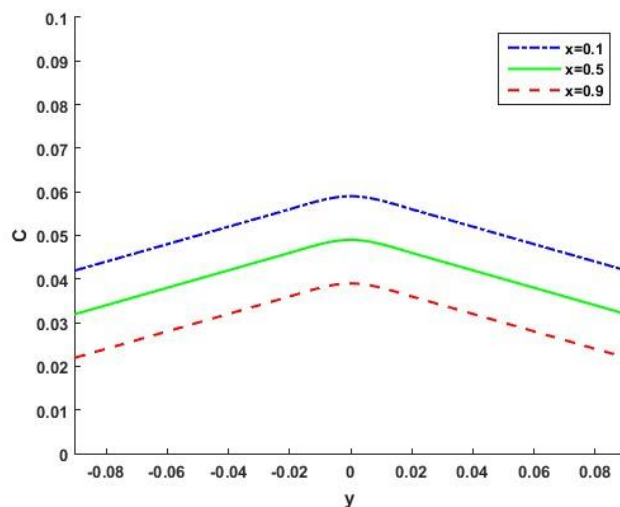


Fig 3: Dimensionless concentration $C(x, y, 0.2)$ plotted against cross-wind distance y .

5. Conclusion

In this study, a mathematical model for the steady state dispersion of non-buoyant air pollutants emitted from a continuous point source has been constructed and an analytical solution is obtained. We have shown how non-buoyant air pollutants are dispersed by a continuous point source, with the wind velocity represented by a power function that varies with cross-wind and downwind distances. For various parametric values, the concentration profile of non-buoyant air pollutants is examined against the cross-wind and downwind distances. The three curves have illustrated uniform behavior for $0 \leq x \leq 1$ with various vertical distances while maintaining the crosswind distance constant. The concentration profiles consistently get smaller as the downwind distance grows. Again, three curves have demonstrated uniform behavior for $0 \leq x \leq 1$ with varying values of the crosswind distances while maintaining the vertical distance constant. Peak concentrations have found along the plume's centerline, and concentration profiles are symmetric for $-1 \leq y \leq 1$ with varying downwind distance values while maintaining a constant vertical distance value. Also, it is shown that as the downwind distance increases, the crosswind concentrations converge toward a uniform distribution.

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