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An Evacuation Vehicle's Shortest Toll-free Routing across Smart Cities using the Fuzzy Weighted Graph Technique

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Abstract:

Rapid technical improvements and rising consumer demands are the two challenges facing in the transport sector. India's growing economy has raised demand for infrastructure and transport services. Maharashtra, while being a sizable city, has inadequate public transportation infrastructure and poor pathway blockades. In smart cities, an integrated public transportation network optimizes time, cost, comfort, safety, accessibility, and convenience for citizens through incorporating many forms of transportation. The fuzzy weighted graph technique is applied in this article to address the demands of the shortest distance with toll-free route accessibility for paramedics and private shuttles to major events in the smart cities of Maharashtra. This initiative will persuade the government to levy a fine on the public for individuals who use the shortest distance designated only for special vehicles.

Keywords: Fuzzy Spanning tree, Fuzzy Weighted Graph Algorithm, Smart City, Smart mobility, Emergency Vehicle routing.

Introduction

In the modern era, urbanization and economic growth have necessitated the transformation of cities into smart cities to address challenges in infrastructure, sustainability and quality of life. Transportation system plays a pivotal role in smart city development by ensuring efficient mobility and accessibility. However, the real-world transportation networks face numerous challenges, including congestion, environmental impact and delays particularly for emergency vehicles. The growing population and increasing traffic density in cities like Maharashtra have made it important to optimize urban transit systems. Transportation networks are inherently complex and dynamic where variables such as traffic condition, travel duration and environmental factors influence route efficiency.

A "smart city" concept seeks to improve public benefits, energy, conveyance, wellness, cultivation and garbage disposal [1]. Proactive planning of social amenities by the government is necessary to prevent regrettable events and raise individual understanding of societal issues for evaluating the key sociological variables affecting women's safety initiatives in a city [2]. Researchers have been drawn to the topic of smart transportation because it has the potential to completely change how we transport people and things [3]. Sumi. L. et al., presents an example of a few smart management systems that have been developed to aid with traffic congestion reduction through the Internet of Things (IoT) [4].

IoT offers motorists in a smart city a number of advantages, such as better traffic management, better logistics, effective parking systems, and increased safety measures. The incorporation of all these advantages into applications for transportation systems is known as "smart transportation" [5,6]. The key challenge of 1-to-n ride-sharing is matching a minimal number of passengers with a driver while maintaining their security, as well as connecting vehicle holders' routes with their riders' spots without suffering a delay. This system can use the

record for analyzing the previously visited locations to suggest the best routes and pick-up locations for drivers and passengers [7]. To limit the number of Road-Side Units, the suitable strategy was utilized, which involved selecting locations at intersections that maximized the surface covered by the urban area while minimizing the number of overlapping zones [8]. A two-feed, computer vision-based framework for 3D convolutional neural network (CNN)-based NDA/DA recognition was proposed in [9]. The conduct of the driver, including head movement and hand movement, is taken into account by the CNN framework. It has been advised choosing a Vehicle-to-Infrastructure (V2I) design and positioning RSUs at junctions to ensure effective data transmission in vehicle networks [10]. In order to support quick re-optimization in the dynamic environment was described which provides an agile optimization algorithm. The algorithm is put to the test using a number of existing benchmark examples, and the results of the experiments demonstrate that it is capable of producing high-quality results quickly in dynamic Internet of Vehicles. The design idea helps to make assessments by using sophisticated techniques to optimize traffic congestion in immediate response and addresses research in science in autonomous vehicles, such as ideas skills, vehicle Internet of Things, communication organization, and transmission procedures [11]. Finding the shortest paths with toll-free route in weighted graphs is one of the classic problems in the field of combinatorial optimization, and it has been extensively studied over the past fifty years. Fuzzy weighted graphs, together with generalizations of techniques for locating optimal pathways within them, have recently emerged as a viable modelling tool for systems that are excessively complex and/or inherently wrong.

In an effort to fulfil the demands of the expanding population, accelerated urban expansion has resulted in a variety of problems, including pollution, more and crowded traffic, poor sustainability, and effects on the environment. Intelligent convergence systems have been proposed as a potential solution to these problems in the context of smart cities. Information, communications, and technology (ICT) have been used to develop the idea of a "smart city" to mitigate the disadvantages of increasing urbanization. To make cities smarter and more sustainable, significant efforts have been made in this area [12].

Fuzzy graph theory has been instrumental in addressing smart city challenges, with Gunasekar et al. exploring concepts like triple connected perfect domination, n-connected total perfect k-domination, and strong and weak-connected total perfect domination [13]–[17]. These studies emphasize its role in optimizing urban systems and promoting sustainability.

However, there hasn't been enough research done on the challenges of developing India's smart city implementation and assessment, particularly in Maharashtra. Traditional approaches often fail to account for this variability, making it difficult to provide precise and reliable solution. Under these circumstances, fuzzy graph theory has emerged as a promising tool to model and address uncertainties in such systems. Despite advancements in algorithms for shortest path optimization, the unique requirements of emergency and special- purpose vehicles remain unexplored.

1. Preliminaries

Definition 2.1. Fuzzy graph with the elementary set S is a pair $G: (\sigma, \mu)$ $where\ \sigma : S \rightarrow [0,1]$ is a fuzzy subset, μ: SxS $\rightarrow$ [0,1] is a fuzzy relation on the fuzzy subset σ such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in S$.

Definition 2.2. The fuzzy tree that connects each and every single vertex in a fuzzy graph is a spanning tree. If it has n vertices, then every spanning tree of a fuzzy graph has $n - 1$ edges.

Definition 2.3. A subset of the edges of a connected, edge-weighted, undirected fuzzy graph that connects all the vertices together, without any cycles, and with the least amount of total arc weightage is known as a fuzzy spanning tree with minimum weight.

Definition 2.4 A closed Hamiltonian fuzzy path is called a Hamiltonian fuzzy cycle in the fuzzy graph. In other words, a Hamiltonian fuzzy path is a spanning fuzzy path and a Hamiltonian fuzzy cycle is a spanning fuzzy cycle.

Definition 2.5. Fuzzy Hamiltonian Cycle are circuits that visit each vertex in a fuzzy graph only once, without repeating.

Definition 2.6. A fuzzy minimum spanning tree or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected fuzzy graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight. The example is shown below in fig 1,

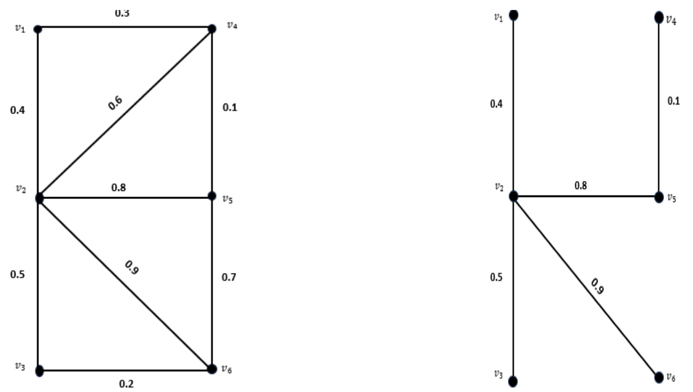

Fig 1. Undirected Fuzzy Graph G and Minimum Weight Spanning Tree

Definition 2.7 The distance between two vertices, u and v, in a fuzzy graph G is represented as $d(u, v)$ and indicates the length of the fuzzy graph G's shortest path. Figure 2 below displays the distance between two vertices.

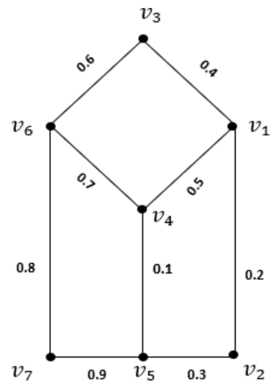

Fig 2. Distance between two vertices

The vertex v_4 can be reached from vertex v_5 by numerous routes, consider the distance between v_4 and v_5.

$v_4 v_1 - v_1 v_2 - v_2 v_5$ = 0.5, 0.2, 0.3

$v_4 v_6 - v_6 v_7 - v_7 v_5$ = 0.7, 0.8, 0.9

$v_4 v_6 - v_6 v_3 - v_3 v_1 - v_1 v_2 - v_2 v_5$ = 0.7, 0.6, 0.4, 0.2, 0.3

$v_4 v_1 - v_1 v_3 - v_3 v_6 - v_6 v_7 - v_7 v_5$ = 0.5, 0.4, 0.6, 0.8, 0.9.

Therefore, the distance between the two vertices $v_4 v_5$ with minimum weight 0.1 is the shortest path.

2. Problem Statement

Maharashtra's smart cities face critical issues including traffic congestion and insufficient infrastructure for toll-free emergency vehicle transit. The lack of an efficient system for determining shortest routes for ambulances and other rescue vehicles contributes to delays in emergency response which can be life-threatening. Existing models primarily focus on general transit without prioritizing the distinct needs of special-purpose vehicles, leaving a critical gap in the implementation of smart city solutions. The below Table 1 shows the list of smart cities in Maharashtra.

Table 1. List of Maharashtra's smart cities.

1	Nashik
2	Thane
3	Solapur
4	Nagpur
5	Kalyan-Dombivli
6	Aurangabad
7	Pune
8	Pimpri Chinchwad

This paper proposes an innovative approach utilizing fuzzy weighted graph theory to design a model for the shortest path determination in integrated transport systems. The aim is to optimize transit for emergency and special vehicles which addressing the broader challenges of smart city mobility and contribute to Maharashtra's urban planning efforts, enhancing the efficiency and sustainability of transportation network.

Therefore, we analyze and formalize the fuzzy weighted algorithms with a focus on the path comparison ranking techniques. In this methodology, shortest path of the network can be traced so that conveyance can be provided by the government among these cities. This work strives to provide a model based on a fuzzy weighted graph with a shortest route technique that is both effective and easy to apply in scenarios in the real world. This increases the possibility of safeguarding each person's life while also significantly lowering the number of emergency deaths caused by traffic congestions. This study furthermore efforts to rank individual obstacles within the categories and highlight them with the objective to pinpoint the most significant challenge that impeding the growth of smart cities in India. The majority of studies have concentrated on shortest distances using various models with algorithms but there has been no research on particular access for emergency vehicles that has been published currently. Observe the map of Maharashtra in Fig 3, which displays the routes that connect the smart cities.

3. Shortest Route Algorithm

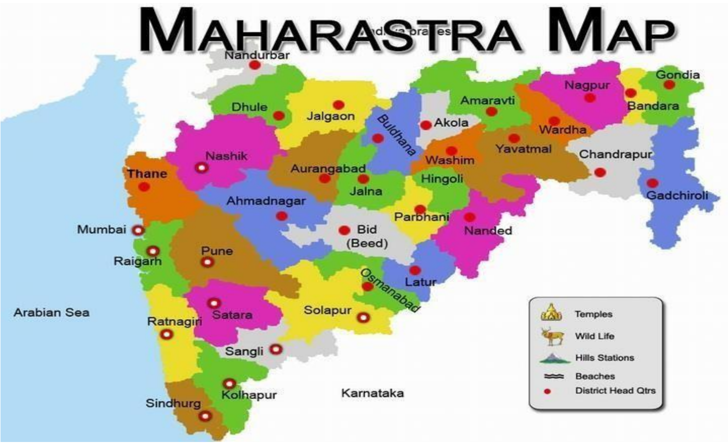

Fig. 3 Roads connecting the smart cities

Assume that $G\ (V, E, W)$ is an undirected, linked fuzzy network with n vertices.

V – the collection of vertices that make up the fuzzy graph,

E - Edges of the fuzzy graph

W − Appropriate weights of the fuzzy graph.

Finding the path that incurs the demands of the shortest distance with toll-free route accessibility for rescuers and private shuttle services between the vertices in fuzzy graph G can be done using the procedures given below.

4.1. Algorithm for the Conceptual Weighted Matrix

It is suggested that the matrix having a minimum of weight in a weighted fuzzy graph is G.
Step 1: The fuzzy graph G's edges have been categorized according to their increasing weight.
Step 2: An edge G with the least amount of weight is selected in order to develop an appropriate spanning tree.
Step 3: The least weighted edges that are not linked to the previously chosen edges are incorporated one subsequent to the other.

Step 4: Proceed with this until all (n -1) edges were successfully observed.
Step 5: We discover the fuzzy network's shortest path.

The result is the undirected weighted fuzzy graph G's minimum spanning tree T.To illustrate the integrated transit system, each smart city of Maharashtra can be utilized as a vertex and the length connecting the two smart cities as an arc or an edge. The distance among the two smart cities should be taken into account in the following way while developing the aforementioned problem as a fuzzy graph.

The following are the vertices for the corresponding smart cities of Maharashtra which is shown in Table 2.

Table 2. Vertices of Smart cities

Vertices	Smart Cities
a	Nashik
b	Thane
c	Solapur
d	Nagpur
e	Kalyan-Dombivli
f	Aurangabad
g	Pune
h	Pimpri Chinchwad

The table 3. below displays the range of distances and their corresponding membership grades.

Table 3. Distances and their corresponding membership values

Distance in km	Membership Values
0-100	0.16
100-200	0.24
200-300	0.19
300-400	0.21
400-500	0.28
500-600	0.17
600-700	0.25
700-800	0.12
800-900	0.23

The roads linking nearby cities are listed in table 4 along with their respective distances in kilometers.

Table 4. Distances between the smart cities

Edge	From	To	Distance (in km)	Membership values	Edge	From	To	Distance (in km)	Membership Values
1	**a**	**b**	145	0.24	15	**b**	**h**	145	0.24
2	**b**	**c**	402	0.28	16	**c**	**e**	396	0.21
3	**a**	**c**	381	0.21	17	**e**	**f**	347	0.21

4	**c**	**d**	633	0.25	18	**f**	**g**	235	0.19
5	**a**	**d**	679	0.25	19	**c**	**f**	309	0.21
6	**b**	**d**	824	0.23	20	**c**	**g**	254	0.19
7	**d**	**e**	818	0.23	21	**c**	**h**	280	0.19
8	**a**	**e**	114	0.24	22	**d**	**f**	491	0.28
9	**b**	**e**	12	0.16	23	**d**	**h**	731	0.12
10	**b**	**f**	371	0.21	24	**h**	**e**	132	0.24
11	**b**	**g**	152	0.24	25	**e**	**g**	142	0.24
12	**a**	**f**	183	0.24	26	**d**	**g**	761	0.12
13	**a**	**g**	210	0.19	27	**f**	**h**	246	0.19
14	**a**	**h**	199	0.24	28	**g**	**h**	15	0.16

The distance between the smart cities indicates their level of membership value is given in the fig.4.

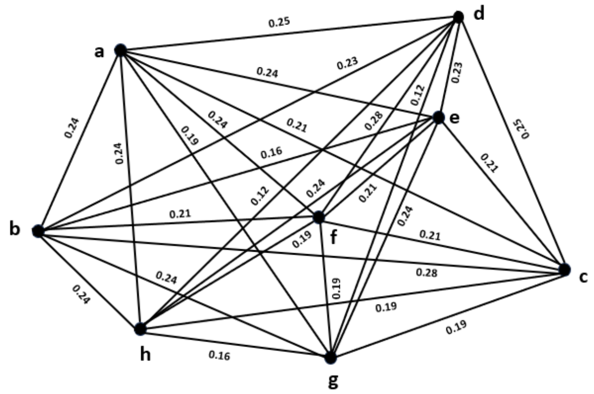

Fig 4. Complete Weighted Fuzzy Graph

Integrated Transport Systems have emerged as a promising solution for managing shortest distance for paramedics and VIP vehicles in developing smart cities. If this facility is implemented effectively, public transportation as well as the special- purpose vehicles will run more efficiently, there will be less traffic, and the public will be delighted. The corporation must determine the shortest route between each pair of cities in order to rectify the roadblocks between them. To determine the shortest path between the smart cities providing the services with the lowest cost, the proposed weighted matrix method is used in the manner described below.

5. Fuzzy Weighted Graph's Distance Matrix

Let us tabulate the vertex positions of each edge connecting each pair of vertex positions as shown below in table 5.

Table 5. Connecting edge between the pair of vertices

	a	b	c	d	e	f	g	h
a	-	0.24	0.21	0.25	0.24	0.24	0.19	0.24
b	0.24	-	0.28	0.23	0.16	0.21	0.24	0.24
c	0.21	0.28	-	0.25	0.21	0.21	0.19	0.19

d	0.25	0.23	0.25	-	0.23	0.28	0.12	0.12
e	0.24	0.16	0.21	0.23	-	0.21	0.24	0.24
f	0.24	0.21	0.21	0.28	0.21	-	0.19	0.19
g	0.19	0.24	0.19	0.12	0.24	0.19	-	0.16
h	0.24	0.24	0.19	0.12	0.24	0.19	0.16	-

The below Table 6 illustrates that the edges c-d, c-g, b-e, e-f, and e-a create a fuzzy cycle and cannot be taken into consideration for a minimal spanning tree.

Table 6. Membership values of the Edges

S. No.	Edges	Membership Grades
1.	a – b	0.24
2.	a – d	0.25
3.	d - e	0.23
4.	e – c	0.21
5.	c – d	0.25(Forms the fuzzy circuit)
6.	c – g	0.19(Forms the fuzzy circuit)
7.	c – f	0.12
8.	f - g	0.19
9.	g – h	0.16
10.	b - e	0.16(Forms the fuzzy circuit)
11.	e – f	0.21(Forms the fuzzy circuit)
12.	e – a	0.24(Forms the fuzzy circuit)

The below is the weighted fuzzy matrix which helps to investigate the irregularity of devices employed in big networks using fuzzy graph theoretical methods. Additionally, the network's nodes efficiency has been increased. The minimum weight is chosen from the preceding figure, and their related edges were drawn again and again until the procedure is finished. If a fuzzy circuit emerges when generating the graph, it is removed, and the weight is marked as 0 in the table.The fuzzy graph in the below matrix contains eight vertices, and a Minimum Spanning Tree can be created by sequentially joining its edges.

	a	b	c	d	e	f	g	h
a	-	0.24	0	0.25	0	0	0	0
b	0.24	-	0	0	0	0	0	0
c	0	0	-	0	0.21	0.12	0	0
d	0.25	0	0	-	0.23	0	0	0
e	0	0	0.21	0	-	0	0	0
f	0	0	0.12	0	0	-	0.19	0
g	0	0	0	0	0	0.19	-	0.16
h	0	0	0	0	0	0	0.16	-

Weighted Matrix of Fuzzy Graph

The tree is made up of the chosen edges which is the necessary fuzzy spanning tree. The fuzzy circuit's forming edges were removed. The Figure 5 shows the smallest spanning tree for the given fuzzy graph.

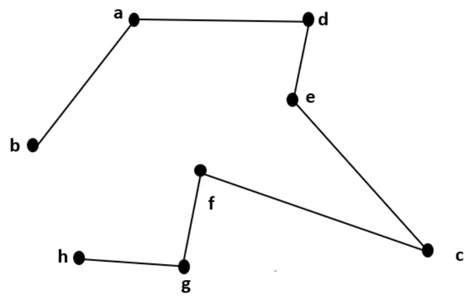

Fig. 5. Minimum Spanning Tree

The shortest overall distance between Maharashtra's smart cities was calculated using the matrix algorithm

= (b, a) + (a, d) + (d, e) + (e, c) + (c, f) + (f, g) + (g, h)

= 0.24 + 0.25 + 0.23 + 0.21 + 0.12 + 0.19 + 0.16

= 1.4

6. Implications of the Suggested Model for Special Vehicles

A discussion was held about an innovative way to improve urban mobility and emergency vehicle transit through Integrated Transport Systems (ITS) in our developing smart cities. The insights of the network diagram were shown to depict the actual pathways and shortest distances between Maharashtra's smart cities. It is a specialized transportation network which is shown in fig 6 that allows emergency vehicles like medical vehicles, rescue engines, and law enforcement cars to move quickly and efficiently around the smart cities. This allows us to accurately portray the journey times for rescue vehicles travelling from a source to a destination. The shortest distance network often contains designated roadways, stoplights, and procedures for routing that prioritize emergency vehicle movement, allowing them to respond swiftly and efficiently. An emergency vehicle transit system aims to enhance community security by minimizing the duration that required for rescue workers to get at the spot of an occurrence.

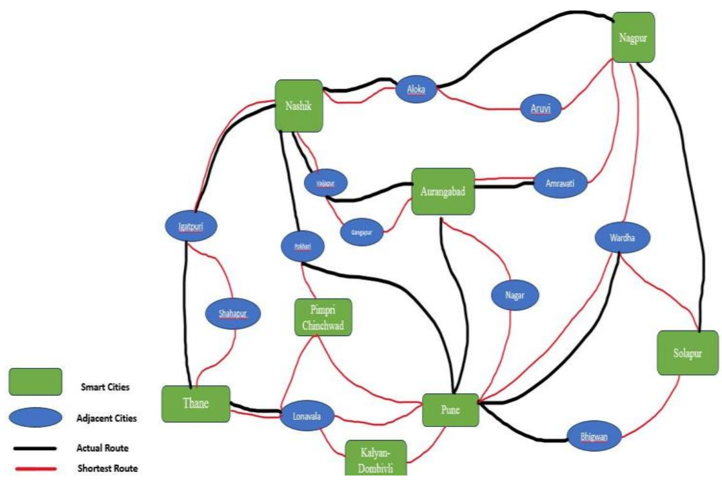

Fig 6. Illustration of Maharashtra’s Integrated Transit System

The investigation was undertaken to emphasize the relevance of taking into account the overall distance between these cities while developing a fuzzy graph model. This analysis will include the routes that connect the surrounding cities, as well as their respective distances in kilometers. The lack of response time in Maharashtra is exacerbated by rescue vehicles such as lifesaving vehicles, fire brigade vehicles, and security vehicles, that need to react to crises as soon as possible. The purpose is to ensure that the shortest paths are easily accessible to special vehicles. The plan of action is based on the fuzzy spanning tree concept, which involves minimizing all edge weights in order to optimize fuzzy graph outcomes. The primary goal of this strategy is to aid the government in defining the key access routes for emergency and special vehicles, thereby allowing smoother operations and removing potential barriers to the development of Maharashtra's smart cities. This opportunity is to examine the notion further and explore how we might work together to improve infrastructure and emergency response capabilities in our smart cities.

To demonstrate the shortest route in integrated transit system mainly for special vehicles, each smart city in Maharashtra can be employed as a vertex, with the length combining the two smart cities as an edge. The overall distance between the two smart cities should be considered in the following way while creating the aforementioned problem as a fuzzy graph. The roads that connect surrounding cities are listed, along with their relative distances in kilometre. Based on the weights of the vertices of the Fuzzy weighted graph can represent for the duration that consumes to commute the rescue vehicles from a source to destination. In addition, this model may signify as an indicator, which means the distinct routes between the areas. The simplest spanning tree of an interconnected weighted undirected fuzzy graph can be identified by numerous methods, allowing any specific vehicle to access the shortest path. This algorithm is based on the fuzzy spanning tree, which minimises all of the fuzzy tree's edge weights which encompasses the outcome of the fuzzy graph. The proposed approach aims to aid the government in determining the primary access routes for special vehicles, thereby removing any potential impediments to the development of Maharashtra's smart cities.

7. Conclusion

Efficient and well-connected transportation system can significantly contribute nation's growing urbanization. In many instances, transportation is the root cause of congestion rather than a problem with the road infrastructure. By offering an integrated transportation system that is easy to use and trend-setting, this suggested algorithm will be beneficial to all types of problems that give a network to smart cities through the quickest way. The shortest path approach is used in this article to determine a weighted fuzzy graph's minimum spanning tree. This effort supports the country's ongoing economic progress, lessens resident annoyance, and also highlights the benefits of direct routes for the special vehicles. Consequently, the fuzzy weighted graph model solves the sustainable and environmentally friendly transportation with innovations that can enhance our healthy lifestyle by lowering tension, anxiety, prices, and expenses. Through this methodology, local government must endeavor to take initiative on the sector which undergoes significant reforms in the smart cities of Maharashtra.

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