

## Jacobi Matrix Polynomial and its Integral Results

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### Article History:

**Received:** 18-10-2024

**Revised:** 02-12-2024

**Accepted:** 10-12-2024

### Abstract:

**Introduction:** Recent advancements in matrix polynomial structures associated with special functions have gained significant traction, showcasing a diverse array of applications across various engineering disciplines. This paper primarily aims to explore and derive multiple integral representations for the modified Jacobi Matrix Polynomial. Specifically, we will delve into both finite and infinite single integral representations, as well as double integral representations of the polynomial. By examining these integral forms, we seek to enhance the understanding of their mathematical properties and implications, thereby providing valuable insights that can be leveraged in practical engineering scenarios. The exploration of these representations not only enriches the theoretical framework surrounding modified Jacobi Matrix Polynomials but also highlights their potential utility in solving complex problems within the engineering domain.

**Keywords :** Finite and Infinite Single Integral Representation, Hypergeometric function, Jacobi Polynomial (Modified), Jacobi matrix polynomial

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## 1. Introduction

In the past few years, many authors extended classical polynomials to matrix polynomials. Many authors generalized the hypergeometric series, Appell's hypergeometric functions also to matrix version of the functions. Recently, a good number of authors studied matrix version of Jacobi, Hermite, Legendre and other polynomials [1]-[4]. The theory of matrix polynomials provides a way to solve many problems in mathematical physics which have real time applications. L.Jodar [5]-[8] introduced matrix form of Laguerre and Hermite and Hypergeometric function. Subhi Khan and others

extended Laguerre polynomials with two variables [9]. Parihar and Patel [10] introduced the modified Jacobi polynomials and derived generating function and recurrence relations of modified Jacobi polynomial. Later on, Sri Lakshmi, V [4] introduced matrix form of modified Jacobi polynomial and derived generating function, recurrence relations of matrix version of modified Jacobi polynomial. Srimanarayana, N et.al [11], [12], derived integral representations for Generalized Hypergeometric function and modified Konhauser’s polynomial. In the present study finite and infinite integral representations and double integral representations has been established for  $J_n^{(A)}(x, w)$ .

**Useful functions and Integral Formulae:**

In the present article, we abide by the rules of matrix theory. Assume  $P_0, P_1, \dots, P_n \in C^{m \times m}$ , where  $P_i$  be the matrix of size ‘m’ and the matrix polynomial of degree ‘n’ as  $f_n(z) = P_n z^n + P_{n-1} z^{n-1} + \dots + P_1 z + P_0$ , where  $P_n$  is not a null matrix. Also,  $I$  and  $O$  be the identity and null matrices in  $C^{m \times m}$ . For a matrix  $P \in C^{m \times m}$ ,  $\sigma(P)$  is the spectrum of P and the matrix  $P$  is a positive stable matrix.

In [13], for all  $P \in C^{m \times m}$  and  $P + nI$  is invertible, where ‘n’ is an integer, then the Pochhammer symbol is defined by

$$(P)_n = \begin{cases} P(P+I)\dots(P+(n-1)I); n \neq 0 \\ \Gamma(P+nI)\Gamma^{-1}(P); n \neq 0 \\ I; n = 0 \end{cases} \tag{1.1}$$

For preliminary matrix version of Gamma, Beta, and Hypergeometric functions one can refer [2], [5], [6], [7], and [13]. In [10], Parihar and Patel defined modified Jacobi Polynomial using a difference operator [14]. Later, Sri Lakshmi, V, et.al extended the same as modified Jacobi Matrix polynomial,  $J_n^{(A)}(x, w)$  as follows [4].

$$J_n^{(A)}(x, w) = \frac{(A+I)_n}{n!} {}_2F_1 \left[ -nI, \frac{x}{w}; I+A; w \right] \tag{1.2}$$

$$= \frac{(A+I)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r (w)^r}{r!(I+A)_r} \tag{1.3}$$

To obtain different integral representations, the following well-known results have been used:

Maclaurin’s theorem is

$$f(s) = \sum_{l=0}^{\infty} \frac{f^{(l)}(0)s^l}{l!} \tag{1.4}$$

so that the coefficients  $f^{(l)}(0); l = 0, 1, 2, \dots$  are obtained by the integrals

$$f^{(l)}(0) = \frac{l!}{2\pi i} \int_{(0^+)} \frac{f(s) ds}{s^{n+1}} ; s = 0, 1, 2, \dots \quad (1.5)$$

For  $\text{Re}(m)$  and  $\text{Re}(n - m) > 0$ , we have

$$\frac{(m)_j}{(n)_j} = \frac{\Gamma(n)}{\Gamma(m)\Gamma(n-m)} \int_0^1 \xi^{m+j-1} (1-\xi)^{n-m-1} d\xi \quad (1.6)$$

$$\Gamma(n) = \int_{-\infty}^{\infty} e^{-t^2} t^{2n-1} dt \quad (1.7)$$

If  $\text{Real}(m, n) > 0$ , then

$$\int_0^1 \int_0^1 \frac{(1-x)^{k-1} y^k (1-y)^{l-1}}{(1-xy)^{k+l-1}} dx dy = \beta(k, l) \quad (1.8)$$

If  $\text{Real}(k, l) > 0$ ,  $\text{Real}(\lambda) > 0$  then

$$\iint_A u^{k-1} v^{l-1} (1-u-v)^{\lambda-1} dudv = \frac{\Gamma(k)\Gamma(l)\Gamma(\lambda)}{\Gamma(k+l+\lambda)} \quad (1.9)$$

where 'A' is the area lies between  $u, v \geq 0$  and  $u+v \leq 1$ .

$$\beta(a, b) = 2 \int_0^{\pi/2} (\sin t)^{2a-1} (\cos t)^{2b-1} dt \quad (1.10)$$

$$(\lambda)_{2r} = 2^{2r} \left(\frac{\lambda}{2}\right)_r \left(\frac{\lambda+1}{2}\right)_r \quad (1.11)$$

provided  $\text{Re}(s) > 0$  and  $\text{Re}(\alpha) > 0$  [15], [16]

## 2. INTEGRAL REPRESENTATION FOR $J_n^{(A)}(x, w)$

### Integral Representation by a Contour Integral

By the generating function of  $J_n^{(A)}(x, w)$ , we have

$$\sum_{n=0}^{\infty} \frac{J_n^{(A)}(x, w)}{(I+A)_n} t^n = e^t {}_1F_1\left(\frac{x}{w}, I+A; -wt\right) \quad (2.1)$$

$$\text{Assume } f(u) = e^u {}_1F_1\left(\frac{x}{w}, I+A; -wu\right) \quad (2.2)$$

Using Maclaurin's theorem (1.4) and using (1.5), we arrive at the following:

### Theorem:

Assume 'A' be any square matrix of order 'n',  $A \neq 0$

$$J_n^{(A)}(x, w) = \frac{(A+I)_n}{2\pi i} \int u^{-n-1} e^u {}_1F_1\left(\frac{x}{w}, I+A; -wu\right) du$$

Where the contour integral is around the u-plane in anti-clockwise sense.

**Proof:**

Using Maclaurin's Theorem, we have

$$f(u) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)u^n}{n!}$$

$$\Rightarrow f^{(n)}(0) = \frac{n!}{2\pi i} \int \frac{f(u)}{u^{n+1}} du; n = 0, 1, \dots$$

$$\text{If } e^u {}_1F_1\left[\frac{x}{w}; I+A; -wu\right] = \sum_{n=0}^{\infty} \frac{J_n^{(A)}(x, w)}{(I+A)_n} u^n$$

$$\Rightarrow \frac{n! J_n^{(A)}(x, w)}{(I+A)_n} = \frac{n!}{2\pi i} \int \frac{f(u)}{u^{n+1}} du, \text{ which leads to}$$

$$J_n^{(A)}(x, w) = \frac{(A+I)_n}{2\pi i} \int u^{-n-1} e^u {}_1F_1\left(\frac{x}{w}, I+A; -wu\right) du$$

## 2. Real Integral Representation

**Theorem:**

If 'A' be any square matrix of order  $n \times n$  and  $J_n^{(A)}(x, w)$  be the modified Jacobi matrix polynomial, then the real integral representations of this polynomial are as follows:

$$J_n^{(A)}(x, w) = \frac{(A+I)_n}{\pi} \sum_{r,m=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r (-w)^r}{r!(I+A)_r} \int_0^\pi \text{cis}(m+r-n)\theta d\theta \tag{3.1}$$

**Proof:**

Using (1.14), it reduces to (by choosing the contour  $u = e^{i\theta}$ )

$$\begin{aligned} J_n^{(A)}(x, w) &= \frac{(A+I)_n}{n! (2\pi i)} \int_0^{2\pi} e^{e^{i\theta}} e^{i\theta(-n-1)} \sum_{r=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r w^r (e^{i\theta})^r}{r!(I+A)_r} i e^{i\theta} d\theta \\ &= \frac{(A+I)_n}{n! (2\pi)} \int_0^{2\pi} e^{-ni\theta} \sum_{m=0}^{\infty} \frac{(e^{i\theta})^m}{m!} \sum_{r=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r (-w)^r e^{ri\theta}}{r!(I+A)_r} d\theta \end{aligned}$$

By changing the order of summation and integration, we obtain

$$\begin{aligned}
 &= \frac{(A+I)_n}{n! (2\pi)} \sum_{r,m=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r (-w)^r}{r! (I+A)_r} \int_0^{2\pi} e^{(m-n+r)i\theta} d\theta \\
 &= \frac{(A+I)_n}{(2\pi)} \sum_{r,m=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r (-w)^r}{r! (I+A)_r} \int_0^{2\pi} \text{cis}(m+r-n) d\theta
 \end{aligned}$$

Consequently, we arrive at

$$J_n^{(A)}(x, w) = \frac{(A+I)_n}{\pi} \sum_{r,m=0}^{\infty} \frac{\left(\frac{x}{w}\right)_r (-w)^r}{r! (I+A)_r} \int_0^{\pi} \text{cis}(m+r-n)\theta d\theta$$

#### 4. Finite Single Integral Representation

##### Theorem 1:

Assume ‘A’ be any square matrix of order ‘n’,  $A \neq 0$ . If  $\text{Re}(a)$  and  $\text{Re}(b-a)$  are positive, then

$$J_n^{(A)}(x, w) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^{a-1} (1-t)^{b-a-1} {}_3F_2 \left[ \begin{matrix} -nI, b, \frac{x}{w}; \\ I+A, a; \end{matrix} wt \right] dt \tag{4.1}$$

**Proof:** By using (1.3)

$$\begin{aligned}
 J_n^{(A)}(x, w) &= \frac{(A+I)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r}{r! (I+A)_r} \\
 &= \frac{(A+I)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r (b)_r (a)_r}{r! (I+A)_r (a)_r (b)_r}
 \end{aligned}$$

On using (1.5), we have

$$= \frac{(A+I)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r (b)_r \Gamma(b)}{r! (I+A)_r (a)_r \Gamma(a) \Gamma(b-a)} \int_0^1 t^{a+r-1} (1-t)^{b-a-1} dt$$

By interchanging the order of integration and summation, we have

$$= \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^{a-1} (1-t)^{b-a-1} \frac{(A+I)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r (b)_r (wt)^r}{r! (I+A)_r (a)_r} dt$$

$$= \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 t^{a-1} (1-t)^{b-a-1} {}_3F_2 \left[ \begin{matrix} -nI, b, \frac{x}{w}; \\ a, (I+A); \end{matrix} ; wt \right] dt$$

**Theorem 2:**

Assume ‘A’ be any square matrix of order ‘n’,  $A \neq 0$

If real part of  $a, b > -\frac{1}{2}$ , then

$$J_n^{(A)}(x, w) = \frac{2\Gamma(a+b)(I+A)}{n!\Gamma(a)\Gamma(b)} \int_0^{\pi/2} (\sin t)^{2a-1} (\cos t)^{2b-1} {}_4F_3 \left[ \begin{matrix} -nI, \frac{x}{w}, \frac{a+b}{2}, \frac{a+b+1}{2}; \\ (I+A), a, b; \end{matrix} ; w \sin^2 t \cos^2 t \right] dt$$

**Proof:**

By the equation (1.3) of  $J_n^{(A)}(x, w)$ , we have

$$\begin{aligned} J_n^A(x, w) &= \frac{(I+A)_n}{n!} {}_2F_1 \left[ -nI, \frac{x}{w}; I+A; w \right] \\ &= \frac{(I+A)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r}{(I+A)_r r!} \frac{\Gamma(a+b+2r)(a)_r (b)_r}{(a)_r (b)_r \Gamma(a+b+2r)} \\ &= \frac{(I+A)_n}{n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r}{(I+A)_r r!} \frac{2\Gamma(a+b+2r)}{\Gamma(a)\Gamma(b)} \int_0^{\pi/2} (\sin t)^{2a+2r-1} (\cos t)^{2b+2r-1} dt \end{aligned}$$

On interchanging the order of integration and summation, we have

$$\begin{aligned} &= 2 \frac{(I+A)_n \Gamma(a+b)}{n!\Gamma(a)\Gamma(b)} \int_0^{\pi/2} (\sin t)^{2a-1} (\cos t)^{2b-1} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r \left(\frac{a+b}{2}\right)_r \left(\frac{a+b+1}{2}\right)_r}{(I+A)_r r! (a)_r (b)_r} (\sin^2 t)^r (\cos^2 t)^r w^r dt \\ &= \frac{2\Gamma(a+b)(I+A)}{n!\Gamma(a)\Gamma(b)} \int_0^{\pi/2} (\sin t)^{2a-1} (\cos t)^{2b-1} {}_4F_3 \left[ \begin{matrix} -nI, \frac{x}{w}, \frac{a+b}{2}, \frac{a+b+1}{2}; \\ (I+A), a, b; \end{matrix} ; w \sin^2 t \cos^2 t \right] dt \end{aligned}$$

Hence the proof.

**5. Infinite Single Integral Representation**

If ‘A’ be any square matrix of order  $n \times n$  and  $J_n^{(A)}(x, w)$  be the modified Jacobi matrix polynomial, then infinite single integral representation of this polynomial is as follows:

**Theorem** Assume A be any square matrix of order ‘n’  $A \neq 0$ .

$$J_n^{(A)}(x, w) = \frac{(I + A)_n}{\Gamma(a)n!} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} {}_2F_2 \left[ \begin{matrix} -nI, \frac{x}{w}; \\ a, I + A; \end{matrix} ; wt^2 \right] dt \quad (5.1)$$

**Proof:**

From the equation (1.3) of  $J_n^{(A)}(x, w)$ , we have

$$\begin{aligned} J_n^A(x, w) &= \frac{(I + A)_n}{n!} {}_2F_1 \left[ -nI, \frac{x}{w}; I + A; w \right] \\ &= \frac{(I + A)_n}{n! \Gamma(a)} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r \Gamma(a+r)}{(I + A)_r r! (a)_r} \\ &= \frac{(I + A)_n}{n! \Gamma(a)} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r w^r}{(I + A)_r r! (a)_r} \int_{-\infty}^{\infty} e^{-t^2} t^{2a+2r-1} dt \end{aligned}$$

On interchanging the order of integration and summation, we have

$$\begin{aligned} &= \frac{(I + A)_n}{\Gamma(a)n!} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r (wt^2)^r}{(a)_r (I + A)_r r!} dt \\ &= \frac{(I + A)_n}{\Gamma(a)n!} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} {}_2F_2 \left[ \begin{matrix} -nI, \frac{x}{w}; \\ a, I + A; \end{matrix} ; wt^2 \right] dt \end{aligned}$$

Hence the theorem.

## 6. Double integral representation

**Theorem:** Assume ‘A’ be any square matrix of order ‘n’,  $A \neq 0$ . If real part of a, b,  $\lambda > 0$  then

$$J_n^{(A)}(x, w) = \frac{\Gamma(\lambda)(I + A)_n}{n! \Gamma(a) \Gamma(b) \Gamma(\lambda - a - b)} \iint_A u^{a-1} v^{b-1} (1-u-v)^{\lambda-a-b-1} {}_4F_3 \left[ \begin{matrix} -nI, \frac{x}{w}, \frac{\lambda}{2}, \frac{\lambda}{2} + \frac{1}{2}; \\ a, b, I + A; \end{matrix} ; uvw \right] dudv \quad (6.1)$$

**Proof:** From the equation (1.3) of  $J_n^{(A)}(x, w)$  we have

$$\begin{aligned} J_n^{(A)}(x, w) &= \frac{(I + A)_n}{n!} {}_2F_1 \left[ -nI, \frac{x}{w}; I + A; w \right] \\ &= \frac{\Gamma(\lambda)(I + A)_n}{n! \Gamma(a) \Gamma(b) \Gamma(\lambda - a - b)} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{\lambda}{2}\right)_r \left(\frac{\lambda}{2} + \frac{1}{2}\right)_r \left(\frac{x}{w}\right)_r w^r}{(I + A)_r r! (a)_r (b)_r} \iint_A u^{a+r-1} v^{b+r-1} (1-u-v)^{\lambda-a-b-1} du dv \end{aligned}$$

On interchanging the order of integration and summation, we have

$$= \frac{\Gamma(\lambda)(I+A)_n}{n!\Gamma(a)\Gamma(b)\Gamma(\lambda-a-b)} \iint_A u^{a-1} v^{b-1} (1-u-v)^{\lambda-a-b-1} {}_4F_3 \left[ \begin{matrix} -nI, \frac{x}{w}, \frac{\lambda}{2}, \frac{\lambda}{2} + \frac{1}{2} \\ a, b, I+A \end{matrix} ; uvw \right] dudv$$

Hence the theorem.

**Special Cases:**

If ‘ A ’ be any square matrix of order  $n \times n$  and  $J_n^{(A)}(x, w)$  be the modified Jacobi matrix polynomial, then the applications of the above theorems of this polynomial are as follows:

**Theorem:**

$$J_n^{(A)}(x, w) = \int_0^1 \frac{t^{a-1}}{(1-t)} J_n^{(A)}(x, wt) dt$$

**Proof:**

By setting  $a = b$ , in (1.15), we come across at

$$\begin{aligned} J_n^{(A)}(x, w) &= \int_0^1 \frac{t^{a-1} (A+I)_n}{(1-t) n!} \sum_{r=0}^n \frac{(-nI)_r \left(\frac{x}{w}\right)_r (wt)^r}{r! (I+A)_r} dt \\ &= \int_0^1 \frac{t^{a-1}}{(1-t)} J_n^{(A)}(x, wt) dt \end{aligned}$$

**Theorem:**

$$J_n^{(A)}(x, w) = \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{(A)}(xt) dt$$

**Proof:**

By considering  $w \rightarrow 0$  and setting  $a = b$  in (1.15), we arrive at the following:

$$\begin{aligned} J_n^{(A)}(x, w) &= \int_0^1 \frac{t^{a-1} (A+I)_n}{(1-t) n!} \sum_{r=0}^n \frac{(-nI)_r (xt)^r}{n! (I+A)_r} dt \\ &= \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{(A)}(xt) dt \end{aligned}$$

Where  $L_n^{(A)}(x)$  is the Laguerre matrix polynomial of one variable [2].

**Theorem:**

$$J_n^{(A)}(x, w) = \frac{(I+A)_n}{n! \Gamma(a)} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} {}_1F_2 \left[ \begin{matrix} -nI; \\ a, I+A; \end{matrix} \middle| xt^2 \right] dt$$

**Proof:**

By assuming  $w \rightarrow 0$ , we have

$$J_n^{(A)}(x, w) = \frac{(I+A)_n}{n! \Gamma(a)} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} \sum_{r=0}^n \frac{(-nI)_r (xt^2)^r}{(a)_r (I+A)_r r!} dt$$

$$= \frac{(I+A)_n}{n! \Gamma(a)} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} {}_1F_2 \left[ \begin{matrix} -nI; \\ a, I+A; \end{matrix} \middle| xt^2 \right] dt$$

**CONCLUSION**

Recent advancements in matrix polynomial structures associated with special functions have gained significant traction, showcasing a diverse array of applications across various engineering disciplines. This paper primarily intended to explore and derive multiple integral representations for the modified Jacobi Matrix Polynomial. Specifically, we will delve into both finite and infinite single integral representations, as well as double integral representations of the polynomial. By examining these integral forms, we seek to enhance the understanding of their mathematical properties and implications, thereby providing valuable insights that can be leveraged in practical engineering scenarios like applying Laplace Transforms. The exploration of these representations not only enriches the theoretical framework surrounding modified Jacobi Matrix Polynomials but also highlights their potential utility in solving complex problems within the engineering domain [17, 18].

**ACKNOWLEDGMENT**

The authors are very thankful to the reviewers for their comments and suggestions. And they are very thankful to their corresponding organizations for their encouragement

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