

## Pythagorean Neutrosophic Cubic Set

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### Article History:

**Received:** 19-10-2024

**Revised:** 03-12-2024

**Accepted:** 11-12-2024

### Abstract:

The purpose of this work is to broaden the definition of Neutrosophic Cubic Set (NCS) and Pythagorean Cubic Set (PCS) to Pythagorean Neutrosophic Cubic Set (PNCS). Related qualities are examined and the concepts of T-external, I-external, and F-external Pythagorean Neutrosophic Cubic Set (PNCS) and T-internal, I-internal, and F-internal PNCS are conveyed.

**Keywords:** Pythagorean Neutrosophic Cubic Set Pythagorean Neutrosophic Internal Cubic Set, Pythagorean Neutrosophic External Cubic Set.

## 1. Introduction

Zadeh[8] established a foundation for fuzzy mathematics in 1965. Soon after, in 1975, Zadeh[8] proposed a reconfiguration of fuzzy sets with interval valued function membership. Neutrosophic Sets were initially defined in 1995 by Florentin Smarandache[5]. This enables ambiguity and uncertainty to be handled more thoroughly. In the year 2012, the introduction of the important theory of Cubic Sets was defined by Jun et al[2]. In 2017, Chang Su Kim, Florentin Smarandache, and Young Bae Jun[3] presented the an idea of Neutrosophic Cubic Sets. Yagar[7] first presented the Pythagorean Fuzzy set's evolution in 2013. In 2019, F. Khana, M. S. Ali Khana, M. Shahzada, and S. Abdullah[4] gave a concept of the Pythagorean Cubic Fuzzy Set. R. Jhansi & K. Mohana[1] proposed the Neutrosophic Pythagorean Sets. The interval-valued Neutrosophic Pythagorean Sets were presented by Stephy et al[6]. This article tries to establish an innovative concept known as Pythagorean Neutrosophic Cubic Sets (PNCS) which is a combination of PNS and PNIVS.

## 2. Preliminaries

**Definition 2.1[2]** If  $\hat{I} \neq \phi$ . Then a cubic set,  $\hat{\mathbb{D}}_{CS} = \{(\hat{i}, \hat{A}_{FIS}(\hat{i}), \hat{\mu}_{FS}(\hat{i})) : \hat{i} \in \hat{I}\}$  where  $\hat{A}_{FIS}(\hat{i})$  is IVFS,  $\hat{\mu}_{FS}(\hat{i})$  is a fuzzy set.

**Definition 2.2[2]** Let  $\hat{I} \neq \phi$ . A cubic set  $\hat{E}_{CS} = \langle \hat{A}_{FIS}(\hat{i}), \hat{\mu}_{FS}(\hat{i}) \rangle$  in  $\hat{I}$  is called an ICS if  $\hat{A}_{FIS}^-(\hat{i}) \leq \hat{\mu}_{FS}(\hat{i}) \leq \hat{A}_{FIS}^+(\hat{i})$  for all  $\hat{i} \in \hat{I}$ .

**Definition 2.3 [2]** If  $\hat{I} \neq \phi$  set. A cubic set  $\hat{E}_{CS} = \langle \hat{D}_{FIS}(\hat{i}), \hat{\nu}_{FS}(\hat{i}) \rangle$  in X is called an ECS if  $\hat{\mu}_{FS}(\hat{i}) \notin (\hat{D}_{FIS}^-(\hat{i}), \hat{D}_{FIS}^+(\hat{i}))$ .

**Definition 2.4[1]** Let  $\hat{I} \neq \phi$ . A PNS in  $\hat{I}$  is  $\hat{A}_{PNS} = \{(\hat{i}, \mathcal{T}_{\hat{A}}(\hat{i}), \mathcal{J}_{\hat{A}}(\hat{i}), \mathcal{F}_{\hat{A}}(\hat{i})) : \hat{i} \in \hat{I}\}$  where  $\mathcal{T}_{\hat{A}}(\hat{i}), \mathcal{J}_{\hat{A}}(\hat{i}), \mathcal{F}_{\hat{A}}(\hat{i}) : \hat{I} \rightarrow [0,1]$  &  $0 \leq (\mathcal{T}_{\hat{A}}(\hat{i}))^2 + (\mathcal{J}_{\hat{A}}(\hat{i}))^2 + (\mathcal{F}_{\hat{A}}(\hat{i}))^2 \leq 2$  where  $\mathcal{T}_{\hat{A}}(\hat{i})$  denotes the degree of membership,  $\mathcal{J}_{\hat{A}}(\hat{i})$  denotes the degree of indeterminacy and  $\mathcal{F}_{\hat{A}}(\hat{i})$  denotes the degree of non-membership.

**Definition 2.5[6]** Let  $\hat{I} \neq \phi$ , PNIVS is of the form,  $\hat{A}_{PNIVS}\{\langle \hat{i}, \mathcal{J}_{\hat{A}}^-(\hat{i}_{PNIVS}), \mathcal{J}_{\hat{A}}^+(\hat{i}_{PNIVS}), \mathcal{F}_{\hat{A}}(\hat{i}_{PNIVS}) \rangle: \hat{i}_{PNIVS} \in \hat{I}\}$

where  $\mathcal{J}_{\hat{A}}(\hat{i}_{PNIVS}) = [\mathcal{J}_{\hat{A}}^-(\hat{i}_{PNIVS}), \mathcal{J}_{\hat{A}}^+(\hat{i}_{PNIVS})]$  and  $\mathcal{F}_{\hat{A}}(\hat{i}_{PNIVS}) = [\mathcal{F}_{\hat{A}}^-(\hat{i}_{PNIVS}), \mathcal{F}_{\hat{A}}^+(\hat{i}_{PNIVS})]$ . Consider the mapping  $\mathcal{J}_{\hat{A}}(\hat{i}_{PNIVS}), \mathcal{J}_{\hat{A}}^-(\hat{i}_{PNIVS}), \mathcal{J}_{\hat{A}}^+(\hat{i}_{PNIVS}), \mathcal{F}_{\hat{A}}(\hat{i}_{PNIVS}): \hat{I} \rightarrow [0,1]$  and  $0 \leq \left[ \frac{\mathcal{J}_{\hat{A}}^-(\hat{i}_{PNIVS}) + \mathcal{J}_{\hat{A}}^+(\hat{i}_{PNIVS})}{2} \right]^2 + \left[ \frac{\mathcal{J}_{\hat{A}}^-(\hat{i}_{PNIVS}) + \mathcal{J}_{\hat{A}}^+(\hat{i}_{PNIVS})}{2} \right]^2 + \left[ \frac{\mathcal{F}_{\hat{A}}^-(\hat{i}_{PNIVS}) + \mathcal{F}_{\hat{A}}^+(\hat{i}_{PNIVS})}{2} \right]^2 \leq 2$

### 3. Pythagorean Neutrosophic Cubic Set (PNCS)

**Definition 3.1** Let  $\hat{\mathcal{D}} \neq \phi$ , a Pythagorean Neutrosophic Cubic Set (PNCS), having a form,  $\hat{\mathbb{D}}_{PNCS} = \{\langle \hat{i}, \hat{\mathcal{D}}_{PNIVS}(\hat{i}), \varpi_{PNCS}(\hat{i}) \rangle: \hat{i} \in \hat{I}\}$  where  $\hat{\mathcal{D}}_{PNIVS}(\hat{i})$  represent the Pythagorean Neutrosophic Interval valued set in  $\hat{I}$ .  $\varpi_{PNCS}(\hat{i})$  represent the Pythagorean Neutrosophic Set. PNCS can be denoted as a pair  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$ .

**Example 3.2** If  $\hat{\mathcal{D}} = \{h, r, v\}$  then  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  with the tabular representation given below,

$\hat{\mathcal{D}}$	$\hat{\mathcal{D}}_{PNIVS}(\hat{i})$	$\varpi_{PNCS}(\hat{i})$
h	([0.2,0.3], [0.3,0.5], [0.4,0.6])	(0.1,0.2,0.3)
r	([0.1,0.4], [0.3,0.7], [0.4,0.8])	(0.3,0.2,0.7)
v	([0.2,0.3], [0.4,0.8], [0.3,0.5])	(0.5,0.2,0.3)

**Table 1 – Example of PNCS**

Therefore,  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$   $\hat{\mathbb{D}}_{PNCS} = \left\{ \langle h, ([0.2,0.3], [0.3,0.5], [0.4,0.6]), (0.1,0.2,0.3) \rangle, \langle r, ([0.1,0.4], [0.3,0.7], [0.4,0.8]), (0.3,0.2,0.7) \rangle, \langle v, ([0.2,0.3], [0.4,0.8], [0.3,0.5]), (0.5,0.2,0.3) \rangle \right\}$  The pair

$\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  is called PNCS.

**Definition 3.3** Let  $\hat{\mathcal{D}} \neq \phi$ . A PNCS  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  in  $\hat{\mathcal{D}}$  is said to be  $\hat{T}$ -internal,

$$\left( \mathbb{T}_{\hat{\mathcal{D}}}^-(\hat{\delta}_{PNCS}) \leq \varpi_{\mathbb{T}}(\hat{\delta}_{PNCS}) \leq \mathbb{T}_{\hat{\mathcal{D}}}^+(\hat{\delta}_{PNCS}) \right) \forall \hat{\delta}_{PNCS} \in \hat{\mathcal{D}}.$$

$\hat{I}$ -internal if  $\left( \mathbb{I}_{\hat{\mathcal{D}}}^-(\hat{\delta}_{PNCS}) \leq \varpi_{\mathbb{I}}(\hat{\delta}_{PNCS}) \leq \mathbb{I}_{\hat{\mathcal{D}}}^+(\hat{\delta}_{PNCS}) \right) \forall \hat{\delta}_{PNCS} \in \hat{\mathcal{D}}$ .  $\hat{F}$ -internal if  $\left( \mathbb{F}_{\hat{\mathcal{D}}}^-(\hat{\delta}_{PNCS}) \leq \varpi_{\mathbb{F}}(\hat{\delta}_{PNCS}) \leq \mathbb{F}_{\hat{\mathcal{D}}}^+(\hat{\delta}_{PNCS}) \right) \forall \hat{\delta}_{PNCS} \in \hat{\mathcal{D}}$ . If a PNCS  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  in  $\hat{\mathcal{D}}$  adheres to the aforementioned shortcomings then,  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  is an IPNCS.

#### Example 3.4

Let  $\hat{I} = \{h, r, v\}$ . Then the pair  $\hat{\mathbb{D}}_{PNCS} = (\hat{\mathcal{D}}, \varpi)$  with the tabular representation given below,

$\hat{\mathcal{D}}$	$\hat{\mathcal{D}}_{PNIVS}(\hat{\delta}_{PNCS})$	$\varpi_{PNCS}(\hat{\delta}_{PNCS})$
h	([0.1,0.3], [0.4,0.6], [0.5,0.8])	(0.25,0.48,0.62)
r	([0.1,0.4], [0.3,0.7], [0.4,0.8])	(0.30,0.58,0.71)
v	([0.2,0.3], [0.4,0.8], [0.3,0.5])	(0.27,0.65,0.47)

**Table 2 – Example of IPNCS**

Therefore,  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$ ,  $\widehat{\mathbb{D}}_{PNCS} = \left\{ \begin{aligned} &\langle h, ([0.1,0.3], [0.4,0.6], [0.5,0.8]), (0.25,0.48,0.62) \rangle, \\ &\langle r, ([0.1,0.4], [0.3,0.7], [0.4,0.8]), (0.30,0.58,0.71) \rangle, \\ &\langle v, ([0.2,0.3], [0.4,0.8], [0.3,0.5]), (0.27,0.65,0.47) \rangle \end{aligned} \right\}$

The pair  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  satisfies,  $(\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \leq \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ ,  $(\mathbb{I}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \leq \mathbb{I}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ ,  $(\mathbb{F}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \leq \mathbb{F}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ . Then the pair  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is called IPNCS.

**Definition 3.5** Let  $\widehat{\mathcal{D}} \neq \phi$ . A PNCS  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  in  $\widehat{\mathcal{D}}$  is said to be  $\widehat{I}$ -external,  $\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS})) \forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}$ ,  $\widehat{I}$ -external if

$\varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{I}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{I}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS})) \forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}$  &  $\widehat{F}$ -external if

$\varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{F}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{F}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS})) \forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}$ . If a PNCS satisfies the above inequalities then,  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is an EPNCS.

**Example 3.6** Let  $\widehat{I} = \{h, r, v\}$ . Then the pair  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  with the tabular representation given below,

$\widehat{\mathcal{D}}$	$\widehat{\mathcal{D}}_{PNIVS}(\widehat{\delta}_{PNCS})$	$\varpi_{PNCS}(\widehat{\delta}_{PNCS})$
h	$([0.1,0.3], [0.4,0.6], [0.5,0.8])$	$(0.4,0.7,0.3)$
r	$([0.1,0.4], [0.3,0.7], [0.4,0.8])$	$(0.5, 0.2, 0.9)$
v	$([0.2,0.3], [0.6,0.8], [0.3,0.5])$	$(0.1, 0.5, 0.6)$

**Table 2 – Example of EPNCS**

Therefore,  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$   $\widehat{\mathbb{D}}_{PNCS} = \left\{ \begin{aligned} &\langle h, ([0.1,0.3], [0.4,0.6], [0.5,0.8]), (0.4,0.7,0.3) \rangle, \\ &\langle r, ([0.1,0.4], [0.3,0.7], [0.4,0.8]), (0.5, 0.2, 0.9) \rangle, \\ &\langle v, ([0.2,0.3], [0.6,0.8], [0.3,0.5]), (0.1, 0.5, 0.6) \rangle \end{aligned} \right\}$ . Then the

pair  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is called EPNCS.

**Theorem 3.7**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be a PNCS in  $\widehat{\mathcal{D}} \neq \phi$  which is not EPNCS. Then there exists  $\widehat{\delta}_{PNCS} \in \widehat{I}$  such that  $\varpi_{PNCS}(\widehat{\delta}_{PNCS}) \in \widehat{\mathcal{D}}_{PNIVS}(\widehat{\delta}_{PNCS})$ .

**Proof:** Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS. Given that  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is not external. Then  $\widehat{\mathbb{D}}_{PNCS}$  does not satisfies the following conditions, i.e.,  $\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ ,  $\varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{I}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{I}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ ,  $\varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{F}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{F}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS})) \forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}$ . Then  $\widehat{\mathbb{D}}_{PNCS}$  must be IPNCS.  $\therefore \varpi_{PNCS}(\widehat{\delta}_{PNCS}) \in \widehat{\mathcal{D}}_{PNIVS}(\widehat{\delta}_{PNCS})$ .

**Theorem 3.8**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS in  $\widehat{\mathcal{D}} \neq \phi$  which is not T-IPNCS then exists  $\widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}, \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \notin \mathbb{T}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$ .

**Proof**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS. Given that  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is not T-IPNCS. Then  $\widehat{\mathbb{D}}_{PNCS}$  does not satisfies,  $(\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \leq \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ . Therefore,  $\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \notin \mathbb{T}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$

**Theorem 3.10**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS in  $\widehat{\mathcal{D}} \neq \phi$  which is not I-IPNCS then exists  $\widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}, \varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \notin \mathbb{I}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$

**Proof**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS. Given that  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is not I-IPNCS. Then  $\widehat{\mathbb{D}}_{PNCS}$  does not satisfies,  $(\mathbb{I}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \leq \mathbb{I}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ . Therefore,  $\varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \notin \mathbb{I}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$

**Theorem 3.11**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS in  $\widehat{\mathcal{D}} \neq \phi$  which is not F-IPNCS then exists  $\widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}, \varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \notin \mathbb{F}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$ .

**Proof**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS. Given that  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is not F-IPNCS. Then  $\widehat{\mathbb{D}}_{PNCS}$  does not satisfies,  $(\mathbb{F}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \leq \mathbb{F}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ . Therefore,  $\varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \notin \mathbb{F}_{\mathcal{D}}(\widehat{\delta}_{PNCS})$ .

**Theorem 3.12**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS  $\widehat{\mathcal{D}} \neq \phi$ . If  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is both T-IPNCS and T-EPNCS then  $(\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}) (\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \in \{\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\} \cup \{\mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\})$

**Proof**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS. If  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is T-IPNCS then  $\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}, (\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) \leq \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \leq \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$ . If  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is T-EPNC then  $\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}} \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \notin (\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}), \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}))$  Then it indicates  $\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}, \varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) = \mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS})$  or  $\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) = \mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS})$  and so that  $(\varpi_{\mathbb{T}}(\widehat{\delta}_{PNCS}) \in \{\mathbb{T}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\} \cup \{\mathbb{T}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\})$

**Theorem 3.13**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS  $\widehat{\mathcal{D}} \neq \phi$ . If  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is both I-IPNCS and I-EPNCS then  $(\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}) (\varpi_{\mathbb{I}}(\widehat{\delta}_{PNCS}) \in \{\mathbb{I}_{\mathcal{D}}^-(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\} \cup \{\mathbb{I}_{\mathcal{D}}^+(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\})$ .

**Proof:** Forthright.

**Theorem 3.14**

Let  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  be PNCS  $\widehat{\mathcal{D}} \neq \phi$ . If  $\widehat{\mathbb{D}}_{PNCS} = (\widehat{\mathcal{D}}, \varpi)$  is both F-IPNCS and F-EPNCS then  $(\forall \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}) (\varpi_{\mathbb{F}}(\widehat{\delta}_{PNCS}) \in \{\mathbb{F}_{\widehat{\mathcal{D}}}^{-}(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\} \cup \{\mathbb{F}_{\widehat{\mathcal{D}}}^{+}(\widehat{\delta}_{PNCS}) | \widehat{\delta}_{PNCS} \in \widehat{\mathcal{D}}\})$ .

**Proof:** Forthright.

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