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New Kinds of Open Sets in Intuitionistic Interval-Valued Fuzzy Topological Space

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Abstract:
In an intuitionistic interval valued fuzzy topological space (inshort IIVFTS) a new sort of open sets termed intuitionistic interval valued fuzzy minimum open(resp.maximal open) have been researched.

Keywords: Intuitionistic interval valued fuzzy minimal open (resp.minimal open), intuitionistic interval valued fuzzy maximal closed (resp.minimal closed) in short IIVFMIO, IIVFMAO, IIVFMAC, IIVFMIC.

1. Introduction

C.L. Chang[2] used the fuzzy set, which Zadeh[1] had established in 1965, to topology in 1968. Once more, Mondal and Samanta [4] used the interval-valued fuzzy set notion used by Zadeh [3] in 1975 to topology in 1999. Additionally, in 1997, Coker[6] applied the intuitionistic fuzzy sets, which Attanassov[5] had introduced in 1986, to topology.In 2012, Pyung Ki Lim et al. [8] used intuitionistic interval-valued fuzzy sets, which Cheong and Hur [7] had introduced in 2010, to topology. In fuzzy topological space and hesitant fuzzy topological space, A. Swaminathan [9,10] and S. Sivaraja established and expanded on new types of open and closed sets.

2. Preliminaries:

Definition 2.1 A function $\vartheta: X \rightarrow \Lambda(I \oplus I)$ is an IIVFS for any nonempty X,denoted by $\vartheta = [\vartheta^L, \vartheta^U] = [(\alpha_\vartheta^L, \beta_\vartheta^L), (\alpha_\vartheta^U, \beta_\vartheta^U)]$. The IIVFS whole (resp.empty) fuzzy set in X, is given by $\tilde{1} = [(1,0),(1,0)]$ (resp $\tilde{0} = [(0,1),(0,1)]$ for $x \in X$.

Definition 2.2 Let $X \neq \phi$, $\zeta \in \Lambda(I \oplus I)^X$ then ζ is an intuitionistic interval valued fuzzy topology (inshort IIVFT) on X if

(i) $\tilde{0}, \tilde{1} \in \zeta$.

(ii) $\vartheta_1 \cap \vartheta_2 \in \zeta, \forall \{\vartheta_1, \vartheta_2\} \in \zeta$.

(iii) $\cup_{p \in K} \vartheta_p \in \zeta, \{\vartheta_p\}_{p \in K} \subset \zeta$.

The pair (X, ζ) is IIVFTS and every $\vartheta \in \zeta$ is open and ϑ is said to be closed in X if $\vartheta^c \in \zeta$.

3 INTUITIONISTIC INTERVAL VALUED FUZZY MINIMAL OPEN SETS

Definition 3.1 A proper IIVFO set ϑ of IIVFTS (X,ζ) is said to be a IIVFMIO iff any IIVFO which is contained in ϑ is either ϑ or $\tilde{0}$.

Lemma 3.1 In a IIVFTS (X,ζ),

(i) If ϑ_1 is a IIVFMIO and ϑ_2 is a IIVFO set in X then, $\vartheta_1 \cap \vartheta_2 = \tilde{0}$ or $\vartheta_1 \subset \vartheta_2$.

(ii) If ϑ_1, ϑ_2 are IIVFMIO sets then $\vartheta_1 \cap \vartheta_2 = \tilde{0}$ or $\vartheta_1 = \vartheta_2$.

Proof. (i) Suppose that $\vartheta_1 \cap \vartheta_2 \neq \tilde{0}$ for any IIVFO set ϑ_2 then $(\vartheta_1 \cap \vartheta_2) \subset \vartheta_1$, a contradiction to minimality of ϑ_1, then $(\vartheta_1 \cap \vartheta_2) = \vartheta_1$ implying that $\vartheta_1 \subset \vartheta_2$.

(ii) As ϑ_1, ϑ_2 are IIVFMIO sets,if $\vartheta_1 \cap \vartheta_2 \neq \tilde{0}$ then $\vartheta_1 \subset \vartheta_2$ and $\vartheta_2 \subset \vartheta_1$, implying that $\vartheta_1 = \vartheta_2$.

Theorem 3.2 If ϑ and ϑ_p are IIVFMIO sets for any $p \in K$. If $\vartheta \subseteq \cup_{p\in K} \vartheta_p$ then $\exists$ an element $p \in K$ such that $\vartheta = \vartheta_p$.

Proof. If $\vartheta \subseteq \cup_{p\in K} \vartheta_p$ then $\vartheta = \vartheta \cap (\cup_{p\in K} \vartheta_p) = \cup_{p\in K} (\vartheta \cap \vartheta_p)$. Clearly $(\vartheta \cap \vartheta_p) = \tilde{0}$ or $\vartheta = \vartheta_p$ as ϑ, ϑ_p are IIVFMIO sets by Lemma-3.1(ii). If $(\vartheta \cap \vartheta_p) = \tilde{0}$ then $\vartheta = \tilde{0}$, a contradiction to minimality of ϑ.

Theorem 3.3 Suppose ϑ, ϑ_p are IIVFMIO sets for any $p \in K$, $\vartheta \neq \vartheta_p$ for any $p \in K$ then $\vartheta \cap (\cup_{p\in K} \vartheta_p) = \tilde{0}$.

Proof. Suppose that $\vartheta \cap (\cup_{p\in K} \vartheta_p) \neq \tilde{0}$, then $\exists$ an element $p \in K$ with $(\vartheta \cap \vartheta_p) \neq \tilde{0}$. By Lemma-3.1(ii) $\vartheta = \vartheta_p$, a contradiction.

Theorem 3.4 A IIVFMIO set ϑ_p for any $p \in K$; $|K| \geq 2$ and $\vartheta_l \neq \vartheta_p$ for any distinct $l, p \in K$, then for any $l \in K, \vartheta_l \cap (\cup_{p\in K} \vartheta_p) = \tilde{0}$.

Proof. Let $\vartheta_l \cap (\cup_{p\in K/l} \vartheta_p) \neq \tilde{0}$, then $\cup_{p\in K/l} (\vartheta_l \cap \vartheta_p) \neq \tilde{0}$. By Lemma-3.1(ii), $\vartheta_l = \vartheta_p$, a contradiction.

Theorem 3.5 If ϑ_p is a IIVFMIO set for any $k \in K$; $|K| \geq 2$ and $\vartheta_l \neq \vartheta_p$ for any distinct $l, p \in K$. If α is a proper IIVF subset of K,then $(\cup_{k\in K/\alpha} \vartheta_l) \cap (\cup_{m\in\alpha} \vartheta_m) = \tilde{0}$.

Proof. By assuming the contrary, we have $\cup (\vartheta_l \cap \vartheta_m) \neq \tilde{0}$ for $k \in K/\alpha$ and $m \in \alpha$ implying $(\vartheta_l \cap \vartheta_m) \neq \tilde{0}$ for some $p \in K, m \in \alpha$.By Lemma-0.1(ii) we have, $\vartheta_l = \vartheta_m$, a contradiction.

Theorem 3.6 If ϑ_p is a IIVFMIOs for any $p \in K$ with $\vartheta_p \neq \vartheta_l$ for any distinct $l, p \in K$, then $[\cup_{p\in K/l} \vartheta_p] \cap [\cup_{l\in T} \vartheta_l] = \tilde{0}$ for any proper IIVF subset T of K.

Proof. By assuming the contrary, we have $\cup [\vartheta_p \cap \vartheta_l] \neq \tilde{0}, \forall p \in K/l, l \in T$ implying $\cup [\vartheta_p \cap \vartheta_l] \neq \tilde{0}$ for some $p \in K; l \in T$. By Lemma-3.1(ii) a contradiction to minimality of ϑ_p.

Theorem 3.7 If ϑ_p, ϑ_l are IIVFMIO sets for any $p \in K; l \in T$ respectively and if $\exists n \in T$ such that $\vartheta_p \neq \vartheta_n$, for any $p \in K$, then $[\cup_{n\in M} \vartheta_n][\cup_{k\in K} \vartheta_p]$.

Proof. By assuming the contrary, then $\exists n \in T$ with $\vartheta_p \neq \vartheta_n$ for any $p \in K$, then

$[\cup_{n\in M} \vartheta_n] \subset [\cup_{p\in K} \vartheta_p]$.

$\vartheta_n \subset [\cup_{p\in K} \vartheta_p]$, for some $n \in M$.

Hence, $\vartheta_p \neq \vartheta_n$, for any $p \in K$, a contradiction.

Theorem 3.8 If ϑ_p is a IIVFMIO for any $p \in K$ with $\vartheta_p \neq \vartheta_m$ for any distinct $p, m \in K$ then $[\cup_{p\in T} \vartheta_p] \subsetneq [\cup_{m\in K} \vartheta_m]$ for any proper subset T of K.

Proof. For any $p \in K/T$, ϑ_l is a IIVFMIO of family $\{\vartheta_l | l \in K/T\}$ of IIVFMIO sets.

Clearly, $\vartheta_l \cap [\cup_{p\in T} \vartheta_p] = \cup_{p\in T} [\vartheta_l \cap \vartheta_p] = \tilde{0}$. Also, $\vartheta_l \cap [\cup_{m\in K} \vartheta_m] = \cup_{m\in K} [\vartheta_l \cap \vartheta_m] = \vartheta_l$.

In case $[\cup_{l\in T} \vartheta_p] = [\cup_{m\in k} \vartheta_m]$ then, $\vartheta_l = \tilde{0}$, a contradiction as ϑ_l is a IIVFMIO. Hence proved.

Theorem 3.9 If ϑ_p is a IIVFMIO for any $p \in T$ such that $\vartheta_p \neq \vartheta_m$ for any $p, m \in T$, then

(i) $\vartheta_m \subset [\cup_{l\in T/m} \vartheta_l]^c$, for some $m \in K$.

(ii) $\cup_{l\in K/m} \vartheta_m \neq \tilde{1}$, $\forall m \in K$.

Proof. (i) Given that $\vartheta_p \neq \vartheta_m$ for any $p, m \in T$,

$\Rightarrow \cup_{p\in T} [\vartheta_p] \cap \vartheta_m = \tilde{0}$.

$\Rightarrow \cup_{p\in T} [\vartheta_p \cap \vartheta_m] = \tilde{0}$.

$\Rightarrow [\vartheta_p \cap \vartheta_m] = \tilde{0}$.

$\Rightarrow \vartheta_p \subset \vartheta_m^c$.

$\Rightarrow \vartheta_m \subset [\cup_{l\in T/m} \vartheta_l]^c$.

(ii) Suppose that, $\cup_{l\in T/m} \vartheta_l = \tilde{1}$.

$\Rightarrow \vartheta_l = \tilde{0}$, a contradiction for minimality of ϑ_l.

This completes the proof.

Corollary 3.10 If ϑ_p is a IIVFMIO for any $p \in T$ such that $\vartheta_p \neq \vartheta_m$ for any $p, m \in T$, then

$\vartheta_p \cup \vartheta_m \neq \tilde{1}$.

Proof. Similar to the Previous theorem.

Theorem 3.11 If ϑ_p is a IIVFMIO for any $p \in T$ such that $\vartheta_p \neq \vartheta_m$ for any $p, m \in T$, then

$\vartheta_m = [\cup_{p\in T} \vartheta_p] \cap [\cup_{p\in T/m} \vartheta_p]^c$, for any $m \in T$.

Proof. $\Rightarrow [\cup_{p\in T} \vartheta_p] \cap [\cup_{p\in T/m} \vartheta_p]^c = [\cup_{p\in T/m} \vartheta_p \cup \vartheta_m] \cap [\cup_{p\in T/m} \vartheta_p]^c$.

$= (\cup_{p\in T/m} \vartheta_p \cap [\cup_{p\in T/m} \vartheta_p]^c)$.

$=\cup (\vartheta_m \cap [\cup_{p\in T/m} \vartheta_p]^c)$.

$= \tilde{0} \cup \vartheta_m = \vartheta_m$.

4. INTUITIONISTIC INTERVAL VALUED FUZZY MAXIMAL OPEN SETS

Definition 4.1 A proper IIVFO sets ϑ of a IIVFTS (X, ζ) is said to be IIVFMAO set if any IIVFO set containing ϑ is either $\tilde{1}$ or itself.

Example 4.2 Let $X = \{l, m, n\}$ and (X, ζ) is IIVFTS with IIVFs ϑ given by:
$\vartheta(l) = [(0.55,0.45), (0.65,0.25)], \vartheta(m) = [(0.45,0.35), (0.55,0.35)]$,

$\vartheta(n) = [(0.25,0.55), (0.45,0.35)]$ and $\zeta = \{\tilde{0}, \vartheta, \tilde{1}\}$ then ϑ is both IIVFMIO and IIVFMAO in (X, ζ).

Lemma 4.2 Let (X, ζ) be a IIVFTS then,

(i) If ϑ_1 is IIVFMAO and ϑ_2 is a IIVFO set in X, then either $\vartheta_2 \subset \vartheta_1$ or $(\vartheta_1 \cup \vartheta_2) = \tilde{1}$.

(ii) If ϑ_1, ϑ_2 are IIVFMAOs in X, then eithet $\vartheta_1 = \vartheta_2$ or $(\vartheta_1 \cup \vartheta_2) = \tilde{1}$.

Proof. (i) Suppose that $(\vartheta_1 \cup \vartheta_2) \neq \tilde{1}$ for any IIVFMAO set ϑ_1 and IIVFO ϑ_2, if $\vartheta_2\vartheta_1$ then $\vartheta_1 \subset (\vartheta_1 \cup \vartheta_2)$, a contradiction. Hence, $\vartheta_2 \subset \vartheta_1$.

(ii) Suppose that $(\vartheta_1 \cup \vartheta_2) \neq \tilde{1}$ for any IIVFMAOs ϑ_1, ϑ_2, then $\vartheta_1 \subset \vartheta_2$ and $\vartheta_2 \subset \vartheta_1$ then $\vartheta_1 = \vartheta_2$.

Theorem 4.3 If $\vartheta_l, \vartheta_m, \vartheta_p$ are IIVFMAO sets such that $\vartheta_l \neq \vartheta_m$ and $(\vartheta_l \cap \vartheta_m) \subset \vartheta_p$, then either $\vartheta_l = \vartheta_p$ or $\vartheta_m = \vartheta_p$.

Proof. Suppose that $(\vartheta_l \cap \vartheta_m) \subset \vartheta_p$ and $\vartheta_l \neq \vartheta_p$ then, $(\vartheta_m \cap \vartheta_p) = (\vartheta_m) \cap (\vartheta_p \cap \tilde{1})$

$=(\vartheta_m) \cap [(\vartheta_p) \cap (\vartheta_l \cup \vartheta_m)]$
$=(\vartheta_m) \cap [(\vartheta_p) \cap \vartheta_l) \cup (\vartheta_p) \cap \vartheta_m)]$
$=(\vartheta_m \cap \vartheta_p \cap \vartheta_l) \cup (\vartheta_m \cap \vartheta_p \cap \vartheta_m)]$
$=(\vartheta_m \cap \vartheta_l) \cup (\vartheta_m \cap \vartheta_p)$
$=\vartheta_m \cap (\vartheta_l \cup \vartheta_p)$
$=\vartheta_m \cap \tilde{1}$
$=\vartheta_m$.
$(\vartheta_m \cap \vartheta_3) = \vartheta_m \Rightarrow \vartheta_m \subset \vartheta_p$. As ϑ_m is a IIVFMAO set $\vartheta_p \subset \vartheta_m$
This implies $\vartheta_m = \vartheta_p$.
Theorem 4.4 $[\vartheta_1 \cap \vartheta_2][\vartheta_1 \cap \vartheta_3]$ for any distinct IIVFMAO sets $\vartheta_1, \vartheta_2, \vartheta_3$.

Proof. Let us assume the contrary, $[\vartheta_1 \cap \vartheta_2] \subset [\vartheta_1 \cap \vartheta_3]$ for any distinct IIVFMAO sets then,

$[\vartheta_1 \cap \vartheta_2] \cup [\vartheta_2 \cap \vartheta_3] \subset [\vartheta_1 \cap \vartheta_3] \cup [\vartheta_2 \cap \vartheta_3]$.

$\Rightarrow [\vartheta_1 \cup \vartheta_3] \cap \vartheta_2 \subset [\vartheta_1 \cup \vartheta_2] \cap \vartheta_3$

$\Rightarrow \tilde{1} \cap \vartheta_2 \subset \tilde{1} \cap \vartheta_3$

$\Rightarrow \vartheta_2 \subset \vartheta_3$, a contradiction.

Remark 4.5 These proofs are left out because Theorem 4.5, Corollary 4.6, Theorem 4.7, and Theorem 4.8 are comparable to Theorem 3.9, Corollary 3.10, Theorem 3.11, and Theorem 3.8. the 4.6 Theorem.

Theorem 4.5 If ϑ_p is a IIVFMAO for any $p \in K$, a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$ then

(i) $[\bigcap_{K/m} \vartheta_p]^c \subset \vartheta_m$ for any $m \in K$.

(ii) $[\bigcap_{K/m} \vartheta_p] \neq \tilde{0}$ for any $m \in K$.

Corollary 4.6 If ϑ_p is a IIVFMAO for any $p \in K$, a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$ then $[\vartheta_p \cap \vartheta_m] \neq \tilde{0}$.

Theorem 4.7 If ϑ_p is a IIVFMAO for any $p \in K$,a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$, then $\vartheta_m = [\bigcap_{p\in K} \vartheta_p] \cup [\bigcap_{p\in K/m} \vartheta_p]^c$ for any $m \in K$.

Theorem 4.8 If ϑ_p is a IIVFMAO for any $p \in K$,a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$, and if T is a proper nonempty subset of K, then $\bigcap_{p\in K} \vartheta_p \subset \bigcap_{t\in T} \vartheta_t$.

Theorem 4.9 If ϑ_p is a IIVFMAO for any $p \in K$,a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$ and if $\bigcap_{p\in K} \vartheta_p$ is a IIVFC set, then ϑ_m is a IIVFC set for any $j \in K$.

Proof. By Theorem-4.7, we have $\vartheta_m = [\bigcap_{p\in K} \vartheta_p] \cup [\bigcap_{p\in K/m} \vartheta_p]^c$ for any $m \in K$

$\Rightarrow \vartheta_m = [\bigcap_{p\in K} \vartheta_p] \cup [\bigcap_{p\in K/m} (\vartheta_p)^c]$.

As K is finite , $[\bigcap_{p\in K/m} (\vartheta_p)^c]$ is IIVFC. Hence, ϑ_m is IIVFC.

Theorem 4.10 If ϑ_p is a IIVFMAO for any $p \in K$,a finite set and $\vartheta_p \neq \vartheta_m$ for any distinct $m, p \in K$. If $\bigcap_{p\in K} \vartheta_p = \tilde{0}$, then $\{\vartheta_p | p \in K\}$ is a set of all IIVFMAO sets of X.

Proof. Suppose $\exists \vartheta_n$ another IIVFMAO set of X such that $\vartheta_n \neq \vartheta_p \forall p \in K$. Clearly, $\tilde{0} = \bigcap_{p\in K} \vartheta_p = \bigcap_{p\in (K\cup n)/n} \vartheta_p \neq \tilde{0}$, a contradiction. Hence, proved.

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