

## Mathematical Models: Unlocking Solution to Two Biological Frontiers

Bevara Kondala Rao<sup>1,2</sup> Biplab Kumar Rath<sup>1</sup> and A. Manickam<sup>3\*</sup>

<sup>1</sup>Department of Mathematics, GIET University, Gunupur, Rayagada 765022, Odisha, India.

<sup>2</sup>Department of Mathematics, SRI GCSR Degree College, Rajam 532127, Andhra Pradesh, India.

<sup>3</sup> School of Sciences, Division of Mathematics, SRM Institute of Science and Technology, Tiruchirappalli Campus, SRM Nagar, Trichy – Chennai Highway, Near Samayapuram, Tiruchirappalli –621105. Tamilnadu, India.

Email: <sup>1</sup>bevarakondala.rao@giet.edu, <sup>2</sup>biplabkr@giet.edu, <sup>3\*</sup>manickammaths2011@gmail.com

---

### Article History:

**Received:** 20-10-2024

**Revised:** 04-12-2024

**Accepted:** 11-12-2024

### Abstract:

In this paper, we study two different biological problems by using mathematical formulations. In the first problem, our study is based on hyperprolactinemia of non-cycling African elephant which is not associated with hyperestrogenism. There is no distinguishable pattern in the moderated and noticed hypoprolactemic group, and moderate group is in base line. Time-based profile is absorbed in cyclic females, and elevations observed during follicular phase. In mathematical model, Weibull distribution is used. Medical results are analysed with corresponding mathematical model. In the second problem, a four-parameter generalized log-logistic distribution is introduced, using a quadrature rank transmutation map to create a transmuted four-parameter log-logistic distribution. The reliability function for the four distributions is calculated. The standard model applies to the concentration of prolactin in dairy cows, with continuous probability and cumulative distribution functions for the four-parameter log-logistic distributions. Finally, the implementation corresponds to the probability distribution, with results closely tied to medical reports.

**Keywords:** Bromocriptine; Domperidone; Log-logistic distribution; Prolactin Hyperprolactinemia; non– Cycling elephant; Least square distribution; Simple percentile estimators

**Mathematical Subject Classification:** 62H<sub>xx</sub>; 62N0<sub>5</sub>; 90B25.

---

### I) First problem: A mathematical model for hyperprolactinemia in noncycling African elephants

In applied mathematics and statistics, the Weibull distribution may be continued probability distribution. In 2020, Manickam [19] observed that the Weibull distribution is widely used in reliability and life data analysis due to its versatility. It was first identified by Frechet in 1927 [27]. In 1933, to describe a particle size distribution, researchers applied this distribution for the first time. It is named after Swedish mathematician Walodi Weibull, who described it in detail in 1951 [26]. It is a complementary cumulative distribution function which is also a stretched exponential function [18]. In 2021, Gayathri et al. in their study have said that depending on the values of the parameters, the distribution can be used to model a variety of life behaviour [12].

## 1. Mathematical model

The probability density function of a two-parameter Weibull random variable is:

$$f(x; \rho, \tau) = \frac{\tau}{\rho} \left(\frac{x}{\rho}\right)^{\tau-1} \exp\left(-\left(\frac{x}{\rho}\right)^\tau\right),$$

The Weibull distribution has two parameters:  $\tau$  (shape) and  $\rho$  (scale),  $\tau > 0$  and  $\rho > 0$ . It is related to various other distributions, including the exponential and Rayleigh distributions. If  $\tau < 1$ , the failure rate decreases over time; if  $\tau = 1$ , the failure rate remains constant; and if  $\tau > 1$ , the failure rate increases with time [6,11,20].

### 1.1 Estimation procedure:

Estimation procedure is of 4 types: (a) method of moment estimators, (b) linear estimator like least – square type estimators, (c) estimators supported few order statistics and (d) maximum likelihood estimators [23].

#### 1.1(a) Process of Moments

Karl Pearson was the first person to introduce this method. Mathematical support for this procedure comes from the principal of moment [7]. This principle states that two distributions with a finite number of common lower moments will approximate each other.

$k$ th moments is given by:

$$M_k = \int_{-\infty}^{\infty} x^k f(x) dx$$

The  $k$ th moment about the mean for the population is known to be  $k$ th central moment and is given by:

$$CM_k = \int_{-\infty}^{\infty} (x - M_1)^k f(x) dx$$

A transformation of the extreme value distribution is commonly used to generate moment estimators as follows: If  $X \sim \text{Weibull}(\rho, \tau)$ , then  $F(x) = 1 - \exp\left[-\left(\frac{x}{\rho}\right)^\tau\right]$ , then the variable  $T = \ln(x)$  will have a extreme value scattering with distribution function.

$$F(t) = 1 - \exp\left[\exp\left\{\frac{t - \rho}{\tau}\right\}\right]$$

The parameter  $\rho$  and  $\tau$  are parameters that be able to be estimated by the first two moments  $m_1$  and  $cm_2$  of the data on this scale [11,12,23].

#### 1.1(b) Least Square Distribution

To get standardized variable, the observed transformation is  $V_r = \frac{(T_r - \rho)}{\tau}$ .

Let  $V_r$  represent the  $r$ th order statistic from a sample of standardized variables. The expected mean, variance, and covariance are denoted by:

$$E(V_r) = A_r, \text{Var}(V_r) = W_{rr} \text{ and } \text{Cov}(V_i, V_j) = W_{ij}, \text{ where } i \text{ is the row and } j \text{ is the column [8].}$$

Then the least square estimate of the parameter  $B$  are the ordinary weighted least square regression and is given by:

$$B_{est} = [d'W^{-1}d]^{-1}d'W^{-1}T_{(r)}$$

The resulting estimates will be best linear unbiased estimates and thus have very good properties [21]. The Weibull distributions were calculated for only limited sample size for an extreme value distribution [15,18,19,20].

### 1.1(c) Simple Percentile Estimators

Let  $t_i, t_j$  and  $t_k$  be sample percentiles and  $p_i, p_j$  and  $p_k$  be approximate cumulative probability.

Then  $t_s = \delta + \rho[-\ln(1 - p_s)]^{1/\hat{\tau}}$ , where  $s = i, j, k$ .

After simplifying little, 
$$\frac{t_k - t_j}{t_j - t_i} = \frac{[-\ln(1 - p_k)]^{1/\hat{\tau}} - [-\ln(1 - p_j)]^{1/\hat{\tau}}}{[-\ln(1 - p_j)]^{1/\hat{\tau}} - [-\ln(1 - p_i)]^{1/\hat{\tau}}}$$

Further choose  $p_j$  such that  $-\ln(1 - p_j) = \{[-\ln(1 - p_i)][-\ln(1 - p_k)]\}^{1/2}$

$$p_j = 1 - \exp\{-[\ln(1 - p_i) \ln(1 - p_k)]^{1/2}\}$$

Therefore the shape parameter  $\hat{\tau} = \frac{1}{2} \ln \left( \frac{[-\ln(1 - p_k)]}{[-\ln(1 - p_i)]} \right) / \ln \frac{t_k - t_j}{t_j - t_i}$

$$\hat{\rho} = (t_s - t_r) / \left\{ [-\ln(1 - p_k)]^{1/\hat{\tau}} - [-\ln(1 - p_j)]^{1/\hat{\tau}} \right\}$$

Where  $(r < s) \in (i, j, k)$

### 1.1(d) Maximum Likelihood Estimation (MLE)

The MLE is usually used in large sample efficiency. For two – parameter Weibull family, the density function is represented by:

$$f(x; \rho, \tau) = \frac{\tau}{\rho} \left( \frac{x}{\rho} \right)^{\tau-1} \exp \left( - \left( \frac{x}{\rho} \right)^\tau \right)$$

Study a random trial of  $n$  observations. The likelihood purpose of this trial is:

$$L(x_1, \dots, x_n; \rho, \tau) = \prod_{i=1}^n \left( \frac{\tau}{\rho} \right) \left( \frac{x_i}{\rho} \right)^{\tau-1} \exp \left( - \left( \frac{x_i}{\rho} \right)^\tau \right)$$

By taking the logarithm of  $L$  and differentiating with respect to  $\rho$  and  $\tau$ , then setting the derivatives to zero, we obtain two equations.

$$\frac{\partial \ln L}{\partial \tau} = \frac{n}{\tau} + \sum_1^n \ln x_i - \frac{1}{\rho^\tau} \sum_1^n x_i^\tau \ln x_i$$

$$\frac{\partial \ln L}{\partial \rho} = \frac{n}{\rho^\tau} + \sum_I^n \ln x_i - \frac{I}{\rho^{2\tau}} \sum_I^n x_i^\tau$$

Eliminating  $\rho$  from the two equations and simplifying yields:

$$\left[ \frac{\sum_I^n x_i^\tau \ln x_i}{\sum_I^n x_i^\tau} - \frac{I}{\tau} \right] = \frac{I}{n} \sum_I^n \ln x_i$$

This can then be solved iteratively to estimate the shape parameter, denoted by  $\hat{\tau}$ . Then

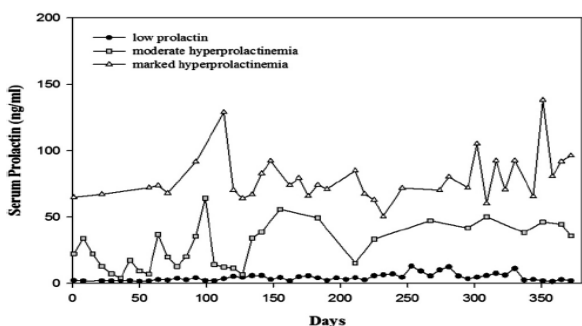
$$\hat{\rho} = \sum_I^n \frac{x_i^{\hat{\tau}}}{n}$$

## 2. Applications

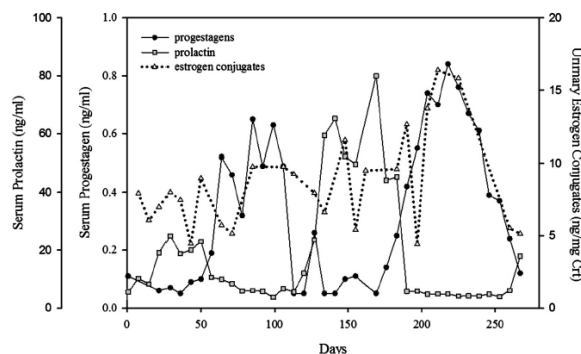
Prolactin is a protein and its main action is to convince caring behaviour and make the breast tissue for lactation [16], it is produced by lactotroph cells of the anterior pituitary [9]. PRL is concealed in a cyclic form in African elephants, with elevation in non – luteal phase [1] [3]. Hypothalamic dopamine controls PRL secretion by inhibitory mechanism via tuberoinfundibular dopaminergic [25]. Lyons and the team in 2012, Mohan Kumar and his team in 1997 and Simpkins and Gabriel in 1984 concluded that any intervention with dopamine’s mixture, release can control prolactin secretion [17]. 10 – 40% of the women’s foremost disorder of the hypothalamic – pituitary axis is hyperprolactinemia [22]. Same suspected problems are faced by zoo – managed elephants [1] [16] [28].

For long term survival, elephants are facing many threats. Zoo elephants are facing challenges associated with their population too [24]. Currently in US zoo, three calves are natural every year. To maintain this situation, six offspring are needed [2]. North American population may face a decline of 2.3% in next 30 years and also female get aged and does not reproduce [24]. Thus, efforts are centred on increased reproductive output, decrease mortality and breeding all elephants which are reproductively viable may improve the populations long – term sustainability [2]. Breeding in zoo are logistical, a main problem because of inadequate housing to maintain large breeding group, expense of transporting female for breeding facilities [10].

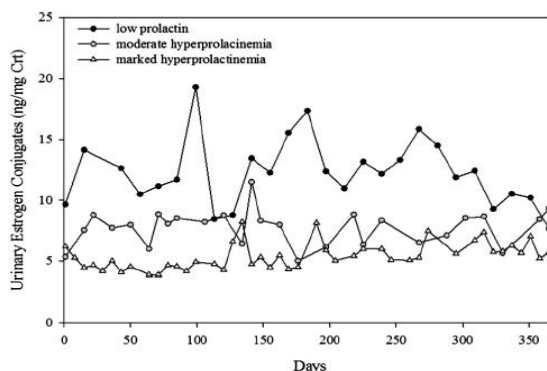
Most of the elephants face physiological problem associated with fertility [4] [13]. On observing the stable baseline concentrations of serum progestogens, most of the elephants are acyclic reveals Elephant Taxon Advisory Group/ Species Survival Plan [14]. In 2004, Brown and his team found that a third of elephants with abnormal cycles produced excessive prolactin, a condition known as hyperprolactinemia, which causes infertility in other species. A 2008 survey revealed that 11% of females had irregular ovarian cycles, while 31% did not cycle at all. Only 72% of the population were of reproductive age (11 to 35 years). In 2012, a study conducted by Dow and Brown [10] concluded that infertility problem has increased significantly. 71% of the American elephants are acyclic with hyperprolactinemia. After monitoring the African elephants for 20 years, the consequence of the study is only non – cycling elephants are identified through hyperprolactinemia [2] [10].



**Figure 1 (a). PRL of three non-cycling African Elephant**



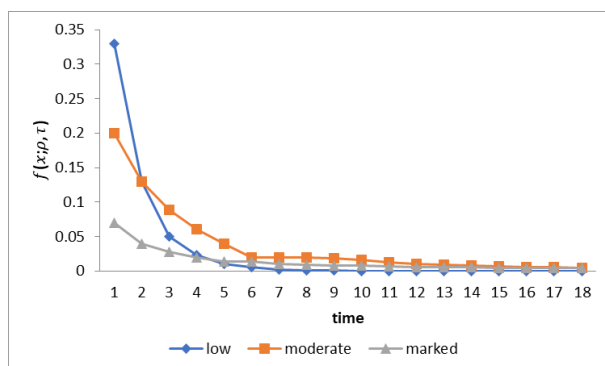
**Figure 1 (b) Progesterone, PRL and Estrogen profile of three cycling African Elephant**



**Figure 1 (c) Estrogen conjugate of three abnormal ovarian cycles**

In figure 1(a), PRL secretion remained varied with no distinct pattern in the reasonable and clear hyperprolactinemia group and concentration in low prolactin relatively stable baseline in non – cycling. In figure 1(b), there is elevation in the concentration were observed in follicular phase and also temporal profile observed in cycling female. In figure 1(c), secretion remained varied by no distinguished shape and infrequently got baseline.

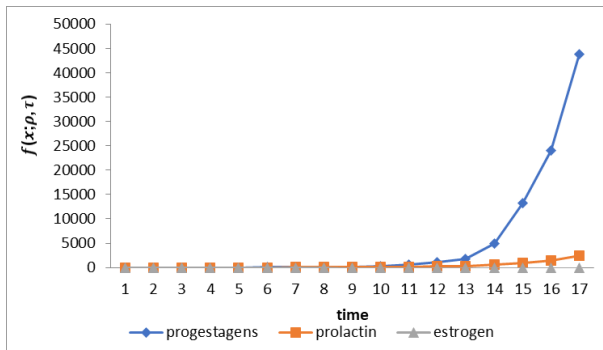
### 3. Mathematical result



**Figure 2(a) probability density function PRL of three non-cycling African Elephant**

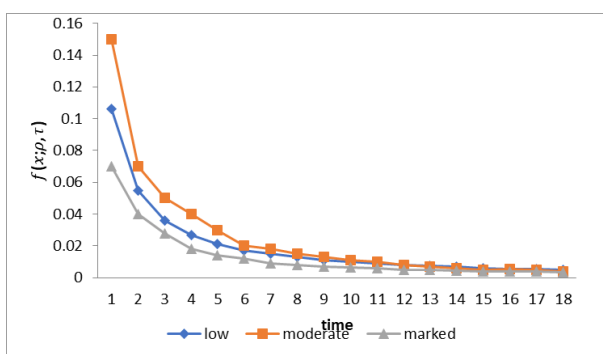
The plot of probability density function of PRL of three non-cycling African Elephant in figure 2(a) indicates low prolactin shows its superiority than the functions and moderate hyperprolactinemia

control. The plot of initially low prolactin monotonically increasing up to  $t=7$  hrs and low prolactin then decreasing monotonically. The rate of decreasing is comparatively good than and control functions.



**Figure 2(b) probability density function Progesterone, PRL and Estrogen profile of three cycling African Elephant**

The plot of probability density function of Progesterone, PRL and Estrogen profile of three cycling African Elephant in figure 2(b) indicates Progesterone, shows its superiority than the functions and prolactin control. The plot of initially Progesterone, monotonically increasing up to  $t=13$  hrs and Progesterone, then decreasing monotonically. The rate of decreasing is comparatively good than and control functions.



**Figure 2(c) probability density function estrogen conjugate of three abnormal ovarian cycles**

The plot of probability density function of estrogen conjugate of three abnormal ovarian cycles in figure 2(c) indicates low prolactin shows its superiority than the functions and moderate hyperprolactinemia control. The plot of initially low prolactin, monotonically increasing up to  $t=9$  hrs and low prolactin, then decreasing monotonically. The rate of decreasing is comparatively good than and control functions.

#### 4. Conclusion

Individual PRL concentrations in hyperprolactinemic elephants were higher than those observed during gestation, although not as high. The examination of a putative temporal effect in non-cycling, hyperprolactinemic elephants revealed no significant correlation between oestrogen conjugate and PRL secretion. Meanwhile, lag time effect models for cycling elephants discovered that oestrogen may influence subsequent PRL secretion, i.e., higher oestrogen conjugate concentration associated with increased PRL. The function's monotonicity has discovered using a mathematical model.

## II) Second problem: Impact of mixing prolactin levels in dairy cows using a generalised Log-Logistic distribution model

The logistics distribution is very useful in the field of analyzing survival. The probability density function (pdf) in its simplest form is given by

$$\phi(z) = \frac{1}{(1+z)^2} \quad z > 0 \quad (1)$$

We studied [30,41,44,46], the logistical model with pdf given by the distribution

$$\phi(z; \gamma, \delta) = \frac{\delta \gamma^\delta z^{\delta-1}}{(\gamma^\delta + z^\delta)^2} \quad z > 0, \gamma > 0, \delta \geq 1 \quad (2)$$

Where  $\gamma$  is the parameter of the scale and  $\delta$  is the shape parameter of the form.

Four parameters widespread logistics distribution is given by probability density function

$$\varphi(z) = \frac{\delta \omega \alpha^\omega z^{\delta-1}}{(\delta + z^\delta)^{\omega+1}} \quad z > 0, \gamma > 0, \delta \geq 1, \omega > 0 \quad (3)$$

Where  $\omega > 0$  is the extra shape parameter. He established its properties and stated that some theorems relate it to some other distributions [30].

### 1. Log Logistic Model and descriptions

The random variable  $Z$  implements four wide-ranging logistical classification parameters with the probability function noted in [42, 44] and the accumulated probability function  $F(z)$  in [44].

$$F(z) = 1 - \left( \frac{\gamma}{\gamma + \gamma^\delta} \right)^\omega \quad \gamma > 0 \quad (4)$$

In the above equation,  $\gamma$  is the scale parameter whereas  $\omega$  and  $\delta$  are the shape parameters. Using the quadratic rand map, an appropriate transmuted four parameters widespread log-logistic distribution is obtained as

$$M(z) = (1 + \rho) M(z) - \rho M^2(z), [\rho] \leq 1 \quad (5) \text{ is given by}$$

$$M(z) = \frac{(\gamma + \gamma^\delta)^\omega z^{\delta-1} \{ (1 - \rho)(\gamma + \gamma^\delta)^\omega + 2\rho\gamma^\omega \}}{(\alpha + \alpha^\beta)^{2\omega+1}}, z > 0 \quad (6)$$

and the corresponding pdf is given by

$$g(z) = \frac{\delta \omega \gamma^\omega z^{\delta-1} \{ (1 - \rho)(\gamma + \gamma^\delta)^\omega + 2\rho\gamma^\omega \}}{(\alpha + \alpha^\beta)^{2\omega+1}}, z > 0 \quad (7)$$

The parameter  $\rho > 0$  is the transmuted exponential distribution [38,39].

The reliability function  $r(t)$  and cumulative distribution function  $F(t)$  represent the probability that a component will not fail before time  $t$ .

$$r(t) = 1 - F(t).$$

The widespread log-logistic distribution's reliability function (four parameters) is given by

$$r(t) = \frac{(1-\rho)\gamma^\omega(\gamma+t^\delta)^\omega + \lambda\alpha^{2\omega}}{(\gamma+t^\delta)^{2\omega}} [32,33]$$

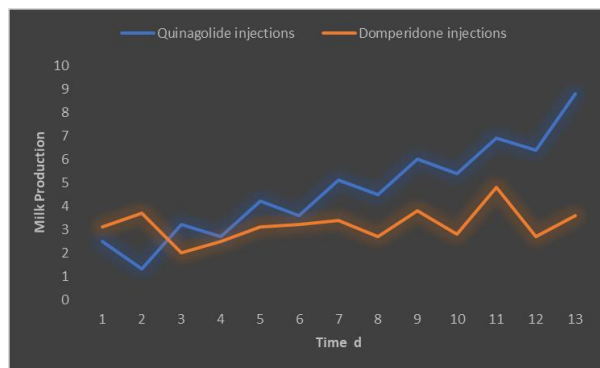
## 2. Applications of widespread log-logistic distribution

In this study, the dataset for the numerical illustration is taken from the reference [47].

Prolactin (PRL) is required for the maintenance of lactation in most warm-blooded creatures, hence restricting PRL inhibits lactation [37]. In many well-developed species, including livestock, rodents, and primates, prolactin intake during lactation prevents or reduces production. But this isn't always the case with large lactating species; if the yield is minimal, it's as if it's a small sum. However, the importance of PRL in managing lactation in ruminants is uncertain, as short-term inhibition of PRL with bromocriptine had inconsistent effects on drain production [38].

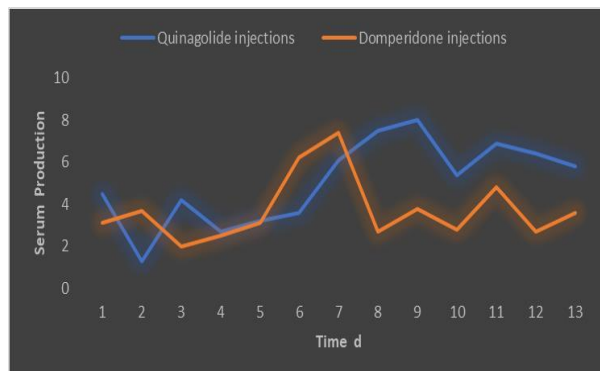
In a subsequent stage, it appeared that daily infusions of dopamine antagonists domperidone for 5 weeks improved PRL discharge. Drain generation increased continuously and was higher in domperidone-treated bovines. Both Quinagolide and cabergoline have been reduced drain generation on the first day of treatment in late-lactation bovines and generated more rapid alterations in a few udder indicators [40]. The role of prolactin in galactose production (PRL) in ruminants has been debated for decades. In dairy bovines, Knight (1993) found that restricting PRL with quinagolide (QN) reduced drain generation [36]. Meanwhile, Lacasse and Ollier discovered in 2015 that infusing the dopamine antagonist domperidone (DOMP) into dairy cows increased drain generation and baseline PRL concentration [38]. As a result, there is currently significant evidence that PRL is a film-forming specialist in dairy cows.

Standard PRL concentrations are altered by the environment throughout the year, regardless of drain generation. In 1973, Koprowski and Trucker concluded in their research study that the affectability of the mammary organ to PRL is versatile [37]. Lacasse et al., 2014 discovered that less circulating PRL has increased the drain generation after the dry period [39,41]. In their inquiry ponder in 2013; Tao and Dahl came to a conflicting result. They discovered that a warm push increased PRL concentrations, and that cooling bovins during the drought period accelerated drain development [48]. McKinnon et al. in 1988 witnessed that the mammary gland's capacity to bind PRL may be well expanded by expanding the recurrence of lactation. To extend the reactivity of the mammary organ to PRL, as it were an instrument to extend the number of PRL receptors [45]. Knight (1993) fed drain to goats and discovered that increasing one-way draining recurrence increased the goats' drain reaction to PRL organization [36].



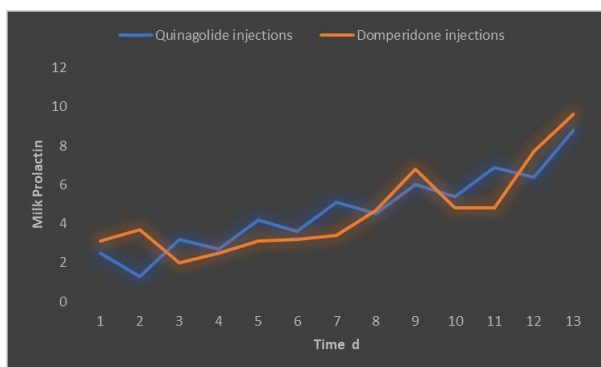
**Figure 3:** CTL and QN cow milk production rates

Before the study began, drain production levels were similar in QN and CTL cows. The figure over clearly appears that amid handling, drain abdicate diminished but was not influenced. Amid the DPKO period, bovines infused with QN created more drain than the control gather. Compared with control bovines, drain generation in dairy animals infused with QN was higher within the post-treatment period. The drugs had no effect on the drain fat substances during the investigation. During the treatment phase, QN diary animals had higher drain protein than CTL bovines, while the other periods were unaffected. There was no evident change in the lactose composition of the drain over the treatment period. Fat, protein, and lactose yields were compared during pre-treatment, treatment, and DPKO stages. Fat yield was higher in QN cows than in CTL cows, with protein and lactose also higher in QN cows post-treatment. Additionally, energy-corrected drain was higher in QN cows during the post-treatment period.



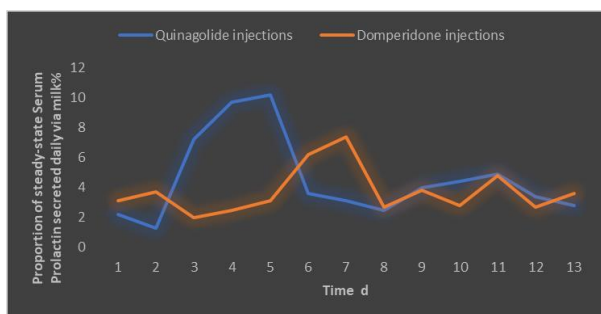
**Figure 4(A):** The serum prolactin CTL and QN cows

In both bunches of bovines, pattern serum PRL concentrations were comparative earlier to the beginning of the dry. Figure 4(A) over clearly appears that PRL levels are moderately exceptionally more in QN bovines compared with CTL diary animals amid the treatment period. Serum PRL levels were higher during the DPKO period. During the post-treatment phase, No significant difference is observed between the CTL and QN groups.



**Figure 4(B):** The prolactin levels in the milk of CTL and QN cows

Figure 4(B) clearly shows that the PRL concentration of milk was unaffected throughout the experiment. However, there was a shift in PRL levels in milk during DPKO and after treatment.

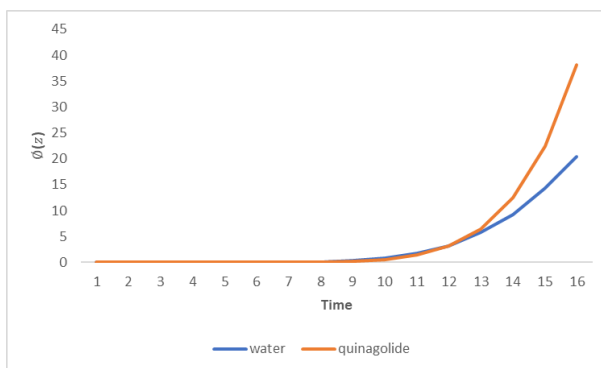


**Figure 4(C):** The Serum PRL levels of QN and CTL cow milk

Figure 4(C) shows no difference in daily steady-state serum PRL proportions between QN and CTL cows during pre- and post-treatment. However, QN cows showed a significant increase during treatment, while CTL cows tended to be higher. During the DPKO period, PRL levels decreased in both groups.

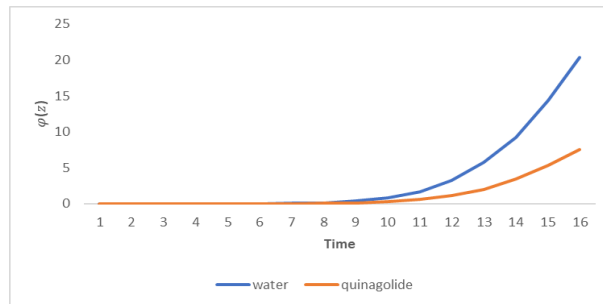
### 3. Mathematical Result

The comparison results are shown in Figure 3(A) as a monotonic function to depict the ratio of milk production over time. The amount of milk production during the initial lactating period is same for both of the groups such as control group (CTL) and the quinagolide injected experimental group (QN). After time period 13, the rate of milk production for the QN group is higher than the CTL group.



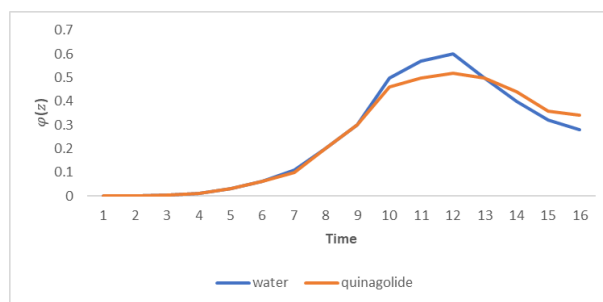
**Figure 5(A):** Probability density functions of production of milk: Control Group (water injected cow) Versus Experimental Group (quinagolide injected cow)

The comparison results are shown in Figure 5(B) as a monotonic function to depict the rate of serum prolactine production over time. The rate of serum prolactine production during the initial lactating period is the same for both of the groups such as the control group (CTL) and the quinagolide injected experimental group (QN). After time period 9, the rate of serum prolactine production for the CTL group is higher than the QN group.



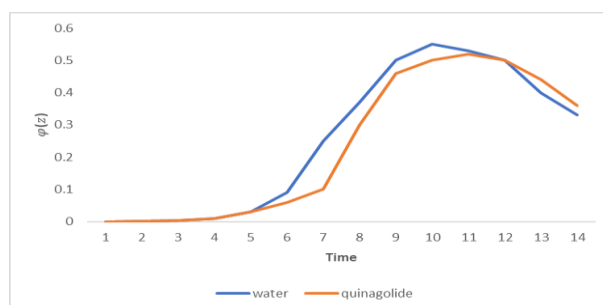
**Figure 5(B):** Probability density functions of Serum Prolactin Secretion: Control Group (water injected cow) Versus Experimental Group (quinagolide injected cow)

The comparison results are shown in Figure 5(C) as an exponential function to depict the rate of milk prolactine production over time. The rate of serum prolactine production during the initial lactating period in terms of function growth for both of the groups such as the control group (CTL) and the quinagolide injected experimental group (QN). After time period 13, the rate of milk prolactine production becomes a sudden drop in the CTL group than the QN group.



**Figure 5(C):** Probability density functions of Milk Prolactin Secretion: Control Group (water injected cow) Versus Experimental Group (quinagolide injected cow)

The comparison results are shown in Figure 5(D) as an exponential function to depict the steady-state serum PRL secreted milk proportions on a daily basis. No significant difference is observed between the CTL and QN groups.



**Figure 5(D):** Probability density functions of daily steady-state serum PRL secretion in milk: Control (water-injected) vs. Experimental (quinagolide-injected) cows.

#### 4. Discussion and Resolution

The mammary gland is a complex tissue with roles in development, secretion, and growth. The organ is controlled by a complex system of hormones and endocrine components, as well as surrounding auto endocrine movement. Prolactin may be the major component of these complexes, in a few species the foremost critical single component. The study findings support the theory that PRL levels influence the mammary gland's response to the hormone. The numerical demonstration using a generalized log-logistic distribution model, the mammary gland response for the quinagolide injected experimental group is gotten the PRL levels of theoretical study. Drain prolactin and the steady-state serum PRL proportion emitted day by day from cow's milk were also constant and mitotically reduced. This article will soon prove highly valuable in medicine and prolactin capacity development.

**Acknowledgement:** NA

**Funding information:** This research received no specific grant from any funding agency, commercial or non-profit sectors.

**Conflict of interest:** The authors have no conflicts of interest to disclose.

**Ethical approval:** This research did not require ethical approval.

#### References:

- [1] Bechert, U., S., Swanson, L., Wasser, S., K., Hess, D., L., Stormshak, F, (1999). "Serum Prolactin Concentrations in the Captive Female African Elephant: Potential effects of season and Steroid Hormone Interaction". *General Comp. Endocrinology*. 114, 269 – 278.
- [2] Brown, J.L., Lehnhardt, J. (1995). "Serum and Urinary hormones during Pregnancy and the peri and post partum period I an Asian Elephant". *Zoo Biology*. 14, 555 – 564.
- [3] Brown, J.L., Lehnhardt, J. (1995). "Secretory Patterns of Serum PRL in Asian and African Elephant during different Reproductive States: Comparison with concentrations in a non – cycling African Elephant". *Zoo Biology*. 16, 149 – 159.
- [4] Brown, J.L., Walker, S.L., Moeller, T. (2004, a). "Comparative Endocrinology of Cycling and Non – cycling Asian and African Elephants". *General Comp. Endocrinology*. 136, 360 – 370.
- [5] Brown, J.L. Olson, D., Keele, M., Freeman, E., W. (2004, b). "Survey of the Reproductive Cyclicity status of Asian and African Elephant in North America". *Zoo Biology*. 23, 309-321.
- [6] Cameron, A., C., Trivedi, P., K. (2005). "Microeconometrics: Methods & Application". P:584
- [7] Chandrasekar, S. (1943). "Stochastic Problems in Physics and Astronomy". *Reviews of Modern Physics*. 15(1): 86.
- [8] Collett, David. (2015). "Modelling Survival Data in Medical Research". Boca Raton: Chapman and Hall.
- [9] Crosignani, P.G. (2012). "Management of Hyperprolactinemic Infertility". *M. E. Fertility Soc Journal*. 17, 63 – 69.
- [10] Dow, T.L., Brown, J., L. (2012). "Evidence that Hyperprolactinemia is Associated with Ovarian Acyclicity in Female Zoo African Elephants" *Reproductive Fertility Developmen*. 24, 1019 – 1027.
- [11] Gayathri, M., Manickam, M., Malar, M., Kumar, K., S. (2021). "A common logistic system for the receptor of mineralocorticoids mediated inhibition of the hypothalamic pole of the adrenal pituitary in older people". *European Journal of Molecular & Clinical Medicine*. 8(2): 1375-1380
- [12] Gayathri, M., Malar, C.M. (2021). "A Mathematical Model for the effect of Prolactin on Dairy Cow's". *Stochastic Modeling and Applications*. 125(1): 211 – 220.
- [13] Hermes, R., Olson, D., Goritz., Brown, J.L. Schmith, D.L, Hagan, D., et al. (2000). "Ultrasonography of the estrous Cycle of Female African Elephants". *Zoo Biology*. 19, 369 – 382.
- [14] Hermes, R., Hildebrandt, T.B., Goritz, F. (2004). "Reproductive problems directly attributable to Long – term Captivity: Asymmetric Reproductive Aging". *Animal Reproduction Science*. 82 – 83, 49 – 60.

- [15] Jiang, R., Murthy, D., N., P. (2011). “A Study of Weibull Shape Parameter: Properties and Significance”. *Reliability Engineering and System Safety*. 96(12): 1619 – 1626.
- [16] Leong, D.J., Frawley, S. Neill, J.D. (1983). “Neuroendocrine Control of PRL secretion”. *Ann.Rev. Physiology*. 45, 109 – 127.
- [17] Lyons, D.J., Hellysaz, A., Brogger, C. (2012). “PRL Regulates tuberoinfundibular Dopamine Neuron Discharge Pattern: Novel Feedback Control Mechanisms in the Lactotrophic axis”. *Journal on Neuroscience*. 32, 8074 – 8083.
- [18] Manickam, A. “A Mathematical model for stimulated hypothalamic pituitary adrenal reactivity dynamic changes in healthy patients” *Advances in Mathematics: Scientific Journal* 10 (Jan 2021), 629-636 ISSN: 1857-8365(p); 1857-8438(e) Volume.10, issue.1, page: 637-644
- [19] Manickam, A. (2020) “A Study On Weibull-G Exponential Distribution Model For Secretion Of GnRH In Beef Cows” *Advances in Mathematics*. 8 (9): 7477–7482.
- [20] Manickam, A. (2021). “A Mathematical Model For Pulsatile Gonadotropin-Releasing Hormone Release From Hypothalamic Explants Of Male Marmoset Monkeys Compared With Male Rats”. *Advances in Mathematics: Scientific Journal*. 10 (1)1: 629-636.
- [21] Muraleedharan, G., Rao, A., D., Kurup, P., G., Nair, N., Unnikrishna, Sinha, Mourani. (2007). “Modified Weibull Distribution for Maximum and Significant Wave Height Simulation and Prediction”. *Costal Engineering*. 54(8): 630 – 638.
- [22] Neill, J., D., Freeman, M.E., Tilson, S.A. (1971). “Control of the Prestrous surge of Prolactin and Luteinizing Hormone Secretion by Estrogens in the Rat”. *Endocrinology*. 89, 1448 – 1453.
- [23] Papoulis, A., Pillai, S., Unnikrishna. (2002). “Probability Random Variables & Stochastic Processes. Boston: McGraw – Hill.
- [24] Patisal, h.,B., Adewale, H.,B. (2009). “Long – term effects of Environmental endocrine Disruptors on Reproductive Physiology and Behavior”. *Front. Behav. Neuroscience*. 3,10.
- [25] Smith, M.S. McLean, B. K., Neill, J.D. (1976). “PRL: the initial luteotropic stimulus of the Rat”. *Endocrinology*. 98, 1370 – 1377.
- [26] Smith, R. L. (1985). “Maximum Likelihood Estimation with the Weibull Distribution”. *Biometrika*. 72: 67 – 90.
- [27] Weibull, W, (1939). “A Statistical Theory of the Strength of Materials”. *Ing. Vetenskaps Akad. Handl*. 153:17ff
- [28] Zanakis, S., H., Kyparisis, J. (1986). “A Review of Maximum Likelihood Estimation Methods for three – parameter Weibull Distribution”. *Journal of Statistical Computation and Simulation*. 25: 53 – 73.
- [29] Auchtung, T. L., Rius, A. G., Kendall, P. E., McFadden, T. B., and Dahl, G.E. (2005). “Effects of photoperiod during the dry period on prolactin, prolactin receptor, and milk production of dairy cows”. *Journal of Dairy Science*. 88: 121 – 127.
- [30] Balakrishnan, N., Malik, H.J., and Puthenpura, S. (1987). Best linear unbiased estimation of location and scale parameters of the log-logistic distribution. *Communications in Statistics –Theory and methods*. Vol 12. Pp 3477-3495.
- [31] Bernier – Dodier, P., Delbecchi, L., Wagner, G. F., Talbot, B. G., and Lacasse, P. (2010). “Effect of milking frequency on lactation persistency and mammary gland remodelling in mid–lactation cows”. *Journal of Dairy Science*. 93: 77 – 104.
- [32] Gayathri, M., Manickam, M., Malar, M., Kumar, K., S. (2021). “A common logistic system for the receptor of mineralocorticoids mediated inhibition of the hypothalamic pole of the adrenal pituitary in older people”. *European Journal of Molecular & Clinical Medicine*. 8(2): 1375-1380
- [33] Kalaiselvi, R., Manickam A., Agrawal, M. (2021). “A mathematical model for the pulsatile gonadotropin- Releasing Hormone Release from Hypothalamic Explants of Male Marmoset Monkeys compared with Male Rats”. *Advances in Mathematics: Scientific Journal*. 10(1): 629-636
- [34] Lacasse, P., and Ollier, S. (2015). “The dopamine antagonist domperidone increases prolactin concentration and enhances milk production in dairy cows”. *Journal of Dairy Science*. 98: 7856 – 7864.
- [35] Lacasse, P., C. Vinet, M., and Petitclerc, D. (2014). “Effect of prepartum photoperiod and melatonin feeding on milk production and prolactin concentration in dairy heifers and cows. *Journal of Dairy Science*. 97: 3589 – 3598.
- [36] Lacasse, P., Lollivier, V., Bruckmaier, R. M., Boisclair, Y. R., Wagner, G. F., and Boutinaud, M. (2011). “Effect of the Prolactin release inhibitor quinagolide on lactating dairy cows”. *Journal of Dairy Science*. 94: 1302 – 1309.

- [37] Lollivier, V. P., Lacasse, P., Angulo Arizala, J., Lamberton, P., Wiart, S., Portanguen, J., Bruckmaier, R., and Boutinau, M. (2015). “In vivo inhibition followed by exogenous supplementation demonstrates galactopoietic effects of prolactin on mammary tissue and milk production in dairy cows”. *Journal of Dairy Science*. 98: 8775 – 8787.
- [38] Manickam, A. (2021). “A Mathematical Model For Pulsatile Gonadotropin-Releasing Hormone Release From Hypothalamic Explants Of Male Marmoset Monkeys Compared With Male Rats”. *Advances in Mathematics: Scientific Journal* 10 (1): 629-636
- [39] Manickam, A. (2021). “A Mathematical Model For Stimulated Hypothalamicpituitary Adrenal Reactivity Dynamic Changes In Healthy Patients”. *Advances in Mathematics: Scientific Journal* 10 (1): 637-644.
- [40] McKinnon, J. C., Knight, H., Flint, D. J. and Wilde, C. J. (1988). “Effect of milking frequency and efficiency on goat mammary prolactin receptor number”. *Journal of Endocrinology* 119(suppl.): 116.
- [41] Ragab, A. and Green, J. (1984). On order statistics from the Log-logistic distribution and their properties. *Communications in Statistics –Theory and methods*. Vol 13. Pp 2713-2724.
- [42] Rose, L.A., Manickam, A., Agrawal, M. (2021). “A Mathematical model for stimulated hypothalamic-pituitary-adrenal reactivity dynamic changes in healthy patients”. *Advances in Mathematics: Scientific Journal*. 10(1): 637-644
- [43] Sellers, K. S. and Shmueli, G. (2010). “A flexible Regression Model for Count Data”. *Annals of Applied Statistics*, 4(2), 943 – 961.
- [44] Shah, B.K and Dave, P.H. (1963), A note on log-logistic distribution. *Journal of Mathematical Sciences of the University of Baroda*. Vol 12. Pp 21-22.
- [45] Tao, S., and Dahl, G. E. (2013). Invited review: heat stress effects during late gestation on dry cows. *Journal of science*. 96: 4079 – 4093.
- [46] Tadikamalla, P.R and Johnson, N.L (1982). Systems of frequency curves generated by the transformation of logistic variables. *Biometrika* Vol 69 Pp 461-465.
- [47] Thompson, I. M., Ollier, S., Zhao, X., and Lacasse, P. (2015). “Effects of milking frequency and domperidone on milk production and expression of prolactin receptors in the mammary gland of dairy cows”. *Journal of Dairy Science*. 98(Suppl. 1): 407.
- [48] Wu, G., Holan, S. H., and Wilkie, C. K. (2013). “Hierarchical Bayesian Spatio-temporal Conway-Maxwell Poisson models with dynamic dispersion”. *Journal of Agricultural and Biological Environment*. 18: 335 – 356.