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# Contra $M$-continuous Maps in Pythagorean Fuzzy Topological Spaces

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**Abstract:**

In this paper, we introduce and investigate Pythagorean fuzzy contra $M$-continuous maps in Pythagorean fuzzy topological spaces and also discuss about some properties and characterization of Pythagorean fuzzy contra $M$-irresolute maps.

**Keywords**: Pythagorean fuzzy $M$-closed sets, Pythagorean fuzzy contra $M$-continuous maps and Pythagorean fuzzy contra $M$-irresolute maps.

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## 1. Introduction

Considering the imprecision in decision-making, Zadeh [36] introduced the idea of fuzzy set which has a membership function, $\mu$ that assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. The notion of fuzzy sets generalizes classical sets theory by allowing intermediate situations between the whole and nothing. In a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class. For fuzzy sets, the membership function replaced the characteristic function in crisp sets. The concept of fuzzy set theory seems to be inconclusive because of the exclusion of nonmembership function and the disregard for the possibility of hesitation margin.

Atanassov critically studied these shortcomings and proposed a concept called intuitionistic fuzzy sets ($IFS$s) [1, 2, 4, 5]. The construct (that is, $IFS$'s) incorporates both membership function, $\mu$ and nonmembership function, $\nu$ with hesitation margin, $\pi$ (that is, neither membership nor non-membership functions), such that $\mu+\nu\leq 1$ and $\mu+\nu+\pi=1$. Atanassov [3] introduced intuitionistic fuzzy sets of second type ($IFSST$) with the property that the sum of the square of the membership and non-membership degrees is less than or equal to one. This concept generalizes $IFS$'s in a way. The notion of $IFS$'s provides a flexible framework to elaborate uncertainty and vagueness. The idea of $IFS$ seems to be resourceful in modelling many real-life situations like medical diagnosis [7, 8, 12, 28, 29], career determination [10], selection process [11], and multi-criteria decision-making [15, 16, 17], among others.

There are situations where $\mu + \nu \geq 1$ unlike the cases capture in $IFS$'s. This limitation in $IFS$ naturally led to a construct, called Pythagorean fuzzy sets ($pfs$'s). Pythagorean fuzzy set ($pfs$) proposed in [33, 34, 35] is a new tool to deal with vagueness considering the membership grade, $\mu$ and non-membership grade, $\nu$ satisfying the conditions $\mu + \nu \leq 1$ or $\mu + \nu \geq 1$, and also, it follows that $\mu^2 + \nu^2 + \pi^2 = 1$, where $\pi$ is the Pythagorean fuzzy set index. In fact, the origin of Pythagorean fuzzy sets emanated from $IFSST$ earlier studied in the literature. As a generalized set, $PFS$ has close relationship with $IFS$. The construct of $PFS$'s can be used to characterize uncertain information more sufficiently and accurately than $IFS$. Garg [14] presented an improved score function for the ranking order of interval-valued Pythagorean fuzzy sets ($IVPFS$s). Based on it, a Pythagorean fuzzy technique for order of preference by similarity to ideal solution ($TOPSIS$) method by taking the preferences of the experts in the form of interval-valued Pythagorean fuzzy decision matrices was discussed. Other explorations of the theory of $PFS$'s can be found in [6, 9, 13, 18, 19, 25, 26]. Saha [27] defined $\delta$-open sets in topological spaces. Vadivel et al. [31] introduced $\delta$-open sets in a neutrosophic topological space. The notion of M-open sets in topological spaces were introduced by El-Maghrabi and Al-Juhani [23] in 2011 and studied some of their properties. The class of sets namely, $M$-open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. Recently, Jeeva et al. [20, 21, 22] introduced neutrosophic soft $M$-open sets in neutrosophic topological spaces and developed the concepts of neutrosophic soft $M$-Continuity and $M$-Irresolute maps.

## 2. Preliminaries

We recall some basic notions of fuzzy sets, $IFS$'s and $pfs$'s .

**Definition 2.1** *[36] Let $X$ be a nonempty set. A fuzzy set $A$ in $X$ is characterized by a membership function $\mu_A: X \to [0,1]$. That is:*

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ ispartlyin } X. \end{cases}$$

Alternatively, a fuzzy set $A$ in $X$ is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X\right\}$, where the function $\mu_A(x): X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more $x$ belongs to $A$, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval [0,1].

Let us consider two examples:

(i) all employees of $XYZ$ who are over $1.8m$ in height; (ii) all employees of $XYZ$ who are tall. The first example is a classical set with a universe (all $XYZ$ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, $\mu$. If we return to our second example and let $A$ represent the fuzzy set of all tall employees and $x$ represent a member of the universe $X$ (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if $x$ is definitely tall or $\mu_A(x) = 0$ if $x$ is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

**Definition 2.2** *[1, 2, 4, 5] Let a nonempty set X be fixed. An* $IFS\ A$ *in* $X$ *is an object having the form:* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ *or* $A = \left\{\left\langle \frac{\mu_A(x),\nu_A(x)}{x} \right\rangle | x \in X\right\}$*, where the functions* $\mu_A(x): X \to [0,1]$ *and* $\nu_A(x): X \to [0,1]$ *define the degree of membership and the degree of nonmembership, respectively, of the element* $x \in X$ *to* $A$*, which is a subset of* $X$*, and for every* $x \in X$: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$*. For each* $A$ *in* $X$: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ *is the intuitionistic fuzzy set index or hesitation margin of* $x$ *in* $X$*. The hesitation margin* $\pi_A(x)$ *is the degree of nondeterminacy of* $x \in X$ *to the set* $A$ *and* $\pi_A(x) \in [0,1]$*. The hesitation margin is the function that expresses lack of knowledge of whether* $x \in X$ *or* $x \notin X$*. Thus:* $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$*.*

**Example 2.1** *Let* $X = \{x, y, z\}$ *be a fixed universe of discourse and* $A = \left\{\left\langle \frac{0.6,0.1}{x} \right\rangle, \left\langle \frac{0.8,0.1}{y} \right\rangle, \left\langle \frac{0.5,0.3}{z} \right\rangle\right\}$*, be the intuitionistic fuzzy set in* $X$*. The hesitation margins of the elements* $x, y, z$ *to* $A$ *are as follows:* $\pi_A(x) = 0.3, \pi_A(y) = 0.1$ *and* $\pi_A(z) = 0.2$*.*

**Definition 2.3** *[33, 34, 35] Let* $X$ *be a universal set. Then, a Pythagorean fuzzy set* $A$*, which is a set of ordered pairs over* $X$*, is defined by the following:* $A = \{< x, \mu_A(x), \nu_A(x) | x \in X\}$ *or* $A = \left\{\left\langle \frac{\mu_A(x),\nu_A(x)}{x} \right\rangle | x \in X\right\}$*, where the functions* $\mu_A(x): X \to [0,1]$ *and* $\nu_A(x): X \to [0,1]$ *define the degree of membership and the degree of nonmembership, respectively, of the element* $x \in X$ *to* $A$*, which is a subset of* $X$*, and for every* $x \in X$, $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$*. Supposing* $(\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$*, then there is a degree of indeterminacy of* $x \in X$ *to* $A$ *defined by* $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\nu_A(x))^2]}$ *and* $\pi_A(x) \in [0,1]$*. In what follows,* $(\mu_A(x))^2 + (\nu_A(x))^2 + (\pi_A(x))^2 = 1$*. Otherwise,* $\pi_A(x) = 0$ *whenever* $(\mu_A(x))^2 + (\nu_A(x))^2 = 1$*. We denote the set of all* $PFS$*'s over* $X$ *by* $pfs(X)$*.*

**Definition 2.4** *[35] Let A and B be* $pfs$*'s of the forms* $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ *and* $B = \{< a, \lambda_B(a), \mu_B(a) > | a \in X\}$*. Then*

1. $A \subseteq B$ if and only if $\lambda_A(a) \leq \lambda_B(a)$ and $\mu_A(a) \geq \mu_B(a)$ for all $a \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $\bar{A} = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$.
4. $A \cap B = \{< a, \lambda_A(a) \wedge \lambda_B(a), \mu_A(a) \vee \mu_B(a) > | a \in X\}$.
5. $A \cup B = \{< a, \lambda_A(a) \vee \lambda_B(a), \mu_A(a) \wedge \mu_B(a) > | a \in X\}$.
6. $0_X = \{< a, 0,1 > | a \in X\}$ and $1_X = \{< a, 1,0 > | a \in X\}$.
7. $\bar{1} = 0$ and $\bar{0} = 1$.

**Definition 2.5** *[24] An Pythagorean fuzzy topology by subsets of a non-empty set X is a family* $\tau$ *of* $pfs$*'s satisfying the following axioms.*

1. $\phi, X \in \tau$.
2. $G_1 \cap G_2 \in \tau$ for every $G_1, G_2 \in \tau$ and

3. $\bigcup \quad G_i \in \tau$ for any arbitrary family $\{G_i | i \in j\} \subseteq \tau$. The pair $(X, \tau)$ is called an Pythagorean fuzzy topological space ($pfts$ in short) and any $pfs\ G$ in $\tau$ is called an Pythagorean fuzzy open set ($pfos$ in short) in $X$. The complement $\bar{A}$ of an Pythagorean fuzzy open set $A$ in an $pfts(X, \tau)$ is called an Pythagorean fuzzy closed set ($pfcs$ in short).

**Definition 2.6** *[24] Let $(X, \tau)$ be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X\}$ be an $pfs$ in $X$. Then the interior and the closure of $A$ are denoted by $pfint(A)$ and $pfcl(A)$ and are defined as follows: $pfcl(A) = \cap \{K | K\ isan\ pfcs\ and\ A \subseteq K\}$ and $pfint(A) = \cup \{G | G\ isan\ pfos\ and\ G \subseteq A\}$. Also, it can be established that $pfcl(A)$ is an $pfcs$ and $pfint(A)$ is an $pfos$, $A$ is an $pfcs$ if and only if $pfcl(A) = A$ and $A$ is an $pfos$ if and only if $pfint(A) = A$. We say that $A$ is $pf$-dense if $pfcl(A) = X$.*

**Lemma 2.1** *[30] For any Pythagorean fuzzy set $A$ in $(X, \tau)$, we have $X - pfint(A) = pfcl(X - A)$ and $X - pfcl(A) = pfint(X - A)$.*

**Definition 2.7** *[30] Let $(X, \tau)$ be an $pfts$ and $A$ be an $pfs$. Then $A$ is said to be an Pythagorean fuzzy (i) regular open set ($pfros$ in short) if $A = pfint(pfcl(A))$. (ii) regular closed set ($pfrcs$ in short) if $A = pfcl(pfint(A))$. By Lemma 2.1, it follows that $A$ is an $pfros$ iff $\bar{A}$ is an $pfrcs$.*

**Definition 2.8** *[32] Let $(X_I, \Gamma_P)$ (or $X_I$) be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X_I\}$ be an $pfs$ in $X_I$. Then the (i) $pf\delta$-interior of $A$ are denoted by $pf\delta int(A)$ and are defined as follows. $pf\delta int(A) = \cup \{G | G$ is an $pfros$ and $G \subseteq A\}$. (ii) $pf\delta$-closure of $A$ are denoted by $pf\delta cl(A)$ and are defined as follows. $pf\delta cl(A) = \cap \{K | K$ is an $pfrcs$ and $A \subseteq K\}$.*

**Definition 2.9** *[32] Let $(X_I, \Gamma_P)$ be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X_I\}$ be an $pfs$ in $X_I$. A set $A$ is said to be $pf$ [(i)]*

1. $\delta$-open set (briefly, $pf\delta os$) if $A = pf\delta int(A)$,
2. $\delta$-pre open set (briefly, $pf\delta\mathcal{P}os$) if $A \subseteq pfint(pf\delta cl(A))$,
3. $\delta$-semi open set (briefly, $pf\delta\mathcal{S}os$) if $A \subseteq pfcl(pf\delta int(A))$,
4. $e$ open set (briefly, $pfeos$ ) if $A \subseteq pfcl(pf\delta int(A)) \cup pfint(pf\delta cl(A))$,
5. $\delta$ (resp. $\delta$-pre, $\delta$-semi and $e$) dense if $pf\delta cl(A)$ (resp. $pf\delta\mathcal{P}cl(A), pf\delta\mathcal{S}cl(A)$ and $pfecl(A)) = X_1$.

The complement of an $pf\delta os$ (resp. $pf\delta\mathcal{P}os, pf\delta\mathcal{S}os$ and $pfeos$) is called an $pf\delta$ (resp. $pf\delta\mathcal{P}, pf\delta\mathcal{S}$ and $pfe$) closed set (briefly, $pf\delta cs$ (resp. $pf\delta\mathcal{P}cs, pf\delta\mathcal{S}cs$ and $pfecs$)) in $X_1$.

The family of all $pf\delta os$ (resp. $pf\delta cs, pf\delta\mathcal{P}os, pf\delta\mathcal{P}cs, pf\delta\mathcal{S}os, pf\delta\mathcal{S}cs, pfeos$ and $pfecs$) of $X_1$ is denoted by $pf\delta OS(X_1)$, (resp. $pf\delta CS(X_1), pf\delta\mathcal{P}OS(X_1), pf\delta\mathcal{P}CS(X_1), pf\delta\mathcal{S}OS(X_1), pf\delta\mathcal{S}CS(X_1), pfeOS(X_1)$ and $pfeCS(X_1)$).

**Definition 2.10** *[32] Let $(X, \tau)$ be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X_I\}$ be an $pfs$ in $X_I$. Then the (i) $pf\delta$-pre (resp. $pf\delta$-semi and $pfe$)-interior of $A$ are denoted by $pf\delta\mathcal{P}int(A)$ (resp. $pf\delta\mathcal{S}int(A)$ and $pfeint(A)$) and are defined as follows: $pf\delta\mathcal{P}int(A)$ (resp. $pf\delta\mathcal{S}int(A)$ and $pfeint(A) = \cup \{G | G$ in a $pf\delta\mathcal{P}os$ (resp. $pf\delta\mathcal{S}os$ and $pfeos$) and $G \subseteq A\}$, (ii) $pf\delta$-pre (resp. $pf\delta$-semi and $pfe$)-closure of $A$ are denoted by $pf\delta\mathcal{P}cl(A)$ (resp. $pf\delta\mathcal{S}cl(A)$ and $pfecl(A)$) and are defined as follows: $pf\delta\mathcal{P}cl(A)$ (resp. $pf\delta\mathcal{S}cl(A)$ and $pfecl(A)) = \cap \{K | K$ is an $pf\delta\mathcal{P}cs$ (resp. $pf\delta\mathcal{S}cs, pfecs$) and $A \subseteq K\}$.*

**Definition 2.11** *[32] Let $(X_I, \Gamma_P)$ be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X_I\}$ be an $pfs$ in $X_I$. A set $A$ is said to be $pf$*

1. $\theta$-interior of $A$ (briefly, $pf\theta int(A)$) is defined by $pf\theta int(A)$ $=\cup$ $\{pfint(B): B \subseteq A \;\&\; B$ isa $pfcs$ in $X_1\}$.

2. $\theta$-open set (briefly, $pf\theta os$) if $A = pf\theta int(A)$.

3. $\theta$ -semi open set (briefly, $pf\theta\mathcal{S}os$) if $A \subseteq pfcl(pf\theta int(A))$.

4. $M$-open set (briefly, $pfMos$) if $A \subseteq pfcl(pf\theta int(A)) \cup pfint(pf\delta cl(A))$.

The complement of a $pfMos$ (resp. $pf\theta os$ & $pf\theta\mathcal{S}os$) is called an $pfM$ (resp. $pf\theta$ & $pf\theta\mathcal{S}$) closed set (briefly, $pfMcs$ (resp. $pf\theta cs$ & $pf\theta\mathcal{S}cs$)) in $X_1$.

The family of all $pf\theta os$ (resp. $pf\theta cs, pf\theta\mathcal{S}os, pf\theta\mathcal{S}cs, pfMos$ and $pfMcs$) of $X_1$ is denoted by $pf\theta OS(X_1)$, (resp. $pf\theta CS(X_1), pf\theta\mathcal{S}OS(X_1), pf\theta\mathcal{S}CS(X_1), pfMOS(X_1)$ and $pfMCS(X_1)$).

**Definition 2.12** *[32] Let $(X_1, \Gamma_P)$ be an $pfts$ and $A = \{< a, \lambda_A(a), \mu_A(a) > | a \in X_1\}$ be an $pfs$ in $X_1$. Then the $pf$*

1. $M$ (resp. $pf\theta$-semi )-interior of $A$ (briefly, $pfMint(A)$ (resp. $pf\theta\mathcal{S}int(A)$) is defined by $pfMint(A)$ (resp. $pf\theta int(A)$ and $pf\theta\mathcal{S}int(A)$) $=\cup$ $\{B: B \subseteq A$ and $B$ is a $pfMos$ (resp. $pf\theta\mathcal{S}os$) in $X_1\}$.

2. $M$ (resp. $\theta$-semi )-closure of $A$ (briefly, $pfMcl(A)$ (resp. $pf\theta\mathcal{S}cl(A)$) is defined by $pfMcl(A)$ (resp. $pf\theta\mathcal{S}cl(A)$) $=\cap$ $\{B: A \subseteq B$ and $A$ is a $pfMcs$ (resp. $pf\theta\mathcal{S}cs$) in $X_1\}$.

**Definition 2.13** *Let $(X_1, \Gamma_P)$ and $(X_2, \Psi_P)$ be any two $pfts$'s. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy (resp. $\delta$, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $e$, $\theta$, $\theta\mathcal{S}$ and $M$ )-continuous (briefly, $pfCts$ (resp. $pf\delta Cts$, $pf\delta\mathcal{P}Cts$, $pf\delta\mathcal{S}Cts$, $pfeCts$, $pf\theta Cts$, $pf\theta\mathcal{S}Cts$ and $pfMCts$)) if the inverse image of every $pfos$ in $(X_2, \Psi_P)$ is a $pfos$ (resp. $pf\delta os$, $pf\delta\mathcal{P}os$, $pf\delta\mathcal{S}os$, $pfeos$, $pf\theta os$, $pf\theta\mathcal{S}os$ and $pfMos$) in $(X_1, \Gamma_P)$.*

## 3 Pythagorean fuzzy contra $M$-continuous maps

**Definition 3.1** *Let $(X_1, \Gamma_P)$ and $(X_2, \Psi_P)$ be any two $pfts$'s. A mapping $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is said to be a Pythagorean fuzzy contra (resp. $\delta$, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $e$, $\theta$, $\theta\mathcal{S}$ and $M$ )-continuous (briefly, $pfcontraCts$ (resp. $pfcontra\delta Cts$, $pfcontra\delta\mathcal{P}Cts$, $pfcontra\delta\mathcal{S}Cts$, $pfcontraeCts$, $pfcontra\theta Cts$, $pfcontra\theta\mathcal{S}Cts$ and $pfcontraMCts$)) if the inverse image of every $pfos$ in $(X_2, \Psi_P)$ is a $pfcs$ (resp. $pf\delta cs$, $pf\delta\mathcal{P}cs$, $pf\delta\mathcal{S}cs$, $pfecs$, $pf\theta cs$, $pf\theta\mathcal{S}cs$ and $pfMcs$) in $(X_1, \Gamma_P)$.*

**Proposition 3.1** *Let $(X_1, \Gamma_P)$ & $(X_2, \Psi_P)$ be a $pfts$'s. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a mapping. Then the following statements are hold for $pfts$, but not conversely.*

1. Every $pfcontra\theta Cts$ is a $pfcontraCts$.
2. Every $pfcontra\theta Cts$ is a $pfcontra\theta\mathcal{S}Cts$.
3. Every $pfcontra\theta\mathcal{S}Cts$ is a $pfcontraMCts$.
4. Every $pfcontra\delta Cts$ is a $pfcontra\delta\mathcal{S}Cts$.
5. Every $pfcontra\delta Cts$ is a $pfcontra\delta\mathcal{P}Cts$.
6. Every $pfcontra\delta\mathcal{S}Cts$ is a $pfcontraeCts$.
7. Every $pfcontra\delta\mathcal{P}Cts$ is a $pfcontraMCts$.
8. Every $pfcontraMCts$ is a $pfcontraeCts$.
9. Every $pfcontra\delta Cts$ is a $pfcontraCts$.

**Proof.**

1. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\theta Cts$, $h_P^{-1}(B)$ is $pf\theta cs$ in $(X_1, \Gamma_P)$. Since every $pf\theta cs$ is a $pfcs$, $h_P^{-1}(B)$ is a $pfcs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraCts$.

2. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\theta Cts$, $h_P^{-1}(B)$ is $pf\theta cs$ in $(X_1, \Gamma_P)$. Since every $pf\theta cs$ is a $pf\theta\mathcal{S}cs$, $h_P^{-1}(B)$ is a $pfcs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontra\theta\mathcal{S}Cts$.

3. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\theta\mathcal{S}Cts$, $h_P^{-1}(B)$ is $pf\theta\mathcal{S}cs$ in $(X_1, \Gamma_P)$. Since every $pf\theta\mathcal{S}cs$ is a $pfMcs$, $h_P^{-1}(B)$ is a $pfMcs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraMCts$.

4. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\delta Cts$, $h_P^{-1}(B)$ is $pf\delta cs$ in $(X_1, \Gamma_P)$. Since every $pf\delta cs$ is a $pf\delta\mathcal{S}cs$, $h_P^{-1}(B)$ is a $pf\delta\mathcal{S}cs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontra\delta\mathcal{S}Cts$.

5. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\delta Cts$, $h_P^{-1}(B)$ is $pf\delta cs$ in $(X_1, \Gamma_P)$. Since every $pf\delta cs$ is a $pf\delta\mathcal{P}cs$, $h_P^{-1}(B)$ is a $pf\delta\mathcal{P}cs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontra\delta\mathcal{P}Cts$.

6. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\delta\mathcal{S}Cts$, $h_P^{-1}(B)$ is $pf\delta\mathcal{S}cs$ in $(X_1, \Gamma_P)$. Since every $pf\delta\mathcal{S}cs$ is a $pfecs$, $h_P^{-1}(B)$ is a $pfecs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraeCts$.

7. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\delta\mathcal{P}Cts$, $h_P^{-1}(B)$ is $pf\delta\mathcal{P}cs$ in $(X_1, \Gamma_P)$. Since every $pf\delta\mathcal{P}cs$ is a $pfMcs$, $h_P^{-1}(B)$ is a $pfMcs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraMCts$.

8. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontraMCts$, $h_P^{-1}(B)$ is $pfMcs$ in $(X_1, \Gamma_P)$. Since every $pfMcs$ is a $pfecs$, $h_P^{-1}(B)$ is a $pfecs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraeCts$.

9. Let $B$ be a $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontra\delta Cts$, $h_P^{-1}(B)$ is $pf\delta cs$ in $(X_1, \Gamma_P)$. Since every $pf\delta cs$ is a $pfcs$, $h_P^{-1}(B)$ is a $pfcs$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is a $pfcontraCts$.

**Remark 3.1** *We obtain the following diagram from the results are discussed above.*

$$\begin{tikzcd}
pfcontra\theta Cts \arrow[rr] \arrow[d] & & pfcontraCts \\
pfcontra\theta\mathcal{S}Cts \arrow[dd] & & pfcontra\delta Cts \arrow[u] \arrow[dl] \arrow[dd] \\
& pfcontra\delta\mathcal{P}Cts \arrow[dl] & \\
pfcontraMCts \arrow[dr] & & pfcontra\delta\mathcal{S}Cts \arrow[dl] \\
& pfcontraeCts &
\end{tikzcd}$$

Note: $A \rightarrow B$ denotes $A$ implies $B$, but not conversely.

**Example 3.1** *Let $X_1 = X_2 = \{x_1, x_2\}$ and $pfs$'s $A_1, A_2, A_3$ & $A_4$ in $X_1$ and $B_1, B_2, B_3$ & $B_4$ in $X_2$ are defined as,*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$

$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$

$A_3 = \{< x_1, 0.90,0.10 >, < x_2, 0.70,0.30 >\}$

$A_4 = \{< x_1, 0.20,0.80 >, < x_2, 0.30,0.70 >\}$

$B_1 = \{< x_1, 0.80,0.20 >, < x_2, 0.60,0.40 >\}$

$B_2 = \{< x_1, 0.90,0.10 >, < x_2, 0.70,0.30 >\}$

$B_3 = \{< x_1, 0.10,0.90 >, < x_2, 0.30,0.70 >\}$

$B_4 = \{< x_1, 0.80,0.20 >, < x_2, 0.70,0.30 >\}$.

Then, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1, B_2, B_3, B_4\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontraCts$ but not $pfcontra\theta Cts$, because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pf\theta cs$ in $X_1$.

**Example 3.2** *Let* $X_1 = X_2 = \{x_1, x_2\}$ *and* $pfs$*'s* $A_1, A_2, A_3, A_4$ *in* $X_1$ *&* $B_1$ *in* $X_2$ *are defined as,*

$A_1 = \{< x_1, 0.20,0.80 >, < x_2, 0.40,0.60 >\}$

$A_2 = \{< x_1, 0.10,0.90 >, < x_2, 0.30,0.70 >\}$

$A_3 = \{< x_1, 0.90,0.10 >, < x_2, 0.70,0.30 >\}$

$A_4 = \{< x_1, 0.20,0.80 >, < x_2, 0.30,0.70 >\}$

$B_1 = \{< x_1, 0.20,0.80 >, < x_2, 0.40,0.60 >\}$

Here, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontra\theta\mathcal{S}Cts$ (resp. $pfcontra\delta\mathcal{S}Cts$) but not $pfcontra\theta Cts$ (resp. $pfcontra\delta Cts$), because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pf\theta cs$ (resp. $pf\delta cs$) in $X_1$.

**Example 3.3** *Let* $X_1 = X_2 = \{x_1, x_2\}$ *and* $pfs$*'s* $A_1, A_2, A_3, A_4$ *in* $X_1$ *&* $B_1$ *in* $X_2$ *are defined as,*

$A_1 = \{< x_1, 0.20,0.80 >, < x_2, 0.40,0.60 >\}$

$A_2 = \{< x_1, 0.10,0.90 >, < x_2, 0.30,0.70 >\}$

$A_3 = \{< x_1, 0.90,0.10 >, < x_2, 0.70,0.30 >\}$

$A_4 = \{< x_1, 0.20,0.80 >, < x_2, 0.30,0.70 >\}$

$B_1 = \{< x_1, 0.80,0.20 >, < x_2, 0.60,0.40 >\}$.

Here, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontraMCts$ but not $pfcontra\theta\mathcal{S}Cts$, because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pf\theta\mathcal{S}cs$ in $X_1$.

**Example 3.4** *Let* $X_1 = X_2 = \{x_1, x_2\}$ *and* $pfs$*'s* $A_1, A_2, A_3, A_4$ *in* $X_1$ *&* $B_1$ *in* $X_2$ *are defined as,*

$A_1 = \{< x_1, 0.20,0.80 >, < x_2, 0.40,0.60 >\}$

$A_2 = \{< x_1, 0.10,0.90 >, < x_2, 0.30,0.70 >\}$

$A_3 = \{< x_1, 0.90,0.10 >, < x_2, 0.70,0.30 >\}$

$A_4 = \{< x_1, 0.20,0.80 >, < x_2, 0.30,0.70 >\}$

$B_1 = \{< x_1, 0.20,0.40 >, < x_2, 0.40,0.40 >\}$

Now, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontraeCts$ but not $pfcontraMCts$, because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pfMcs$ in $X_1$.

**Example 3.5** *Let $X_1 = X_2 = \{x_1, x_2\}$ and $pfs$'s $A_1, A_2, A_3, A_4$ in $X_1$ & $B_1$ in $X_2$ are defined as,*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$

$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$

$$A_3 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$

$$A_4 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}$$

$$B_1 = \{< x_1, 0.80, 0.20 >, < x_2, 0.70, 0.30 >\}$$

Now, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontraCts$ (resp. $pfcontraeCts$ and $pfcontra\delta\mathcal{P}Cts$ ) but not $pfcontra\delta Cts$ (resp. $pfcontra\delta\mathcal{S}Cts$ and $pfcontra\delta Cts$), because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pf\delta cs$ (resp. $pf\delta\mathcal{S}cs$ and $pf\delta cs$) in $X_1$.

**Example 3.6** *Let $X_1 = X_2 = \{x_1, x_2\}$ and $pfs$'s $A_1, A_2, A_3, A_4$ in $X_1$ & $B_1$ in $X_2$ are defined as,*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$

$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$

$$A_3 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$

$$A_4 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}$$

$$B_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.60 >\}$$

Now, we have $\Gamma_P = \{0_X, 1_X, A_1, A_2, A_3, A_4\}$ and $\Psi_P = \{0_X, 1_X, B_1\}$. Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be an identity mapping. Then, $h_P$ is $pfcontraMCts$ but not $pfcontra\delta\mathcal{P}Cts$, because the set $B_1$ is $pfos$ in $X_2$ but $h_P^{-1}(B_1) = B_1$ is not $pf\delta\mathcal{P}cs$ in $X_1$.

**Theorem 3.1** *A map $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ is $pfcontraMCts$ (resp. $pfcontraCts$, $pfcontra\delta Cts$, $pfcontra\delta\mathcal{S}Cts$, $pfcontra\delta\mathcal{P}Cts$, $pfcontra\theta Cts$, $pfcontraeCts$ and $pfcontra\theta\mathcal{S}Cts$ ) iff the inverse image of each $pfcs$ in $(X_2, \Psi_P)$ is $pfMos$ (resp. $pfos$, $pf\delta os$, $pf\delta\mathcal{S}os$, $pf\delta\mathcal{P}os$, $pf\theta os$, $pfeos$ and $pf\theta\mathcal{S}os$ ) in $(X_1, \Gamma_P)$.*

**Proof.** Let $B$ be a $pfcs$ in $(X_2, \Psi_P)$. This implies $B^c$ is $pfos$ in $(X_2, \Psi_P)$. Since $h_P$ is $pfcontraMCts$, $h_P^{-1}(B^c)$ is $pfMcs$ in $(X_1, \Gamma_P)$. Since, $h_P^{-1}(B^c) = (h_P^{-1}(B))^c$, $h_P^{-1}(B)$ is a $pfMos$ in $(X_1, \Gamma_P)$.

Conversely, let $B$ be a $pfcs$ in $(X_2, \Psi_P)$. Then, $B^c$ is a $pfos$ in $(X_2, \Psi_P)$. By hypothesis $h_P^{-1}(B^c)$ is $pfMcs$ in $(X_1, \Gamma_P)$. Since, $h_P^{-1}(B^c) = (h_P^{-1}(B))^c$, $(h_P^{-1}(B))^c$ is a $pfMcs$ in $(X_1, \Gamma_P)$. Therefore, $h_P^{-1}(B)$ is a $pfMos$ in $(X_1, \Gamma_P)$. Hence, $h_P$ is $pfcontraMCts$. The proof of other cases are similar

**Definition 3.2** *A $pfts$ $(X_1, \Gamma_P)$ is said to be a Pythagorean fuzzy $MU_{1/2}$ (resp.$pf\delta\mathcal{S}U_{1/2}$, $pf\delta\mathcal{P}U_{1/2}$, $pf\theta U_{1/2}$, $pfeU_{1/2}$ and $pf\theta\mathcal{S}U_{1/2}$ )-space, if every $pfMos$ (resp. $pf\delta\mathcal{S}os$, $pf\delta\mathcal{P}os$, $pf\theta os$, $pfeos$ and $pf\theta\mathcal{S}os$ ) in $X_1$ is a $pfos$ in $X_1$.*

**Theorem 3.2** *Let $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ be a $pfcontraMCts$ (resp. $pfcontra\delta\mathcal{S}Cts$, $pfcontra\delta\mathcal{P}Cts$, $pfcontra\theta Cts$, $pfcontraeCts$ and $pfcontra\theta\mathcal{S}Cts$ ), then $h_P$ is a $pfcontraCts$ if $X_1$ is a $pfMU_{1/2}$) (resp. $pf\delta\mathcal{S}U_{1/2}$, $pf\delta\mathcal{P}U_{1/2}$, $pf\theta U_{1/2}$, $pfeU_{1/2}$ and $pf\theta\mathcal{S}U_{1/2}$ )-space.*

**Proof.** Let $B$ be a $Pfos$ in $X_2$. Then, $h_P^{-1}(B)$ is a $pfMcs$ in $X_1$, by hypothesis. Since $X_1$ is a $pfMU_{1/2}$-space, $h_P^{-1}(B)$ is a $pfcs$ in $X_1$. Hence, $h_P$ is a $pfcontraCts$. The proof of other cases are similar.

**Theorem 3.3** *Let* $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ *be a* $pfcontraMCts$ *map and* $g_P: (X_2, \Psi_P) \to (X_3, \Phi_P)$ *be a* $pfCts$*, then* $g_P \circ h_P: (X_1, \Gamma_P) \to (X_3, \Phi_P)$ *is a* $pfcontraMCts$.

**Proof.** Let $K$ be a $pfos$ in $X_3$. Then, $g_P^{-1}(K)$ is a $pfos$ in $X_2$, by hypothesis. Since $h_P$ is a $pfcontraMCts$ map, $h_P^{-1}(g_P^{-1}(K))$ is a $pfMcs$ in $X_1$. Hence $g_P \circ f_P$ is a $pfcontraMCts$ map. The proof of other cases are similar.

**Theorem 3.4** *Let* $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ *be a* $pfcontraMCts$ *map. Then, the following conditions are hold.*

1. $h_P(pfMcl(A)) \geq pfint(h_P(A))$, for all $pfcs\ A$ in $X_1$.
2. $pfMcl(h_P^{-1}(B)) \geq h_P^{-1}(pfint(B))$, for all $pfcs\ B$ in $X_2$.

**Proof.**

1. Since $pfMcl(h_P(A))$ is a $pfMcs$ in $X_2$ and $h_P$ is $pfcontraMCts$, then $h_P^{-1}(pfMcl(h_P(A)))$ is $pfMos$ in $X_1$. Now, since $A \geq h_P^{-1}(pfint(h_P(A))), pfMcl(A) \geq h_P^{-1}(pfMint(h_P(A)))$. Therefore, $h_P(pfMcl(A)) \geq pfint(h_P(A))$.

2. By replacing $A$ with $B$ in (i), we obtain $h_P(pfMcl(h_P^{-1}(B))) \geq pfint(h_P(h_P^{-1}(B))) \geq pfint(B)$. Hence, $pfMcl(h_P^{-1}(B)) \geq h_P^{-1}(pfint(B))$.

**Remark 3.2** *Theorems 3.3 and 3.4 are true for* $pfcontra\delta Cts$, $pfcontra\delta \mathcal{S}Cts$, $pfcontra\delta \mathcal{P}Cts$, $pfcontra\theta Cts$, $pfcontraeCts$ *and* $pfcontra\theta \mathcal{S}Cts$.

## 4 Pythagorean fuzzy contra $M$ -irresolute maps in $pfts$

In this section, we introduce $pfcontraMIrr$ (resp. $pfcontraIrr$, $pfcontra\delta Irr$, $pfcontra\delta \mathcal{S}Irr$, $pfcontra\delta \mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ and $pfcontra\theta \mathcal{S}Irr$) maps and study some of its characterizations.

**Definition 4.1** *A map* $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ *is called a Pythagorean fuzzy contra M (resp.* $pfcontra$, $pfcontra\delta$, $pfcontra\delta \mathcal{S}$, $pfcontra\delta \mathcal{P}$, $pfcontra\theta$, $pfcontrae$ *and* $pfcontra\theta \mathcal{S}$*) -irresolute (briefly,* $pfcontraMIrr$ *(resp.* $pfcontraIrr$, $pfcontra\delta Irr$, $pfcontra\delta \mathcal{S}Irr$, $pfcontra\delta \mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta \mathcal{S}Irr$*)) map if* $h_P^{-1}(B)$ *is a* $pfMcs$ *(resp.* $pf\mathcal{S}cs$, $pf\delta cs$, $pf\delta \mathcal{S}cs$, $pf\delta \mathcal{P}cs$, $pf\theta cs$, $pfecs$ *and* $pf\theta \mathcal{S}cs$*) in* $(X_1, \Gamma_P)$ *for every* $pfMos$ *(resp.* $pf\mathcal{S}os$, $pf\delta os$, $pf\delta \mathcal{S}os$, $pf\delta \mathcal{P}os$, $pf\theta os$, $pfeos$ *and* $pf\theta \mathcal{S}os$*)* $B$ *of* $(X_2, \Psi_P)$.

**Theorem 4.1** *Let* $h_P: (X_1, \Gamma_P) \to (X_2, \Psi_P)$ *be a* $pfcontraIrr$ *(resp.* $pfcontraMIrr$, $pfcontra\delta Irr$, $pfcontra\delta \mathcal{S}Irr$, $pfcontra\delta \mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta \mathcal{S}Irr$*), then* $h_P$ *is a* $pfcontra\mathcal{S}Cts$ *(resp.* $pfcontraMCts$, $pfcontraCts$, $pfcontra\delta \mathcal{S}Cts$, $pfcontra\delta \mathcal{P}Cts$, $pfcontraCts$, $pfcontraeCts$ *and* $pfcontra\theta \mathcal{S}Cts$ *) map. But not conversely.*

**Proof.** (i) Let $h_P$ be a $pfcontraIrr$ map. Let $B$ be any $pfos$ in $X_2$. Since every $pfos$ is a $pf\mathcal{S}os$, $B$ is a $pf\mathcal{S}os$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pf\mathcal{S}cs$ in $X_1$. Hence, $h_P$ is a $pfcontra\mathcal{S}Cts$ map.

(ii) Let $h_P$ be a $pfcontraMIrr$ map. Let $B$ be any $pfos$ in $X_2$. Since every $pfos$ is a $pfMos$, $B$ is a $pfMos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pfMcs$ in $X_1$. Hence, $h_P$ is a $pfcontraMCts$ map.

(iii) Let $h_P$ be a $pfcontra\delta Irr$ map. Let $B$ be any $pf\delta os$ in $X_2$. Since every $pf\delta os$ is a $pfos$, $B$ is a $pfos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pfcs$ in $X_1$. Hence, $h_P$ is a $pfcontraCts$ map.

(iv) Let $h_P$ be a $pfcontra\delta\mathcal{S}Irr$ map. Let $B$ be any $pf\delta os$ in $X_2$. Since every $pf\delta os$ is a $pfos$, $B$ is a $pfos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pf\delta\mathcal{S}cs$ in $X_1$. Hence, $h_P$ is a $pfcontra\delta\mathcal{S}Cts$ map.

(v) Let $h_P$ be a $pfcontra\delta\mathcal{P}Irr$ map. Let $B$ be any $pf\delta os$ in $X_2$. Since every $pf\delta os$ is a $pfos$, $B$ is a $pfos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pf\delta\mathcal{P}cs$ in $X_1$. Hence, $h_P$ is a $pfcontra\delta\mathcal{P}Cts$ map.

(vi) Let $h_P$ be a $pfcontra\theta Irr$ map. Let $B$ be any $pf\theta os$ in $X_2$. Since every $pf\theta os$ is a $pfos$, $B$ is a $pfos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pf\theta cs$ in $X_1$. Hence, $h_P$ is a $pfcontra\theta Cts$ map.

(vii) Let $h_P$ be a $pfcontraeIrr$ map. Let $B$ be any $pfos$ in $X_2$. Since every $pfos$ is a $pfeos$, $B$ is a $pfeos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pfecs$ in $X_1$. Hence, $h_P$ is a $pfcontraeCts$ map.

(viii) Let $h_P$ be a $pfcontra\theta\mathcal{S}Irr$ map. Let $B$ be any $pf\theta os$ in $X_2$. Since every $pf\theta os$ is a $pfos$, $B$ is a $pfos$ in $X_2$. By hypothesis $h_P^{-1}(B)$ is a $pf\theta\mathcal{S}cs$ in $X_1$. Hence, $h_P$ is a $pfcontra\theta\mathcal{S}Cts$ map.

**Example 4.1** *Let* $X = \{x_1, x_2\} = Y = \{y_1, y_2\}$ *and* $pfs$*'s* $A_1, A_2, A_3$ *&* $A_4$ *in* $X$ *and* $B_1$ *&*$B_2$ *in* $Y$ *are defined as,*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$

$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$

$$A_3 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$

$$A_4 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}$$

$$B_1 = \{< y_1, 0.10, 0.90 >, < y_2, 0.30, 0.70 >\}$$

$$B_2 = \{< y_1, 0.20, 0.40 >, < y_2, 0.40, 0.40 >\}.$$

Here, $\tau_1 = \{0_P, 1_P, A_1, A_2, A_3, A_4\}$, $\tau_2 = \{0_P, 1_P, B_1\}$, $\tau_3 = \{0_P, 1_P, A_2^c\}$, $\tau_4 = \{0_P, 1_P, A_4^c\}$ and $\tau_5 = \{0_P, 1_P, A_2^c, A_3^c\}$.

(i) Let $f_1: (X, \tau_1) \to (Y, \tau_2)$ be an identity mapping. Then, $f_1$ is $pfcontraMCts$ but not $pfcontraMIrr$, because the set $B_2$ is a $pfMos$ in $Y$ but $f^{-1}(B_2) = B_2$ is not $pfMcs$ in $X$. (ii) Let $f_2: (X, \tau_1) \to (Y, \tau_3)$ be an identity mapping. Then, $f_2$ is $pfcontra\mathcal{S}Cts$ but not $pfcontraIrr$, because the set $A_2^c$ is a $pf\mathcal{S}os$ in $Y$ but $f_2^{-1}(A_2^c) = A_2^c$ is not $pf\mathcal{S}cs$ in $X$. (iii) Let $f_3: (X, \tau_1) \to (Y, \tau_4)$ be an identity mapping. Then, $f_3$ is $pfcontraCts$ (resp. $pfcontra\delta\mathcal{S}Cts$) but not $pfcontra\delta Irr$ (resp. $pfcontra\delta\mathcal{S}Irr$), because the set $A_4^c$ is a $pf\delta os$ (resp. $pf\delta\mathcal{S}os$) in $Y$ but $f_3^{-1}(A_4^c) = A_4^c$ is not $pf\delta cs$ (resp. $pf\delta\mathcal{S}cs$) in $X$. (iv) Let $f_4: (X, \tau_1) \to (Y, \tau_5)$ be an identity mapping. Then, $f_4$ is $pfcontra\delta\mathcal{P}Cts$ but not $pfcontra\delta\mathcal{P}Irr$, because the set $A_1$ is a $pf\delta\mathcal{P}os$ in $Y$ but $f_4^{-1}(A_1) = A_1$ is not $pf\delta\mathcal{P}cs$ in $X$.

**Example 4.2** *Let* $X = \{x_1, x_2\} = Y = \{y_1, y_2\}$ *and* $pfs$*'s* $A_1, A_2, A_3, A_4$ *&* $A_5$ *in* $X$ *are defined as,*

$$A_1 = \{< x_1, 0.20, 0.80 >, < x_2, 0.40, 0.60 >\}$$

$$A_2 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.70 >\}$$

$$A_3 = \{< x_1, 0.90, 0.10 >, < x_2, 0.70, 0.30 >\}$$

$$A_4 = \{< x_1, 0.20, 0.80 >, < x_2, 0.30, 0.70 >\}$$

$$A_5 = \{< x_1, 0.10, 0.90 >, < x_2, 0.30, 0.60 >\}.$$

Here, $\tau_1 = \{0_P, 1_P, A_2, A_4\}$, $\tau_2 = \{0_P, 1_P, A_1^c, A_2^c, A_3^c, A_4^c\}$. Let $h_P: (X, \tau_1) \to (Y, \tau_2)$ be an identity mapping. Then, $h_P$ is $pfcontraeCts$ but not $pfcontraeIrr$, because the set $A_5^c$ is a $pfeos$ in $Y$ but $h_P^{-1}(A_5^c) = A_5^c$ is not $pfecs$ in $X$.

**Theorem 4.2** *Let* $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ *be a* $pfcontraMIrr$ *(resp.* $pfcontra\delta Irr$, $pfcontra\delta\mathcal{S}Irr$, $pfcontra\delta\mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta\mathcal{S}Irr$*), then* $h_P$ *is a* $pfcontraCts$ *map if* $X_1$ *is a* $pfMU_{1/2}$ *(resp.* $pf\delta U_{1/2}$, $pf\delta\mathcal{S}U_{1/2}$, $pf\delta\mathcal{P}U_{1/2}$, $pf\theta U_{1/2}$, $pfeU_{1/2}$ *and* $pf\theta\mathcal{S}U_{1/2}$ *)-space.*

**Proof.** Let $B$ be a $pfos$ in $X_2$. Then, $B$ is a $pfMos$ in $X_2$. Therefore $h_P^{-1}(B)$ is a $pfMcs$ in $X_1$, by hypothesis. Since $X_1$ is a $pfMU_{1/2}$ -space, $h_P^{-1}(B)$ is a $pfcs$ in $X_1$. Hence, $h_P$ is a $pfcontraCts$ map. The proof of other cases is similar.

**Theorem 4.3** *Let* $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ *and* $g_P: (X_2, \Psi_P) \rightarrow (X_3, \Phi_P)$ *be a* $pfcontraMIrr$ *(resp.* $pfcontra\delta Irr$, $pfcontra\delta\mathcal{S}Irr$, $pfcontra\delta\mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta\mathcal{S}Irr$*) maps, then* $g_P \circ h_P: (X_1, \Gamma_P) \rightarrow (X_3, \Phi_P)$ *is a* $pfMIrr$ *(resp.* $pf\delta Irr$, $pf\delta\mathcal{S}Irr$, $pf\delta\mathcal{P}Irr$, $pf\theta Irr$, $pfeIrr$ *and* $pf\theta\mathcal{S}Irr$*) map.*

**Proof.** Let $K$ be a $pfMos$ in $X_3$. Then, $g_P^{-1}(K)$ is a $pfMcs$ in $X_2$. Since $h_P$ is a $pfcontraMIrr$ map, $h_P^{-1}(g_P^{-1}(K))$ is a $pfMos$ in $X_1$. Hence $g_P \circ f_P$ is a $pfMIrr$ map. The proof of other cases is similar.

**Theorem 4.4** *Let* $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$ *be a* $pfcontraMIrr$ *(resp.* $pfcontra\delta Irr$, $pfcontra\delta\mathcal{S}Irr$, $pfcontra\delta\mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta\mathcal{S}Irr$*) map and* $g_P: (X_2, \Psi_P) \rightarrow (X_3, \Phi_P)$ *be a* $pfcontraMCts$ *(resp.* $pfcontra\delta Cts$, $pfcontra\delta\mathcal{S}Cts$, $pfcontra\delta\mathcal{P}Cts$, $pfcontra\theta Cts$, $pfcontraeCts$ *and* $pfcontra\theta\mathcal{S}Cts$*) map, then* $g_P \circ h_P: (X, \tau_1) \rightarrow (Z, \tau_3)$ *is a* $pfMCts$ *(resp.* $pf\delta Cts$, $pf\delta\mathcal{S}Cts$, $pf\delta\mathcal{P}Cts$, $pf\theta Cts$, $pfeCts$ *and* $pf\theta\mathcal{S}Cts$*) map.*

**Proof.** Let $K$ be a $pfos$ in $X_3$. Then, $g_P^{-1}(K)$ is a $pfMcs$ in $X_2$. Since, $h_P$ is a $pfcontraMIrr, h_P^{-1}(g_P^{-1}(K))$ is a $pfMos$ in $X_1$. Hence, $g_P \circ h_P$ is a $pfMCts$ map. The proof of other cases is similar.

**Theorem 4.5** *Let a map* $h_P: (X_1, \Gamma_P) \rightarrow (X_2, \Psi_P)$*. Then the following conditions are equivalent if* $X_1$ *and* $X_2$ *are* $pfMU_{1/2}$ *-spaces.*

1. $h_P$ is a $pfcontraMIrr$ map.
2. $h_P^{-1}(B)$ is a $pfMos$ in $X_1$, for each $pfMcs$ $B$ in $X_2$.
3. $pfcl(h_P^{-1}(B)) \supseteq h_P^{-1}(pfint(B))$, for each $pfs$ $B$ of $X_2$.

**Proof.** (i) → (ii): Let $B$ be any $pfMcs$ in $X_2$. Then, $B^c$ is a $pfMos$ in $X_2$. Since $h_P$ is $pfcontraMIrr$, $h_P^{-1}(B^c)$ is a $pfMcs$ in $X_1$. But $h_P^{-1}(B^c) = (h_P^{-1}(B))^c$. Therefore, $h_P^{-1}(B)$ is a $pfMcs$ in $X_1$.

(ii) → (iii) : Let $B$ be any $pfs$ in $X_2$ and $pfint(B) \leq B$. Then, $h_P^{-1}(pfcl(B))h_P^{-1}(B) \leq h_P^{-1}(B)$. Since $pfint(B)$ is a $pfos$ in $X_2$, $pfint(B)$ is a $pfMos$ in $X_2$. Therefore, $(pfint(B))^c$ is a $pfMcs$ in $X_2$. By hypothesis, $h_P^{-1}((pfint(B))^c)$ is a $pfMos$ in $X_1$. Since, $h_P^{-1}((pfint(B))^c) = (h_P^{-1}(pfint(B)))^c, h_P^{-1}(pfint(B))$ is a $pfMos$ in $X_1$. Since, $X_1$ is a $pfMU_{1/2}$ -space, $h_P^{-1}(pfint(B))$ is a $pfos$ in $X_1$. Hence, $pfcl(h_P^{-1}(B)) \supseteq pfcl(h_P^{-1}(pfint(B))) = h_P^{-1}(pfint(B))$. That is, $pfcl(h_P^{-1}(B)) \supseteq h_P^{-1}(pfint(B))$.

(iii) → (i) : Let $B$ be any $pfMcs$ in $X_2$. Since $X_2$ is a $pfMU_{1/2}$-space, $B$ is a $pfcs$ in $X_2$ and $pfcl(B) = B$. Hence, $h_P^{-1}(B) = h_P^{-1}(pfMcl(B)) \supseteq pfMint(h_P^{-1}(B))$. But clearly, $h_P^{-1}(B) \supseteq pfint(h_P^{-1}(B))$. Therefore, $pfint(h_P^{-1}(B)) = h_P^{-1}(B)$. This implies, $h_P^{-1}(B)$ is a $pfos$ and hence, it is a $pfMos$ in $X_1$. Thus, $h_P$ is a $pfcontraMIrr$ map.

**Remark 4.1** *Theorem 4.5 is true for* $pfcontra\delta Irr$, $pfcontra\delta\mathcal{S}Irr$, $pfcontra\delta\mathcal{P}Irr$, $pfcontra\theta Irr$, $pfcontraeIrr$ *and* $pfcontra\theta\mathcal{S}Irr$.

## 5 Conclusion

In this paper, using $pfMos$ we have defined $pfcontraMCts$ map and analyzed its properties. After that we have defined $pfcontraM$-irresolute maps. In future, these concepts can be extended to some mathematical applications.

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