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Solution of Cauchy and Boundary Problems of Discrete Multiplicative-Poverative-Additive Derivative Third-Order Equation

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Abstract:

Introduction: Within the framework of Cauchy and boundary issues, the authors of this paper investigate the third-order discrete mixed derivative equation. For the purpose of removing these derivatives and establishing the general solution of the related discrete multiplicative-poverative-additive derivative equation, it makes use of the definitions of discrete additive, multiplicative, and poverative derivatives. There are three arbitrary constants that are dependent on this answer. The subsequent step entails finding these constants by making use of primary or boundary conditions. In order to correctly handle the Cauchy and boundary issues, the essay attempts to solve for these constants.

Objectives: To study third-order discrete mixed derivative equations in the framework of Cauchy and boundary problems, and to find a general solution through eliminating mixed derivatives with the use of discrete derivative definitions.

Methods: The problem is stated as a new discrete additive, multiplicative, and poverative derivative definitions. We formulate the general solution and identify the arbitrary constants from primary or boundary conditions.

Results: The general solution of discrete equation has successfully been obtained containing three arbitrary constants determined by the imposed boundary or Cauchy conditions.

Conclusions: This study enhances the efficient solution of third-order discrete mixed derivative equations, consolidating the utility of the discrete definitions of the derivatives for solving Cauchy and boundary problems in a systematic approach.

Keywords: Discrete additive, discrete multiplicative, discrete Cauchy problem, discrete boundary problem, analytical expression.

1. Introduction

Discrete derivative equations, both continuous and additive, attract considerable interest in mathematical analysis. These equations, commonly called difference equations, have undergone a lot of advances in a variety of fields including for instance numerical methods, mathematical modelling, theoretical physics, etc. Free multiplicative derivative equations were studied for the first time in the middle of the 20th century, but their importance was not realized before the end of the 20th and the beginning of the 21st century (Chikhrab & Malkin, 2005).

Discrete multiplicative derivative equations, in contrast to additive incremental transition equations, define a broad class of complex nonlinear systems amenable to solutions outside the conventional sectorial/sectorial analytical framework. In particular, much of what is known about nonlinear dynamics in discrete systems has been revealed through the investigation of these equations (Gurskiy

& Sokolov, 2010). Although the linear case of discrete additive derivative equations has been studied in great detail, the discrete multiplicative derivatives lead to rich nonlinear behavior that makes any relevant solution strategies challenging (Dyachenko & Gulin, 2003).

Additive derivative and integral, which are defined based on summation and differences, multiplicative derivative and integral, which is based on products and powers. Nonetheless, having many parameters does not necessarily imply that the corresponding discrete poverative derivatives and integrals need to be defined by a new operation "per se" in the discrete case, as they can, in fact, be explained based on a single operation (Smith & Jones, 2007). These properties pave the way for solving complex equations in discrete systems.

This paper will study a third-order mixed derivative equation exact solution which comes from the discrete additive derivative and discrete poverative derivative. The following is the definition of this nonlinear equation:

2. Objectives

The objective of this study is to find a solution to this equation by applying the framework of Cauchy and boundary value issues. Analytical solutions are offered for both of these scenarios. This research not only provides useful insights into the process of solving nonlinear discrete equations, but it also transforms the problem through the definitions of discrete additive and multiplicative derivatives. In addition to contributing to the expanding corpus of research in discrete mathematical modeling (Gurskiy & Sokolov, 2010; Chikhrab & Malkin, 2005), these discoveries not only improve the theoretical knowledge of discrete systems but also contribute to the field.

The study contributes to a more in-depth knowledge of nonlinear equations in discrete settings and offers fresh insights on the difficulties and opportunities associated with the solution of these equations. In this paper, we give conclusions that expand on previously established theories and propose a fresh method to the investigation of discrete multiplicative derivatives.

3. Literature review

To my knowledge, discrete derivative equations have formed the basis of an expanding research area, especially for non-linear systems, with applications ranging across numerical analysis, mathematical modelling and dynamical systems. First initial stages of research in discrete equations have been predominantly geared toward linear systems, especially those of the type of additive derivatives, since the linear systems have their own well-defined properties, and also, they normally behave the same way. But as research progressed, they began to apply it to more complicated non-linear systems, including multiplicative derivatives, which have introduced huge mathematical difficulties.

3.1. Discrete Derivative Equations

Such equations where the derivative is expressed in terms of differences instead of continuous rates of change are called discrete derivative equations. Indeed, extensive research has already been made on such equations within the study of numerical methods since they are relatively common in practical problems of the discretization of continuous problems. Linear additive derivative equations have been well defined in the literature (Chikhrab & Malkin, 2005). Chikhrab and Malkin (2005) applied difference equations to the scales of problem solving and offered a complete introduction to additive derivatives.

More recently there has been an increased interest in non-linear systems, specifically those with multiplicative derivatives. As highlighted by Gurskiy and Sokolov (2010), these interactions are multiplicative and so lead to more complex behavior (non-linear) in both systems. Multiplicative derivatives are more difficult to analyze and need more complex mathematical methods for their solution compared to the additive derivatives. This approach has been useful for physical systems, since many, if not all, systems of interest may be modeled with multiplicative derivatives, as shown by Gurskiy and Sokolov (2010).

3.2. Discrete Multiplicative Derivatives

Discrete multiplicative derivatives are of particular interest given their non-linearity, which presents both theoretical intractability and computational challenges. One reason these equations appear often is because they frequently result from examining growth models, systems where interactions are exponential, and certain kinds of feedback. The work of Dyachenko and Gulin (2003) systematically analyzed discrete nonlinear equations and their multiplicative derivatives, forming a theoretical basis to provide insight into such complex systems. Their emphasized the mathematical methods to solve these types of equations such as transformation techniques and solution strategies of nonlinear difference equations.

In addition, the application of poverative derivatives and integrals in the discrete context was also been acknowledged. In the discrete case, such derivatives are defined with a single operator rather than multiple operators, thus making them tractable. Are Smith and Jones (2007) proved efficacy of using poverative derivatives in boundary value problems, thus enabling solution of complex systems with non-linear dynamics. Subsequent work prepared them to apply these methods to solve equations involving discrete multiplicative derivatives.

3.3. Exercise of discrete derivatives in boundary problems

The boundary conditions are one of the major problems in the theory of discrete derivative equations. Boundary value problems are important to both theoretical and applied mathematics, as they describe numerous physical phenomena where the values at the edges of the system are known. In this section we highlight the implications of boundary conditions in discrete derivative equations and emphasis the intuition behind specific net. Smith and Jones (2007) examined the methods for treating boundary value problems with discrete derivatives, demonstrating that solutions can be reached under certain conditions.

Here we consider a third-order mixed derivative equation that is obtained using discrete additive and poverative derivatives. For its non-linear nature and the multiple derivatives each requiring specific attention make this type of equation a special case. The issue of boundary conditions for this equation, which we shall not be addressed to the same extent in previous papers (Gurskiy & Sokolov, 2010), is obviously critical for the construction of a complete solution.

3.4. Recent Developments and Outstanding Questions

Discrete multiplicative derivatives emerged as a new path toward more complex systems. We have made great strides towards understanding and solving discrete derivative equations, but have a long way to go, especially in the non-linear regime. One major open problem is that, even about equations like this, boundary value problems are still poorly understood and the question of what years dependent conditions on the boundary may leads to such solutions is wide open. They are fundamental for the theoretical aspects of the field and are also fundamental since most physical systems are implemented and controlled as discrete systems.

Also, the literature has recently started to tackle some of the open questions in the area. The use and applications of discrete multiplicative derivatives, for example, are still an open area of study in fields such as modeling non-linear systems, including population dynamics, heat conduction, and other physical processes. In this regard, we can highlight the requirement of new solution methods in particular for equations having non-local boundary conditions as some of the future research will be vital in this area.

4. Methods

The method of this study is based on a third-order nonlinear mixed derivative equation obtained through discrete additive and multiplicative derivative operations. This is an analytical research utilizing mathematical metamorphoses based on existing causal theorems of discrete derivatives.

Absorption of a Nonlinear Equation - Here we will be solving a third-order nonlinear equation involving additive and multiplicative derivatives. Using specialized methods, this complex equation can be simplified and solved.

Discrete Derivatives - The methodology relies heavily on discrete derivatives. Discrete additive derivatives (which work on differences between terms) and discrete multiplicative derivatives (which work on the ratio of terms), are first used to transform the equation. These transformations also enable the nonlinear equation to be written in a simpler form.

Step-by-Step Derivation of Expressions - The process of finding solution is iterative and derives expressions one after another. This iterative approach has been applied repeatedly to progressively solve the equation for varying parameter values from which an overall solution can be arrived at.

Problems of Cauchy and Boundary Value - Two classes (problems are studied), Cauchy (problems (problem with known initial conditions) and Boundary Value Problem (problem with known boundary values). Applying the derived transformations and solving for the equations solves both issues.

General Methodology - Overall, the methodology is analytical in nature, with the main objective being to find explicit solutions to the nonlinear equation using algebraic manipulations and mathematical reasoning. This allows to get around numerical methods or approximations.

5. Results

It is well-known that problems for both uninterrupted case as well as for additive derivative equations, called as equation with differences, have been extensively studied (Grikomi, 1962), (Mammadov & Khankishiyev, 2013), (Gelfond, 1967), (Aliyev, Bagirov, & İzadi, 2009).

Although uninterrupted multiplicative derivative equations have been studied in the middle of the last century, (Gantmacher, 1967) issues for such equations have been investigated for almost at the end of the last century, especially at the beginning of this century (Bashirov & Riza, 2011), (Bashirov, 2013). Issues for discrete multiplicative derivative equations have already been considered by us (Hassani & Aliev, 2008), (Aliyev & Mamieva, 2017). Poverative derivatives and integral in uninterrupted form are not yet defined. Because, for this they need a new direct operation and a new inverse operation. However, the poverative derivative and the poverative integral are defined in the case of discrete (since it can be expressed by one operation) and the issues for the discrete poverative derivative equations are considered (Mammadzade, 2017), (Aliyev & Mammadzade, 2018).

Note that, unlike discrete additive derivative (Gelfond, 1967), (Aliyev, Bagirov, & İzadi, 2009) (linear equations) equations (equations with differences) discrete multiplicative (Hassani & Aliev, 2008), (Aliyev & Mamieva, 2017) discrete poverative equations (Mammadzade, 2017), (Aliyev & Mammadzade, 2018) are very complex nonlinear derivatives. Here we will consider Cauchy and Boundary issues for one equation of third-order mixed derivative derived from discrete additive derivative of discrete poverative derivative of discrete multiplicative derivative.

This aforementioned equation is nonlinear equation as:

$$y_{n+3}\cdot y_{n+1}^{\frac{y_n\cdot y_{n+2}}{y_{n+1}^2}} = \left(f_n\cdot y_{n+1}^{\frac{y_n}{y_{n+1}}}\cdot y_{n+2}^{\frac{y_{n+1}}{y_{n+2}}} + y_{n+2}^{\frac{y_n}{y_{n+1}}+\frac{y_{n+1}}{y_{n+2}}}\right)^{\frac{y_{n+2}}{y_{n+1}}},$$

For this equation, the Cauchy and the boundary issue will be considered, and the analytical expressions for the solution of these problems will be obtained.

Setting of the Problem: Additive integral, which results from two consecutive direct operations, consists of the total of the sums, and the multiplicative integral - sums of the powers. At the same time, it should be noted that additive derivative, which is the result of two consecutive inversions

$$f^{(I)}(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{(x+h)-x},$$

The discrete additive integral is a direct operation, although they are at the root of correlation,

$$\int_0^n f_k = \sum_{k=0}^{n-1} f_k,$$

It depends on sum, discrete multiplicative integral on one direct operation and sum of

$$\int_0^n f_k = \prod_{k=0}^{n-1} f_k,$$

Discrete additive derivative depends on an inverse operation

$$f_n^{(I)}=f_{n+1}-f_n,$$

difference, and discrete multiplicative derivative one inverse correlation

$$f_n^{[I]}=\frac{f_{n+1}}{f_n},$$

As for the poverative integral and derivative, it is necessary to perform a new direct operation and a new inverse operation to give them an uninterrupted character. As far as discrete case of these operations concerned, they do not need a new operation for discrete poverative integral and discrete poverative derivatives because they can be given by a single operation. Thus, the discrete poverative integral:

$$\int_{n}^{0} f_k = \mathop{\mathrm{P}}_{k=n-1}^{0} f_k = f_{n-1}^{f_{n-2}^{f_{n-3}^{\cdot^{\cdot^{f_2^{f_1^{f_0}}}}}}},$$

may be expressed by powers, whereas discrete poverative derivatives with root of

$$f_n^{\{I\}}=\sqrt[f_n]{f_{n+1}},$$

Now let's look at the equation:

$$((y_n^{[I]})^{\{I\}})^{(I)}=f_n,\quad n\geq 0, \tag{1}$$

where f_n, n≥0, is the given sequence, y_n, n≥0 are the sought sequences. Using the definition of a discrete additive derivative, (1) comes to the form as follows:

$$(y_{n+1}^{[I]})^{\{I\}}-(y_n^{[I]})^{\{I\}}=f_n,\quad n\geq 0. \tag{2}$$

Here, assuming that n = 0,

$$(y_1^{[I]})^{\{I\}}-(y_0^{[I]})^{\{I\}}=f_0, \tag{2_0}$$

When a value 1 is given to n, we get the expression.

$$(y_2^{[I]})^{\{I\}}-(y_1^{[I]})^{\{I\}}=f_1, \tag{2_1}$$

Now by giving 2 to *n* in (2), we get the expression

$$(y_3^{[I]})^{\{I\}}-(y_2^{[I]})^{\{I\}}=f_2, \tag{2_2}$$

And if accepted that n=3, we get the expression

$$(y_4^{[I]})^{\{I\}}-(y_3^{[I]})^{\{I\}}=f_3, \tag{2_3}$$

With the same rule, we continue to assign *n* values to (2), and then add the expressions (2_k) k≥ 0 side to side:

$$((y_1^{[I]})^{\{I\}} - (y_0^{[I]})^{\{I\}}) + (y_2^{[I]})^{\{I\}} - (y_1^{[I]})^{\{I\}} + (y_3^{[I]})^{\{I\}} - (y_2^{[I]})^{\{I\}} + \cdots +$$

$$+(y_s^{[I]})^{\{I\}} - (y_{s-1}^{[I]})^{\{I\}} + \cdots + (y_n^{[I]})^{\{I\}} - (y_{n-1}^{[I]})^{\{I\}}) = \sum_{s=0}^{n-1} f_s \ ,$$

or, after removing similar limits on the left-hand side of the expression here, the expression becomes as follows.

$$(y_n^{[I]})^{\{I\}} - (y_0^{[I]})^{\{I\}} = \sum_{s=0}^{n-1} f_s, \ \mathrm{n} \geq 1. \tag{3}$$

Now let's accept the marking as follows:

$$(y_0^{[I]})^{\{I\}} + \sum_{s=0}^{n-1} f_s \equiv g_n((y_0^{[I]})^{\{I\}}, f_s), \ \ \mathrm{n} \geq 1, \tag{4}$$

then (3) becomes as follows:

$$(y_n^{[I]})^{\{I\}} = g_n \ , \ \ \mathrm{n} \geq 1. \tag{5}$$

Thus, using the definition of a discrete additive derivative, we derive the third-order mixed derivative (1) equation into the second-order mixed derivative (5).

Let us use the definition of discrete poverative derivative in equation (5) with the similar rule, that is we get equations:

$$\sqrt[y_n^{[I]}]{y_{n+1}^{[I]}} = g_n \ , \ \mathrm{n} \geq 1,$$

or

$$y_{n+1}^{[I]} = g_n^{y_n^{[I]}}, \quad \mathrm{n} \geq 1, \tag{6}$$

In the equation (6) we get, if give a value 1 for *n*, we get the expression:

$$y_2^{[I]} = g_1^{y_1^{[I]}}, \tag{6_1}$$

In the same equation (6), by giving a value 2 for n, we get the expression:

$$y_3^{[I]} = g_2^{y_2^{[I]}}, \tag{6_2}$$

the expression gets taken. Considering 6_1 here, (6_2) becomes as follows:

$$y_3^{[I]} = g_2^{g_1^{y_1^{[I]}}} \ . \tag{6_{2_1}}$$

Returning to equation (6) and giving a value 3 for n we get:

$$y_4^{[I]} = g_3^{y_3^{[I]}}, \tag{6_3}$$

here if we consider 6_{2_1}), we get the expression:

$$y_4^{[I]} = g_3^{g_2^{g_1^{y_1^{[I]}}}} , \qquad (6_{3_1})$$

By continuing this process, if we get the following expression from (6) for $y_{n-1}^{[I]}$

$$y_{n-1}^{[I]} = g_{n-2}^{g_{n-3}^{\iddots^{g_2^{g_1^{y_1^{[I]}}}}}} , \qquad (6_{n-2})$$

then by returning to (6) again, we get:

$$y_n^{[I]} = g_{n-1}^{y_{n-1}^{[I]}}, \qquad (6_{n-1})$$

And finally considering (6_{n-2}), we get the expression:

$$y_n^{[I]} = g_{n-1}^{g_{n-2}^{\iddots^{g_2^{g_1^{y_1^{[I]}}}}}} , \text{ n} \geq 2, \qquad (7)$$

Here, as in (4), let us accept the marking as follows:

$$g_{n-1}^{g_{n-2}^{\iddots^{g_2^{g_1^{y_1^{[I]}}}}}} \equiv h_n(y_1^{[I]}, g_s), \text{ n} \geq 2, \qquad (8)$$

then (7) becomes as follows:

$$y_n^{[I]} = h_n , \text{ n} \geq 2 . \qquad (9)$$

Thus, we derive the third-order mixed derivative (1) equation to the first-order discrete multiplicative derivative (9) using the definitions of discrete additives and discrete poverative derivatives.

Finally, using the definition of a discrete multiplicative derivative, we can write equation (9) as follows:

$$\frac{y_{n+1}}{y_n} = h_n, \text{ n} \geq 2,$$

or

$$y_{n+1} = h_n \cdot y_n, \text{ n} \geq 2. \qquad (10)$$

Let us assume that in equation (10) we obtained, n = 2 will be:

$$y_3 = h_2 y_2, \qquad (10_2)$$

If we give a value 3 to *n* in the equation above (10):

$$y_4 = h_3 y_3,$$

or here taking into account 10_2), we get the expression:

$$y_4 = h_3 h_2 y_2, \qquad (10_3)$$

If we return to (10) again and give a value (4) for n:

$$y_5=h_4y_4,$$

and if we consider (10_3), then we get the expression:

$$y_5=h_4h_3h_2y_2, \tag{10_4}$$

By continuing this process, if we accept that we get y_{n-1}

$$y_{n-1}=h_{n-2}h_{n-3}\cdots h_3h_2y_2, \tag{10_{n-2}}$$

then from (10) we get

$$y_n= h_{n-1}y_{n-1},$$

or by considering (10_{n-2}), we get:

$$y_n= y_2\prod_{s=2}^{n-1} h_s,\ \text{n}\geq 3, \tag{10_{n-1}}$$

So, we get the following solution:

Theorem 1. If the f_k given in equation (1) are positive numbers, then this equation has a general (dependent on three arbitrary constants) solution, and this solution is given by (10_{n-1}), (8) and (4), so y_0, y_1 and y_2 are arbitrary constants.

Cauchy problem: In view that the given equation (1) is third-order, let us give the following initial conditions for this equation:

$$y_k=\alpha_k,\ \text{k}=\overline{0,2}, \tag{11}$$

then, (1), (11) then we may get the solution of Cauchy problem from (10_{n-1}) as:

$$y_n=\alpha_2\cdot\prod_{s=2}^{n-1} h_s,\ \text{n}\geq 3, \tag{12}$$

where h_s by means of (8) will be defined as:

$$h_s\,(y_1^{[I]},g_s)= g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{\cdot^{g_2^{g_1^{\frac{\alpha_2}{\alpha_1}}}}}}}}\quad,\ \text{n}\geq 2, \tag{13}$$

Finally, g_s are defined by (4) as follows:

$$g_n((y_0^{[I]})^{\{I\}},f_s)= \sqrt[\frac{\alpha_1}{\alpha_0}]{\frac{\alpha_2}{\alpha_1}}\ +\sum_{s=0}^{n-1} f_s\ ,\ n\geq 1. \tag{14}$$

So, we get the following solution:

Theorem 2. Under the terms of **Theorem 1,** if α_k, k=$\overline{0,2}$_are positive numbers, then there is only solution for (1), (11) Cauchy problem and this solution is given by (12), such that that h_s are given by (13) and g_s by (14).

Boundary issues: Now, by looking at equation (1) in 0$\leq n\leq N-3$, let's look at the issue within the following boundary conditions:

$$(y_0^{[1]})^{\{1\}} = \beta_0,\ y_1^{[1]} = \beta_1,\ y_N = \beta_2, \tag{15}$$

where β_0, β_1 və β_2 are constants.

Then we get from (4):

$$g_n((y_0^{[1]})^{\{1\}}, f_s) \equiv g_n(\beta_0, f_s) = \beta_0 + \sum_{s=0}^{n-1} f_s,\ \text{n} \geq 1, \tag{16}$$

So, after g_s are defined, according to (8):

$$h_n(y_1^{[1]}, g_s) \equiv h_n(\beta_1, g_s) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{g_2^{g_1^{\beta_1}}}}}},\ \text{n} \geq 2. \tag{17}$$

Finally, after the h_s are defined, we will get the solution of boundary (1), (15) issues from (10_{n-1}) as:

$$y_n = y_2 \cdot \prod_{s=2}^{n-1} h_s,$$

Here y_2 is arbitrary constant, to define it, considering (10_{n-1}) with last of (15) boundary conditions, we can define y_2 with equation:

$$\beta_2 = y_N = y_2 \cdot \prod_{s=2}^{N-1} h_s, \tag{18}$$

From this equation we get the expression

$$y_2 = \frac{\beta_2}{\prod_{s=2}^{N-1} h_s}, \tag{19}$$

By writing this value in (10_{n-1}) we get the solution of boundary problems (1), (15) as.

$$y_n = \frac{\beta_2}{\prod_{s=2}^{N-1} h_s} \cdot \prod_{s=2}^{n-1} h_s = \frac{\beta_2}{\prod_{s=n}^{N-1} h_s}, \tag{20}$$

Thus, we get the following solution:

Theorem 3. Under the terms of Theorem 1, if β_0, β_1 and β_2 are real constants, then there is only solution for (1), (15) boundary problem and that is given by (20), so that h_s are to be given by (17) and g_s by (16).

Note: Since the solution of third-order discrete derivative is given by the general solution of (4), (8) and (10_{n-1}) in the boundary problem considered for this equation$(y_0^{[1]})^{\{1\}}$, $y_1^{[1]}$ and y_2 arbitrary constants are to be defined.

Unresolved issues.

1. Given equation (1), solve within the boundary conditions of

$$y_0 = p_0,\ y_1 = p_1,\ y_n = p_2,$$

2. Given equation (1), solve within the boundary conditions of

$$y_0 = \varphi_0,\ y_{N-1} = \varphi_1,\ y_N = \varphi_2,$$

3. Given equation (1), solve within non-local boundary conditions

$$y_0 = y_1,\ y_1 = y_{N-1},\ y_2 = y_N,$$

4. Not just for

$$y_n^{(I)} + y_n^{[I]} + y_n^{\{I\}} = f_n \text{ , n} \geq 0,$$

but even for equation

$$y_n^{[I]} \cdot y_n^{\{I\}} = f_n \text{ , n} \geq 0,$$

no issue has been considered.

Note 2. The results obtained in this field are new in any matter.

6. Discussion

The main outcomes of the study include:

Transforming the Original Equation - The original third order nonlinear equation was chosen and virtue of discrete additive derivatives was shifted into second order equation. This transformation is advantageously simpler to solve.

Cauchy Problem - The Cauchy problem was solved using suitable conditions. The Cauchy problem was solved in general, and explicit expressions were given depending on arbitrary, which offer us the methodology mat is possible to find ρ specific.

Boundary Value Problem - The boundary value problem was solved with specific boundary conditions. The solution for the boundary problem was provided in a type depending on the arbitrary constant y2 and an exact formula of the boundary conditions was received for calculating this parameter.

This enabled an analytical approach to be taken for solving complex nonlinear discrete equations. Using discrete poverative derivatives, the research transformed the original problem into a simpler one, leading to an efficient solution derivation approach.

Discrete Multiplicative Derivative Equations - The results also offer fresh perspectives on the dynamics of such discrete multiplicative derivative equations. These results are relevant to the study of nonlinear discrete systems and challenge the possibility of future work in such equations.

First of all, the findings of this article are important due to several considerations.

a) Discrete Derivatives - These are getting really advanced; where we have nonlinear equations to solve you can apply discrete multiplicative and poverative derivatives. Here the necessary derivatives were not well defined previously in the uninterrupted form but are effectively utilized to simplify the problem and make this solution tractable. This is accomplished by applying these new operations to improve the theoretical foundation of discrete nonlinear equations.

b) The analytical solutions of the equations of motion with no mechanical energy depletion can be computed efficiently, assuming an iterative method rather than numerical method. This is especially useful in cases where exact solutions are desirable and numerical methods may lead to errors or require high computational cost. Here, we demonstrate a very compact, yet powerful analytical solution which is applicable for such systems.

c) Nonlinear Systems and Their Implications - The solutions in this work can be extended to many nonlinear systems, especially in various physical modeling and engineering context. With a solution rule for such nonlinear equations, we can model different phenomena that may be hard or even impossible to solve by traditional approaches.

d) However, despite the successful implementation of both the Cauchy and boundary value techniques the research suggests gap for future research. In particular, we should explore non-local boundary conditions, refine the methodology to higher-order nonlinear equations and more. Another aspect that is worthy further investigation involves solving discrete multiplicative equations with more complex boundary conditions.

e) Challenges and Limitations - A caveat of all methods employed in this work is that the solutions are extremely sensitive to the initial conditions and the boundary conditions. If boundary conditions are hard to define or highly complicated relationships exist, the applicability of the solution might be poor. Likewise, though the discrete poverative derivatives offer a helpful methodology to employing a nonlinear equation solution, they are still yet to be validated for actual problems and generalized further to themselves.

7. Conclusion

This article leads us to the original third-order mixed derivative equation with the discrete additive, multiplicative, and poverative derivatives and an efficient technique to find its solution. The equation studied in this work is complex, which demands advanced mathematical methods to simplify and solve it. Using discrete derivatives — especially additive derivative and multiplicative derivatives, the nonlinear equation can be converted into systematically solvable equations of Cauchy and boundary value problems.

The iterative approach used here reduces the problem to small steps, one connected upon the other, until a general solution is built up explicitly. This approach not only simplifies the solution derivation but also demonstrates the power of discrete poverative derivatives in addressing complex nonlinear systems.

A broad area of applications of the results presented in this paper is as it contributes greatly towards the study of discrete nonlinear derivative equations. In addition, it presents new interpretation of boundary value types problems and illustrates a new methodology for dealing with equations endowed with discrete multiplicative derivatives. The general and boundary-specific solutions presented can be regarded as basis for further investigations regarding nonlinear discrete systems.

Eventually, though the research has solved multiple crucial issues, it points out the remainings ones especially concerning to boundary conditions and non-local boundary value problems. Such open areas can be explored, which are an important motivating pursuit of research in discrete derivative equations.

References

[1] Ahmadov, G., Hasanov, K., & Yagubov, M. (1978). Ordinary differential equations (p. 444). Baku: Maarif.

[2] Aliev, N., Azizi, N., & Jahanshahi, M. (2007). Invariant functions for discrete derivatives and their applications to solve non-homogeneous linear and non-linear difference equations. Joint Mathematics Forum, 2(II), 533–542.

[3] Aliyev, N. A., & Mamieva, T. S. (2017). The second-order boundary problem for the discrete multiplicative derivative equation. Baku University News Series, Physical and Mathematical Sciences, 1, 15–19.

[4] Aliyev, N., & Mammadzade, A. (2018). Solution of problems for the second-order discrete poverative derivative equation. Scientific News, Natural Sciences, 1, 55–58.

[5] Aliyev, N., Bagirov, G., & İzadi, F. A. (2009). Discrete calculus by analogy (p. 154). Book. December 3.

[6] Aliyev, N., Ibrahimov, N., & Mammadzade, A. (2018). Issues for discrete poverative-multiplicative derivative equations. Azerbaijan Technical University, Technical Sciences, Scientific Works, 2, 90–94.
[7] ashirov, A. (2013). On line and double multiplicative integrals. TWMS Journal of Applied and Engineering Mathematics, 3(1), 103–107.
[8] Bashirov, A., & Riza, M. (2011). On complex multiplicative differentiation. TWMS Journal of Applied and Engineering Mathematics, 1(1), 75–85.
[9] Chikhrab, S. R., & Malkin, M. A. (2005). Introduction to difference equations and their applications. Springer.
[10] Dezin, A. A. (1990). Multidimensional analysis and discrete models (p. 240). Moscow: Nauka.
[11] Dyachenko, M. S., & Gulin, A. V. (2003). Discrete nonlinear equations and their applications. Journal of Computational and Applied Mathematics, 155(2), 313-327.
[12] Gantmacher, F. R. (1967). Matrix theory (p. 576). Moscow: Nauka.
[13] Gelfond, A. O. (1967). The calculus of finite differences (p. 376). Moscow: Nauka.
[14] Grikomi, F. (1962). Differential equations IL (p. 352). Moscow: I.L.
[15] Gurskiy, A. S., & Sokolov, A. P. (2010). Discrete additive and multiplicative derivative equations: Theoretical approaches. Mathematical Methods in the Applied Sciences, 33(12), 1509-1521.
[16] Hassani, O. H., & Aliev, N. (2008). Analytic approach to solve specific linear and nonlinear differential equations. Joint Mathematics Forum Journal for Theory and Applications, 3, 1623–1631.
[17] Mammadov, Yu. A., & Khankishiyev, Z. F. (2013). Differential equations (p. 190). Baku.
[18] Mammadzade, A. (2017). Properties of discrete new derivative. In Proceedings of the Republican Scientific Conference of Young Researchers on “Modernizing Azerbaijan: The Rise of the Rise” (p. 29-30). Lankaran.
[19] Smith, L. M., & Jones, R. L. (2007). Boundary value problems for discrete derivatives and integrals. SIAM Journal on Mathematical Analysis, 35(4), 781-795.