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Inventory Optimization with Ramp-Type Demand, Time-Dependent Holding Costs, and Discounted Backorders under Inflation

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Abstract:

This study presents an inventory model with a ramp-type demand pattern, where holding costs increase over time, and inflation is taken into account. The model is particularly relevant for inventories like seasonal produce and newly launched fashion items. When stock runs out, the inventory manager offers a discount to customers willing to backorder their demand. The goal is to determine the optimal ordering policy and backorder discount by minimizing the total cost across a replenishment cycle. Finally, numerical results are provided to assess how the optimal policies respond to changes in key system parameters. Graphs are used to visualize the impact of these parameters on the total inventory cost, enhancing the understanding of the model.

Keywords: price discount on backorder, ramp-type demand, shortage, inflation, time dependent holding cost.

1. Introduction

The demand rates are considered to be constant in classical inventory models, but in actuality, time is a significant factor in the inventory system. When the demand rate is ramp type, it rises gradually until it reaches a particular limit, at which point it stabilizes and reaches a constant level. This type of demand commonly occurs with the launch of new consumer goods like mobile phones, clothing, automobiles, fashion items and cosmetics. Hill [7] was the first to investigate inventory models for scenarios with an initial increase in demand, followed by a constant demand rate. Mandal and Pal [10] expanded on Hill's inventory model to include deteriorating items and allowed shortages. Panda et al. [18] presented optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. Mandal [8] developed an EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Chandra [4] introduced an inventory model with ramp type demand, time varying holding cost and price discount on backorders. Saha et al. [19] proposed inventory model with ramp-type demand and price discount on back order for deteriorating items under partial backlogging. Chandra studied [3] two warehouse inventory model for deteriorating Items with ramp type demand and price discount on backorders. Mondal et al. [11] discussed an EOQ model for seasonal product with ramp-type time and stock dependent demand, shortage and partial backorder.

In most models, holding cost is assumed to be known and constant. However, this is not always the case. For example, in the case of seasonal fruits and vegetables, extended storage time requires more advanced storage facilities and services which leads to higher holding costs. Muhlemann and Valtis-Spanopoulous [13] were the first to introduce variable holding costs, developing an EOQ model with constant demand in which the holding cost is a percentage of the average inventory value. Goh [6] explored two types of holding cost variations: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level. Alfares [1] introduced an inventory system with stock-dependent demand, where the holding cost is a step function of storage time, considering two types of holding cost variation based on storage time: retroactive increase and incremental increase. Palanivel and Suganya [19] suggested an inventory model with partial backlogging, demand depend on price and stock level, time varying holding cost and quantity discounts. Sharma and Sharma [21] discussed on the effect of inflation on a deteriorating inventory model with non-linear holding cost and non-linear demand.

Unmet demand is usually considered either completely lost or fully backlogged in inventory models with shortages. Therefore, in practice, some customers may choose to wait for their orders to be fulfilled while others might abandon their purchase. In such cases, inventory managers may implement strategies like offering discounts on backorders or reducing waiting times to encourage customers to wait patiently for their desired items. A continuous review inventory model with negotiable backorders and order quantity as decision variables was presented by Pan and Hsiao [17]. In order to account for more realistic elements of the actual inventory systems, Ouyang et al. [14] created a periodic review inventory model with backorder discounts. Uthayakumar and Parvathi [23] have studied an inventory model with mixture of backorders involving reducible lead time and setup cost. Pal and Chandra [15] investigated a deterministic inventory model with permissible delay in payment and price discount on backorders. Pal and Chandra [16] developed a periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders. Chauhan and Tayal [5] suggested an order quantity scheme for ramp type demand and backlogging during stock out with discount strategy. Mrudul et al. [12] introduced Optimal pricing policy for deteriorating items with continuous compounding under price-sensitive demand and shortages. Kumar P et al. [8] studied a two-warehouse inventory system with time-dependent demand and preservation technology.

In today's business environment, inflation is a significant concern. It represents a persistent increase in the overall price levels of goods and services over time, which leads to reduction in the purchasing power of money. High inflation rates can significantly decrease the purchasing power of businesses. Consequently, when constructing any inventory models, it is crucial to consider for the impact of inflation. Buzacott [2] was among the first to incorporate inflation rates into inventory modelling alongside different pricing strategies. Under the impact of inflation. Economic ordering policy with price and advertisement dependent demand and allowable payment delay under inflation was described by Udayakumar et al. [22] for non-instantaneously deteriorating items.

This paper introduces an inventory model where holding cost increases as linear increase function of time, and the demand rate is represented as a ramp type demand. If the consumer is willing to backorder his demand during a stock-out, the manager offers him a discount. Additionally, the effects of inflation are also incorporated into the model. Numerical results are provided to examine the impact of change in various system parameters on the sensitivity of the optimal policies. Graphical representations clarify the relationships between various parameters and the total inventory cost, thereby enhancing the understanding of the inventory models.

2. Assumptions and Notations:

The following assumptions and notations are used in the development of the model.

2.1 Notations

I(t) = The inventory quantity at a given time t.

b = The fraction of demand fulfilled through backorders when there is a stockout.

P = The cost of purchasing each unit

s_1= The backorder cost per unit on backorder for each time period

s_2 = Cost associated with a lost sale

T = duration of a cycle of replenishing

T_1= time it takes for the available goods to be sold out, $0 < T_1 < T$

S_r = maximum height of stock during a replenishing cycle

s = The inventory shortage at the end of the cycle of replenishment

2.2 Assumptions

- A single inventory item is taken consideration by the model.
- There is no lead time because inventory is replenished instantly upon ordering.
- Shortages are permitted, with b fraction of unfulfilled demands during stockouts being backlogged.
- The holding cost per item per unit time, denoted as h(t), is considered depend on time

$h(t) = h + \alpha t$ where $\alpha > 0$, $b > 0$

- It is assumed that the demand rate R(t) is a ramp-type function of time t.

$$R(t) = D_0[t - (t - \mu)H(t - \mu)]$$

Where D_0 and μ are positive constants and $H(t-\mu)$ is the Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & \text{for } t \geq \mu \\ 0 & \text{for } t < \mu \end{cases}$$

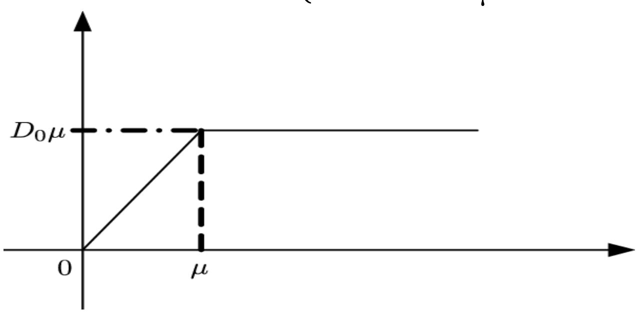

Figure (1): The ramp type demand rate

- The time required for available stock to run out (T_1) is more than μ.

- The fraction b of backorder during the stock out period is directly proportional to the price discount provided by the inventory manager.

3. Mathematical model:

The planning period consists of reorder intervals, each of length T units, with orders placed at T, 2T, 3T, and so on, with the order amount being only enough to raise the stock height to a maximum level S_r. Demand depletes inventory during the times $T_1 < T$ and $(0, T_1)$, and shortages arise during the interval (T_1, T), with a fraction b being backlogged. Therefore, the change in inventory level over time is described by

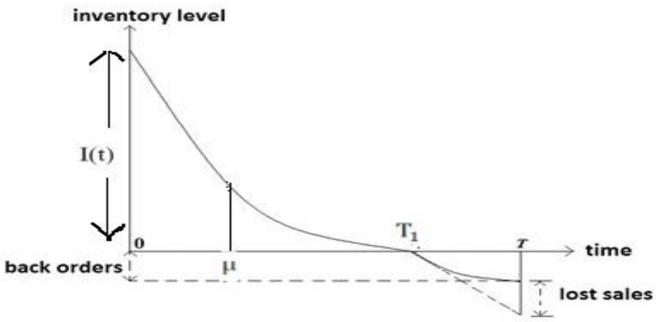

Figure (2): Graphical representation of Inventory model

$$\frac{d}{dt}I(t) = -D_0 t \qquad \text{if } 0 < t < \mu$$

$$\frac{d}{dt}I(t) = -D_0 \mu \qquad \text{if } \mu < t < T_1$$

$$\frac{d}{dt}I(t) = -bD_0 \mu \qquad \text{if } T_1 < t < T$$

With boundary conditions $I(0) = S_r$ and $I(T_1) = 0$,

The solutions to the differential equation for the three different cases (I, II, III) are as follows:

Case I: $0 < t < \mu$

$$I(t) = \frac{D_0 t^2}{2} + S_r \qquad \ldots (1)$$

Case II: $\mu < t < T_1$

$$I(t) = D_0 \mu (T_1 - t) \qquad \ldots (2)$$

Case III: $T_1 < t < T$

$$I(t) = bD_0 \mu (T_1 - t) \qquad \ldots (3)$$

Hence,

$$S_r = \frac{D_0 \mu}{2}(2T_1 - \mu) \qquad \ldots (4)$$

$$s = bD_0 \mu (T - T_1) \qquad \ldots (5)$$

Then,

Ordering cost per cycle (OC) =A

Holding cost for a cycle (HC) $= \int_0^{T_1} h(t)\, I(t)dt$

$$HC = \int_0^{\mu} h(t)\, I(t)dt + \int_{\mu}^{T_1} h(t)\, I(t)dt$$

$HC = \frac{D_0\mu}{r^2}[e^{-rT_1} + e^{-r\mu}(T_1 r - 1 - r\mu)] - \frac{hD_0}{2r^3}[2 - e^{-r\mu}(r^2\mu^2 + 2r\mu + 2) - \mu r^2(e^{-r\mu} - 1)(\mu + 2T_1)] - \frac{aD_0}{2r^4}\left[\{6 - e^{-r\mu}(r^3\mu^3 + 3r^2\mu^2 + 6r\mu + 6)\} + r^2\mu^2\{1 - e^{-r\mu}(r\mu + 1)\} - 2r\mu\left\{e^{-rt}\left(r^2{T_1}^2 + 2rT_1 + 2\right) + e^{-r\mu}r^2\mu^2 + 2r\mu + 2\right\} + 2T_1\mu r^2\{e^{-rT_1}(rT_1 + 1) - 1\}\right]$ … (6)

The backorder cost, at the end of a cycle $= -s_1 \int_{T_1}^{T} I(t)dt$

$BC = \frac{s_1 b D_0 \mu}{r^2}(T_1 r e^{-rT} - rTe^{-rT} - e^{-rT} + e^{-rT_1})$ … (7)

Lost sales cost for a cycle (LS) $= s_2 D_0 \mu(1 - b)(T - T_1)$ … (8)

Purchase cost for a cycle (PC) $= P\left(\frac{D_0\mu}{2}(2T_1 - \mu) + bD_0\mu(T - T_1)\right)$ … (9)

Consequently, the cost per replenishment cycle unit is represented by

$$TC = \frac{1}{T}[OC + HC + BC + LC + PC]$$

$TC = A + \frac{D_0\mu}{r^2}[e^{-rT_1} + e^{-r\mu}(T_1 r - 1 - r\mu)] - \frac{hD_0}{2r^3}[2 - e^{-r\mu}(r^2\mu^2 + 2r\mu + 2) - \mu r^2(e^{-r\mu} - 1)(\mu + 2T_1)] - \frac{aD_0}{2r^4}\left[\{6 - e^{-r\mu}(r^3\mu^3 + 3r^2\mu^2 + 6r\mu + 6)\} + r^2\mu^2\{1 - e^{-r\mu}(r\mu + 1)\} - 2r\mu\left\{e^{-rt}\left(r^2{T_1}^2 + 2rT_1 + 2\right) + e^{-r\mu}r^2\mu^2 + 2r\mu + 2\right\} + 2T_1\mu r^2\{e^{-rT_1}(rT_1 + 1) - 1\}\right] + \frac{s_1 b D_0\mu}{r^2}(T_1 r e^{-rT} - rTe^{-rT} - e^{-rT} + e^{-rT_1}) + s_2 D_0\mu(1 - b)(T - T_1) + P\left(\frac{D_0\mu}{2}(2T_1 - \mu) + bD_0\mu(T - T_1)\right)$ … (10)

4. Numerical Example:

Using the statistical programme MATLAB, we numerically solve the equations for given sets of costs. $A = 500, a = 0.6, P = 5, x = 0.25, s = 6, r = 0.05, D = 100, h = 3, s_2 = 7$ and $b = 0.7$

We obtained the following optimum values

t =3.6354, T = 5.65794 and Total Cost= ₹ 300.04158

5. Sensitive Analysis:

Sensitivity analysis is given by taking one parameter at a time, increasing or reducing it by 10% and 20% while maintaining the original values of the remaining parameters. The results can be shown in the table.

Table (1): Variation in system parameters

Parameters	% Change	Value	T_1	T	TC
A	-20%	400	3.3152	5.0480	281.3582
	-10%	450	3.4810	5.3607	290.9658
	+10%	550	3.7802	5.9422	308.6623
	+20%	600	3.9171	6.2157	316.8876
α	-20%	0.48	3.8605	5.8494	295.8814
	-10%	0.54	3.7416	5.7478	298.0349
	+10%	0.66	3.5395	5.7765	301.9204
	+20%	0.72	3.4524	5.5053	303.6872
p	-20%	4	3.6766	5.6434	278.2421
	-10%	4.5	3.6561	5.6509	289.1500
	+10%	5.5	3.6143	5.6644	310.9176
	+20%	6	3.5930	5.6703	321.7754
μ	-20%	0.2	4.0034	6.3584	255.9501
	-10%	0.225	3.8067	5.9788	278.2303
	+10%	0.275	3.4835	5.6579	321.4527
	+20%	0.3	3.3469	5.3809	342.5141
s_1	-20%	4.8	3.5519	6.0137	293.0917
	-10%	5.4	3.5971	5.8171	296.8455
	+10%	6.6	3.6683	5.5263	302.7976
	+20%	7.2	3.6968	5.4156	305.2000
r	-20%	0.04	3.5847	5.5099	302.1615
	-10%	0.045	3.6096	5.5821	301.1090
	+10%	0.055	3.6622	5.7378	298.9582
	+20%	0.06	3.6902	5.8222	297.8578
D_0	-20%	80	3.9828	6.3489	256.6936
	-10%	90	3.7958	5.9731	278.6353
	+10%	110	3.4955	5.3883	320.9927
	+20%	120	3.3719	5.1542	341.5506
h	-20%	2.4	3.6443	5.6456	297.7484
	-10%	2.7	3.6443	5.6518	298.8970
	+10%	3.3	3.6263	5.6639	301.1818
	+20%	3.6	3.6172	5.6698	302.3180
s_2	-20%	5.6	3.5866	5.9770	296.2199
	-10%	6.3	3.6132	5.8007	298.1477
	+10%	7.7	3.6540	5.5397	301.9011
	+20%	8.4	3.6698	5.4402	303.7259
b	-20%	0.56	3.5866	5.9770	295.9244
	-10%	0.63	3.6132	5.8007	298.1733

	+10%	0.77	3.6540	5.5397	301.6135
	+20%	0.84	3.6698	5.4402	302.9501

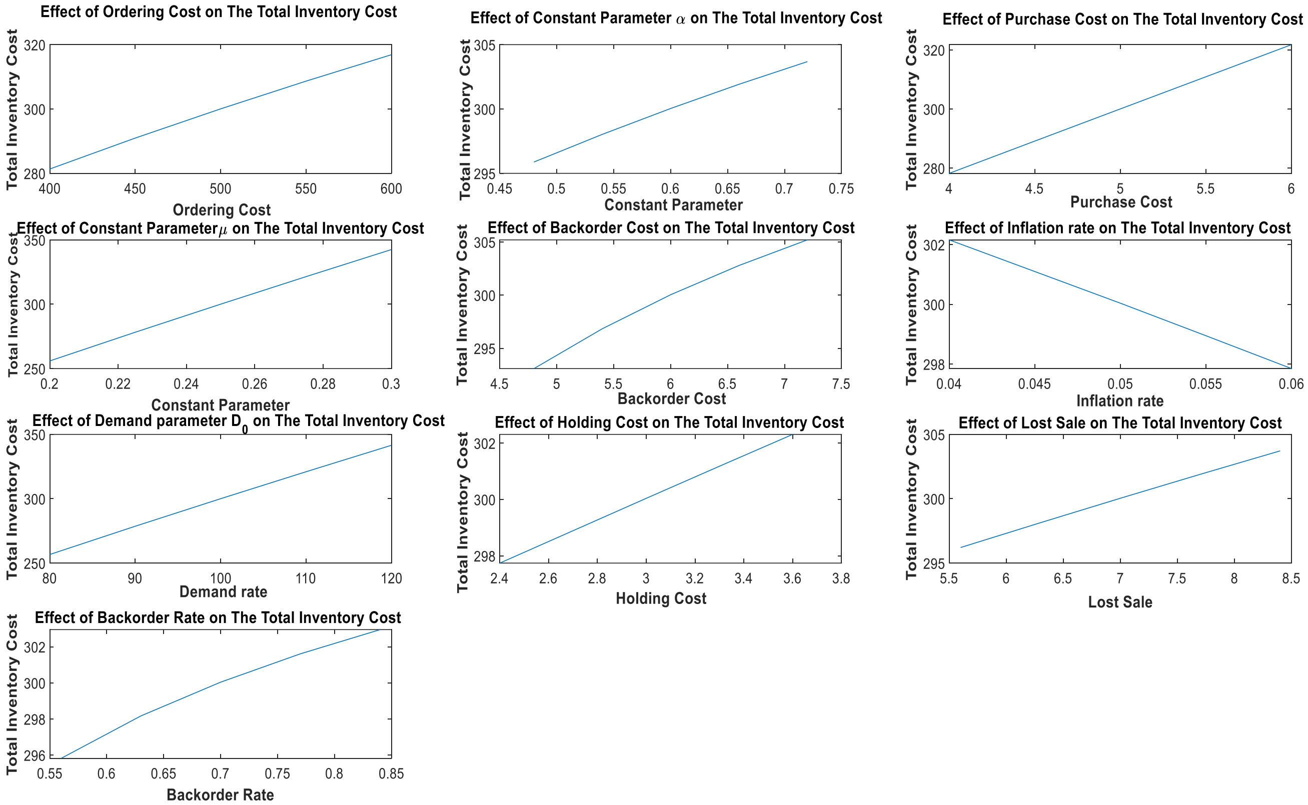

Figure 3: Variation in system parameters table (1)

Table (2): Effect of backorder rate (b) and Inflation (r)

b \ r		0.56	0.63	0.7	0.77	0.84
0.04	T_1	3.5348	3.5621	3.5847	3.6037	3.6199
	T	5.8037	5.641	5.5099	5.4005	5.3081
	TC	298.1047	300.3201	302.1615	303.7112	305.0305
0.045	T_1	3.5602	3.5872	3.6096	3.6284	3.6444
	T	5.8879	5.7191	5.5621	5.4685	5.3726
	TC	297.0230	299.2546	301.1090	302.6697	303.9971
0.05	T_1	3.5866	3.6132	3.6354	3.6540	3.6698
	T	5.9770	5.8007	5.6579	5.5397	5.4402
	TC	295.9244	298.1733	300.0415	301.6135	302.9501
0.055	T_1	3.6042	3.6404	3.6622	3.6805	3.6961
	T	6.0714	5.8869	5.7378	5.6146	5.5110
	TC	294.8075	297.0751	298.9582	300.5421	301.8886
0.06	T_1	3.6432	3.6688	3.6902	3.7082	3.7234
	T	6.1718	5.9782	5.8222	5.6935	5.5855
	TC	293.6711	295.9588	297.8578	299.4547	300.8175

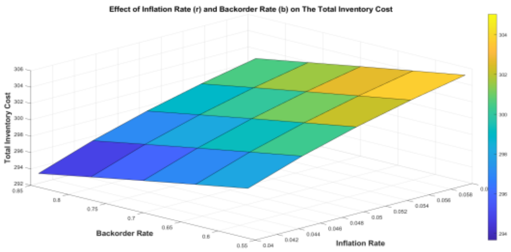

Figure 4: Effect of backorder rate (b) and Inflation (r)

6. Conclusion:

The paper examines an inventory model where demand is considered as ramp-type, inflation and allowing for shortages. The holding cost is assumed to increase linearly over time. This study is relevant to inventories such as seasonal vegetables and fruits, newly launched fashion items, and electronic goods. A portion of the demand is backlogged, and customers who are willing to wait are offered a discount. The goal is to determine the optimal ordering policy and backorder discount that minimizes the total cost over a replenishment interval. Numerical analysis indicates that offering substantial discounts on backorders is beneficial for the inventory manager when backorder costs are low. Sensitivity analysis and numerical examples are used to illustrate the model. When increase parameters A, a, p, b, h, μ, s_1, s_2 and D_0 the total cost increase significantly. When decrease parameter r the total cost decrease marginally.

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