Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 6s (2025)

Homotopy Analysis Method for the Approximate Solution of the SIRC Epidemic Model

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Article History:

Received: *22-10-2024*

Revised: *06-12-2024*

Accepted: *13-12-2024*

Abstract:

This paper investigates the Homotopy Analysis Method (HAM) as a means of approximating a solution to the SIRC epidemic model. This method allows the solution of the governing differential equation to be found as an infinite series with easily calculated components. The HAM uses an auxiliary parameter and a straightforward way to regulate and alter the region where the infinite series solution converges. The outcomes are displayed, and with just six terms, an extremely precise approximation solution can be achieved.

Math. Subject Classification: 34G20; 34A34

Keywords:HAM, SIRC epidemic model, h-curve, Numerical Analysis

1. Introduction

In the study and control of infectious diseases, mathematical models have grown to be crucial tools. In Kermack and McKendrick [1], one of the earliest models in epidemiology was presented to predict how a disease will spread. The total population is separated into three classes in this model: susceptible, infectious, and recovered, with the assumption being that it will remain stable over time. These models are hence known as SIR models. Cross-immune individuals (C) in the population have just recently been introduced in [2], they exist in a state that is between totally protected and unprotected (R). Because of this, the derived SIRC model considers transient partial immunity and might effectively characterise, say, influenza A. This study introduces and develops the homotopy analysis approach for approximately solving the SIRC model. Parameters and variables are presented in Table 1, and the model is in Table 1, and the model is

$$\begin{aligned}
\dot{S} &= \eta(1-S) - \xi SI + \beta C \\
\dot{I} &= \xi SI + \sigma\xi CI - (\eta+\alpha)I \\
\dot{R} &= (1-\sigma)\xi CI + \alpha I - (\eta+\gamma)R \\
\dot{C} &= -\xi CI - (\beta+\eta)C + \gamma R
\end{aligned} \tag{1}$$

Table I. Values of parameters and variables

Parameters and Variables	Meaning
η	Death amount in every division presumed to be equal to the amount of new born in the population
β	Amount of re-susceptibility of the cross-immune population
ξ	Rate of contact
σ	Average probability of reinfection of cross-immune individuals
α	Regaining amount of the infected population
γ	Amount at which the regaining population to the cross-immune population and from fully immunized to partial immunity
S	Susceptible
I	Infected
R	Recovered
C	Cross-immune

Rihan et al., [3] studied the fractional SIRC model with salmonella bacterial infection. Amjad et al., [4] have studied the numerical simulation of SIRC model of fractional order derivative. Geethamalini et al. [5] have published numerical and analytical study of sirc epidemical model using HPM.Ghoreishi et., al. [6] have studied the HAM for solving for CD4+ T-cells of HIV infection. The authors was published a semi-analytical solutions of mathematical models in EIAV (Equine Infectious Anemia Virus) infection using HAM. Also the authors in [7–9] were established an approximate solutions and dynamical analysis of an EIAV infection . Alijhani et al. [10] were studied the numerical solution of Fractional-order HIV model using HAM. Naik et al. [11–13] have studied the utilizing the homotopy analysis method to estimate the approximate analytical solution of the HIV viral dynamic model of CD4+ T-cells.

(Odibat and Baleanu, [14] studied the linearization based approach of HAM for nonlinear time fractional parabolic PDEs. Saad et al. [15] studied the exact solutions for time fractional Burger's equations using HAM. Yepez and Gomez, [16] was introduced, an updated explanation of the Caputo Fabrizio fractional-order derivative, including with its applications to the multistep HAM Deniz, [17]have studied the Semi-analytical approach for solving a model for hiv infection. In recent years, different nonlinear systems of differential equations inmathematics and the sciences have been solved using HAM as a solution technique [18–21]. The HAM was first developed by [22–24]. Chioma et al. [25] studied the Application of Homotopy Analysis method for solving an SEIRS epidemic Model. Abdi-Rahim et al. [26] studied the Analytical Study of Fractional Epidemic Model via Natural Transform Homotopy Analysis Method. The smoking pandemic model with fractional order was investigated by Veeresha et al. [27] using a modified homotopy analysis transforms approach. Bakare et al. [28] studied the Interval-based uncertain SIR epidemic model numerically solved using HAM. Duarte et al. [29] studied the Chaos analysis of SIR epidemic Method.

2. Solution of the SIRC model by the HAM

In order to develop the HAM solutions to (1), first we select

$$S(0)=S_0,\ I(0)=I_0\ ,R(0)=R_0\ ,C(0)=C_0\ . \quad (2)$$

The auxiliary linear operators $L_1, L_{2,}$, and L_3 L_4 are selected as

$$L_1[S(t,s)]=\frac{dS(t,s)}{dt},$$

$$L_2[I(t,s)]=\frac{dI(t,s)}{dt},$$

$$L_3[R(t,s)]=\frac{dR(t,s)}{dt}.$$

$$L_4[C(t,s)]=\frac{dC(t,s)}{dt}.$$

Which satisfy the following properties:

$L_i(\bar{C}_i)=0$, where $\bar{C}_i(i=1,2,3,4)$ are integral constants. Describe

$$N_1[S,I,R,C]=\dot{S}-\eta(1-S)+\xi SI-\beta C,$$

$$N_2[S,I,R,C]=\dot{I}-\xi SI-\sigma\xi CI+(\eta+\alpha)I,$$

$$N_3[S,I,R,C]=\dot{R}-(1-\sigma)\xi CI-\alpha I+(\eta+\gamma)R$$

$$N_4[S,I,R,C]=\dot{C}+\xi CI+(\beta+\eta)C-\gamma R$$

Introduce nonzero auxiliary parameter h and nonzero auxiliary function using Liao's definitions. The embedding parameter is H (t) and s ∈ [0, 1]. We create the zero-order deformation equations using this.

$$(1-s)L_1[S(t;s)-S_0(t)]=sh_1H_1(t)N_1[S,I,R,C], \quad (3)$$

$$(1-s)L_2[I(t;s)-I_0(t)]=sh_2H_2(t)N_2[S,I,R,C], \quad (4)$$

$$(1-s)L_3[R(t;s)-R_0(t)]=sh_3H_3(t)N_3[S,I,R,C], \quad (5)$$

$$(1-s)L_4[C(t;s)-C_0(t)]=sh_4H_4(t)N_4[S,I,R,C], \quad (6)$$

clearly, when s=0 and s=1, then

$$S(t;0)=S_0(t),\quad S(t;1)=S(t),$$

$$I(t;0)=I_0(t),\quad I(t;1)=I(t),$$

$$R(t;0) = R_0(t), \quad R(t;1) = R(t).$$

$$C(t;0) = C_0(t), \quad C(t;1) = C(t).$$

When a result, as s the embedding parameter increases from 0 to 1, the solutions S (t; s), I (t; s) ,R (t; s) & C(t;s) varies continuously from S_0 (t), I_0 (t) , R_0 (t) & C_0((t) to the exact solution S (t), I (t), R(t) & C (t). Using Taylor's series

$$S(t;s) = S_0(t) + \sum_{i=1}^{\infty} S_i(t)s^i, \tag{7}$$

$$I(t;s) = I_0(t) + \sum_{i=1}^{\infty} I_i(t)s^i, \tag{8}$$

$$R(t;s) = R_0(t) + \sum_{i=1}^{\infty} R_i(t)s^i, \tag{9}$$

$$C(t;s) = C_0(t) + \sum_{i=1}^{\infty} C_i(t)s^i, \tag{10}$$

where

$$S_i = \frac{1}{i!}\frac{\partial^i S(t;s)}{\partial s^i}|_{s=0},$$

$$I_i = \frac{1}{i!}\frac{\partial^i I(t;s)}{\partial s^s}|_{s=0},$$

$$R_i = \frac{1}{i!}\frac{\partial^i R(t;s)}{\partial s^i}|_{s=0},$$

$$C_i = \frac{1}{i!}\frac{\partial^i C(t;s)}{\partial s^i}|_{s=0},$$

If h_1, h_2, h_3,h_4, $H_1(t)$,$H_2(t)$, $H_3(t)$ and $H_4(t)$ are selected, and the series converges at p=1.

$$S(t) = S_0(t) + \sum_{i=1}^{\infty} S_i(t),$$

$$I(t) = I_0(t) + \sum_{i=1}^{\infty} I_i(t),$$

$$R(t) = R_0(t) + \sum_{i=1}^{\infty} R_i(t).$$

$$C(t) = C_0(t) + \sum_{i=1}^{\infty} C_i(t).$$

The ith-order deformation equations are obtained by differentiating (3)–(6) 'i' times with regard to s, allocating by i! and setting s = 0.

$$L_1[S_i(t) - \chi_i S_{i-1}(t)] = h\bar{R}_{1,i}(S_{i-1}(t)), \tag{11}$$

$$L_2[I_i(t) - \chi_i I_{i-1}(t)] = h\bar{R}_{2,i}(I_{i-1}(t)), \tag{12}$$

$$L_3[R_i(t) - \chi_i R_{i-1}(t)] = h\bar{R}_{3,i}(R_{i-1}(t)), \tag{13}$$

$$L_4[C_i(t)-\chi_i C_{i-1}(t)] = h\bar{R}_{4,i}(C_{i-1}(t)), \tag{14}$$

where

$$\bar{R}_{1,i}(t) = \frac{dS_{i-1}(t)}{dt} + \xi\sum_{j=0}^{i-1} S_j(t)I_{i-1-j}(t) + \eta S_{i-1}(t) - \beta C_{i-1}(t) - (1-\chi_i)\eta,$$

$$\bar{R}_{2,i}(t) = \frac{dI_{i-1}(t)}{dt} - \xi\sum_{j=0}^{i-1} S_j(t)I_{i-1-j}(t) - \sigma\xi\sum_{j=0}^{i-1} C_j(t)I_{i-1-j}(t) + (\eta+\alpha)I_{i-1}(t),$$

$$\bar{R}_{3,i}(t) = \frac{dR_{i-1}(t)}{dt} - (1-\sigma)\xi\sum_{j=0}^{i-1} C_j(t)I_{i-1-j}(t) - \alpha I_{i-1}(t) + (\eta+\gamma)R_{i-1}(t),$$

$$\bar{R}_{4,i}(t) = \frac{dC_{i-1}(t)}{dt} + \xi\sum_{j=0}^{i-1} C_j(t)I_{i-1-j}(t) + (\beta+\eta)\,C_{i-1}(t) - \gamma R_{i-1}(t),$$

&

$$\chi_i = \begin{Bmatrix} 1 & i>1 \\ 0 & i\le 1 \end{Bmatrix}.$$

When i is higher than or equal to 1, ith-order deformation (11)–(14) becomes

$$S_i(t) = \chi_i S_{i-1}(t) + h\int_0^t \bar{R}_{1,i}(\tau)d\tau,$$

$$I_i(t) = \chi_i I_{i-1}(t) + h\int_0^t \bar{R}_{2,i}(\tau)d\tau,$$

$$R_i(t) = \chi_i R_{i-1}(t) + h\int_0^t \bar{R}_{3,i}(\tau)d\tau.$$

$$C_i(t) = \chi_i C_{i-1}(t) + h\int_0^t \bar{R}_{4,i}(\tau)d\tau.$$

3. Numerical Simulations

Take into account the values below for the numerical outcomes [3].

$$S_0 = 0.3,\ I_0 = 0.5,\quad R_0 = 0, C_0 = 0.6$$

$$\xi = 1.3, \eta = 0.09, \gamma = 0.1, \beta = 0.05, \alpha = 0.36, \sigma = 0.9$$

We obtain the sixth-order for using the Mathematica software. $S(t), I(t), R(t)$ *and* $C(t)$ were obtained, and are given below

$$S(t) = 0.3 + 0.612ht + 1.53h^2t + 2.04h^3t + 1.53h^4t + 0.612h^5t + 0.102h^6t - 0.550575h^2t^2 - 1.4682h^3t^2 - 1.65173h^4t^2 - 0.88092h^5t^2 - 0.183525h^6t^2 - 0.667932h^3t^3 - 1.50285h^4t^3 - 1.20228h^5t^3 - 0.333966h^6t^3 + 0.0627671h^4t^4 + 0.100427h^5t^4 + 0.0418447h^6t^4 + 0.0461811h^5t^5 + 0.0384842h^6t^5 - 0.00050418h^6t^6 + \ldots \tag{15}$$

$$I(t) = 0.5 - 1.926ht - 4.815h^2t - 6.42h^3t - 4.815h^4t - 1.926h^5t - 0.321h^6t - 1.03131h^2t^2 - 2.75016h^3t^2 - 3.09393h^4t^2 - 1.6501h^5t^2 - 0.34377h^6t^2 + 1.63923h^3t^3 - 3.68826h^4t^3 - 2.95061h^5t^3 + 0.819613h^6t^3 + 0.2508h^4t^4 + 0.40128h^5t^4 + 0.1672h^6t^4 - 0.115992h^5t^5 + 0.09666h^6t^5 - 0.00467747h^6t^6 + ... \quad (16)$$

$$R(t) = 0.001 - 1.314ht - 3.285h^2t - 4.38h^3t - 3.825h^4t - 1.314h^5t - 0.219h^6t + 0.511335h^2t^2 + 1.36356h^3t^2 + 1.53401h^4t^2 + 0.81813h^5t^2 + 0.170445h^6t^2 + 0.344179h^3t^3 + 0.774402h^4t^3 + 0.619522h^5t^3 + 0.172089h^6t^3 - 0.0790303h^4t^4 - 0.126448h^5t^4 - 0.0526868h^6t^4 - 0.0173387h^5t^5 - 0.0144489h^6t^5 + 0.000654392h^6t^6 + ... \quad (17)$$

$$C(t) = 0.6 + 2.844ht + 7.11h^2t + 9.48h^3t + 7.11h^4t + 2.844h^5t + 0.474h^6t + 1.09484h^2t^2 + 2.9196h^3t^2 + 3.28455h^4t^2 + 1.7516h^5t^2 + 0.36495h^6t^2 - 1.3145h^3t^3 - 2.95763h^4t^3 - 2.3661h^5t^3 - 0.65725h^6t^3 - 0.23452h^4t^4 - 0.375233h^5t^4 - 0.156347h^6t^4 + 0.0871498h^5t^5 + 0.0726248h^6t^5 + 0.00452726h^6t^6 + ... \quad (18)$$

4. Discussion

We found the solution from (11)-(14) which contain "h" demonstrate a simple method Liao suggests for controlling and adjusting curves to validate series solutions converge.

The graphs of the 5th and 6th term approximations of S, I, R &C is shown in Figures 1–5. From these curves it shows the the horizontal axis forms a valid region of ’”h” Table 2 contains a list of the suitable regions. To get the ideal values for h, an error analysis is performed. We enter Eqs (15) through (18) into (1) and

obtain the corresponding residual functions:

$$E\bar{R}_1(S,I,R,C;h_1) = \frac{d\phi_S(t;h_1)}{dt} - \eta(1-S) + \xi\phi_S(t;h_1)\phi_I(t;h_1) - \beta\phi_C(t;h_1) \quad (19)$$

$$E\bar{R}_2(S,I,R,C;h_2) = \frac{d\phi_I(t;h_2)}{dt} - \xi\phi_S(t;h_2)\phi_I(t;h_2) - \sigma\xi\phi_C(t;h_2)\phi_I(t;h_2) + (\eta+\alpha)\phi_I(t;h_2) \quad (20)$$

$$E\bar{R}_3(S,I,R,C;h_3) = \frac{d\phi_R(t;h_3)}{dt} - (1-\sigma)\xi\phi_C(t;h_3)\phi_I(t;h_3) - \alpha\phi_I(t;h_3) + (\eta+\gamma)\phi_R(t;h_3). \quad (21)$$

$$E\bar{R}_4(S,I,R,C;h_4) = \frac{d\phi_C(t;h_4)}{dt} + \xi\phi_C(t;h_4)\phi_I(t;h_4) - (\beta+\eta)\phi_C(t;h_4) + \gamma\phi_R(t;h_4). \quad (22)$$

Based on the following [27-29], for the 6th order approximation, we estimate the square residual error to be

$$\bar{R}S(h_1) = \int_0^1 (E\bar{R}_1(S,I,R,C;h_1))^2 dt, \tag{23}$$

$$\bar{R}I(h_2) = \int_0^1 (E\bar{R}_2(S,I,R,C;h_2))^2 dt, \tag{24}$$

$$\bar{R}R(h_3) = \int_0^1 (E\bar{R}_3(S,I,R,C;h_3))^2 dt, \tag{25}$$

$$\bar{R}C(h_4) = \int_0^1 (E\bar{R}_4(S,I,R,C;h_4))^2 dt. \tag{26}$$

Table 2.

Figures (1)–(8) shows the values of h.

$S(t)$	$-1.4 \le h \le -0.4$
$I(t)$	$-1.3 \le h \le -0.6$
$R(t)$	$-1.3 \le h \le -0.6$
$C(t)$	$-1.4 \le h \le -0.5$

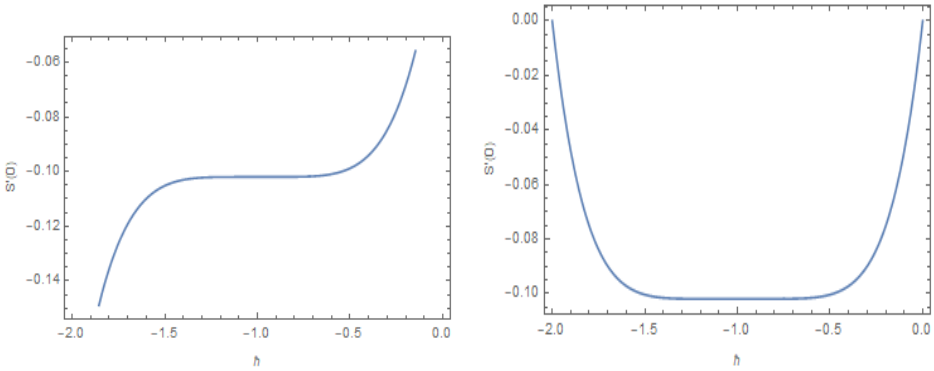


Figure 1: The h-curves of S′(0) obtained by the fifth-order and sixth-order approximations of the HAM

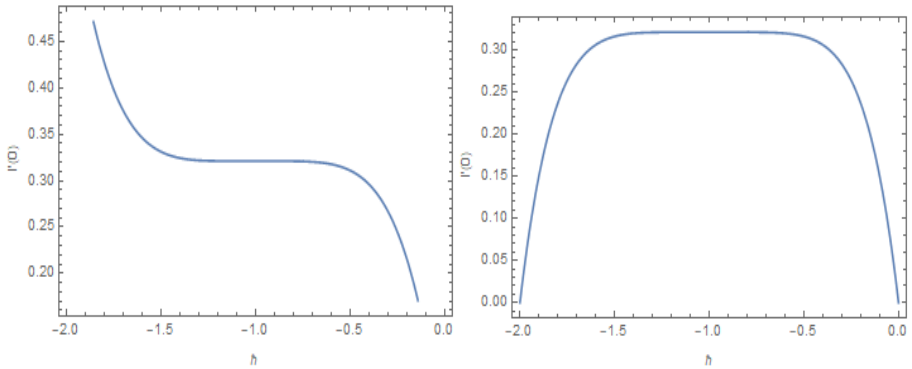


Figure 2: The h-curves of I′(0) obtained by the fifth-order and sixth-order approximations of the HAM.

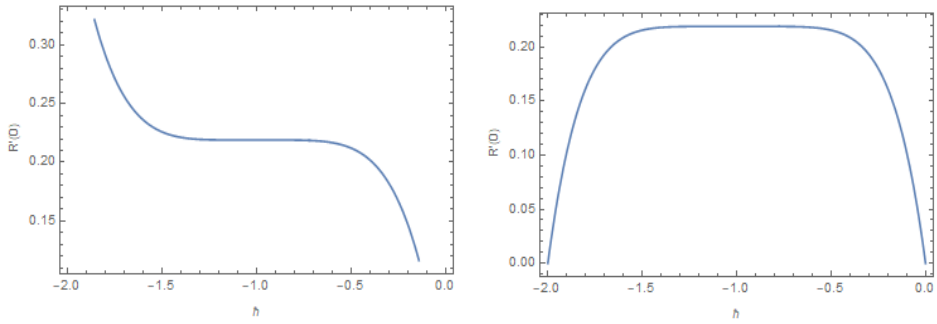


Figure 3: The h-curves of R′(0) obtained by the fifth-order and sixth-order approximations of the HAM.

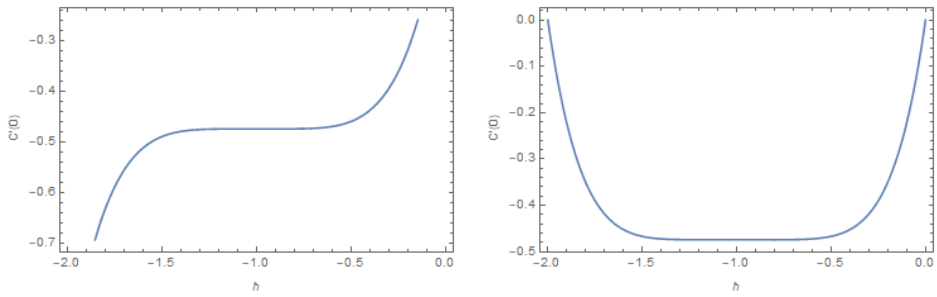
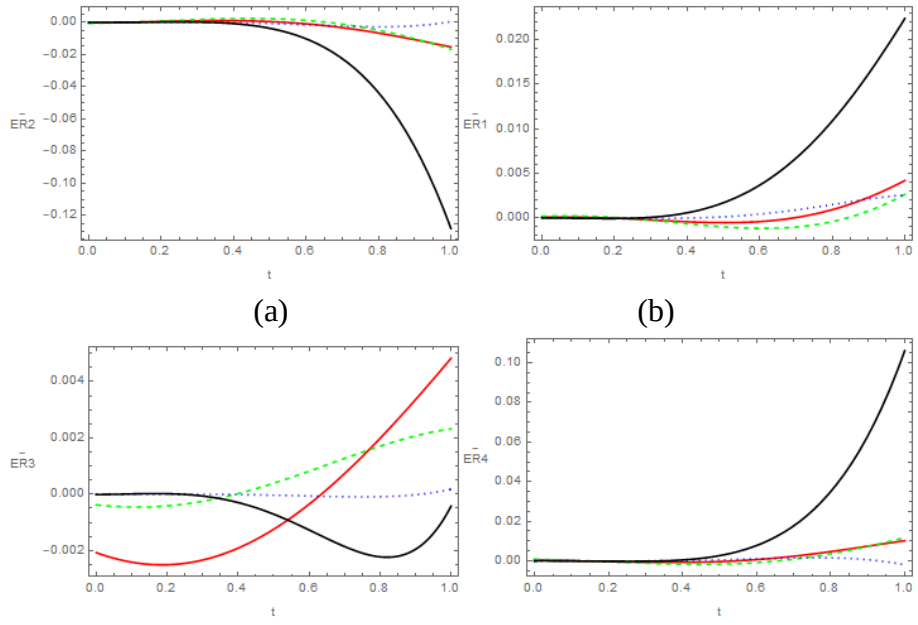


Figure 4: The h-curves of C′(0) obtained by the fifth-order and sixth-order approximations of the HAM.



(a) (b)

(c) (d)

Figure 5: (a) Various h = −0.9, h = −1, h = ideal, h = −1.1 and t ∈ (0, 1) the residual errors function of Eq.(19). (b) Various h = −0.9, h = −1, h = ideal, h = −1.1 and t ∈ (0, 1) the residual errors function of Eq.(20). © Various h = −0.9, h = −1, h = ideal, h = −1.1 and t ∈ (0, 1), the residualerrors function Eq. of (21). (d) Various h = −0.9, h = −1, h = ideal, h = −1.1 and t ∈ (0, 1), the residual errors function Eq. of (22)

Values of h1, h2, h3 and h4 for which $\overline{RS}(h1)$, $\overline{RI}(h2)$, $\overline{RR}(h3)$ & $\overline{RC}(h4)$ are minimum.

Then

$$\frac{d\bar{R}S(h_1^*)}{dh_1}=0,\quad \frac{d\bar{R}I(h_2^*)}{dh_2}=0,\frac{d\bar{R}R(h_3^*)}{dh_3}=0,\frac{d\bar{R}C(h_4^*)}{dh_3}=0..$$

The optimal values h_1,h_2 , h_3 & h_4 for all of the cases considered are obtained as

$$h_1^*=-0.856097,\ h_2^*=-1.0557,\ h_3^*=-0.913549,\ h_4^*=-1.04116.$$

Table 3 shows the minimal values of $\bar{R}S\ (h_1)$, $\bar{R}I\ (h_2)$,$\bar{R}R\ (h_3)$ & $\bar{R}C\ (h_4)$ for the optimal values of h_1, h_2 h_3 and h_4 .

We estimated errors for various t in (0, 1) which is listed in Table 4. This demonstrates that the HAM provides us with a close approximation to the solution for the SIRC model (1).

The residual errors of t in (0,1) and different h are shown in Figures 5(a,b,c &d).

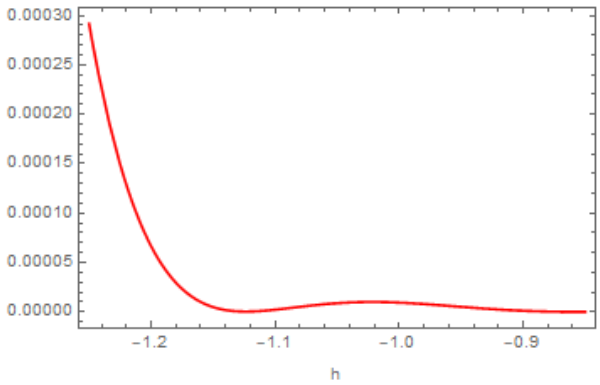


Figure 6: The optimum and Minimum value of S(t).

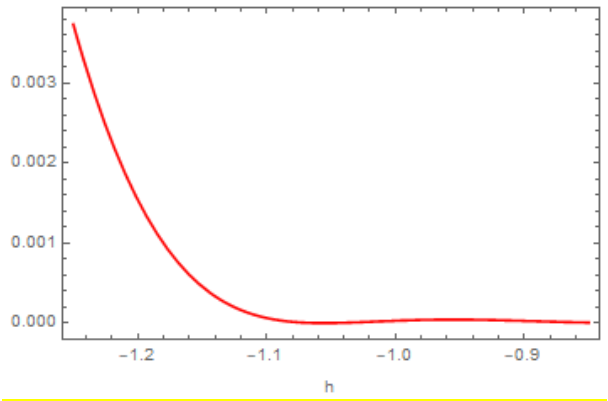


Figure 7 : The optimum and Minimum value of I(t).

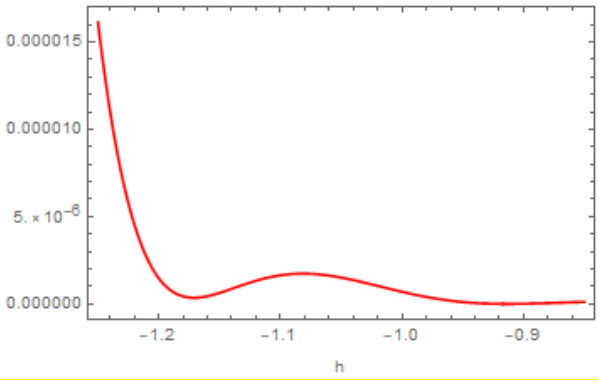


Figure 8: The optimum and Minimum value of R(t).

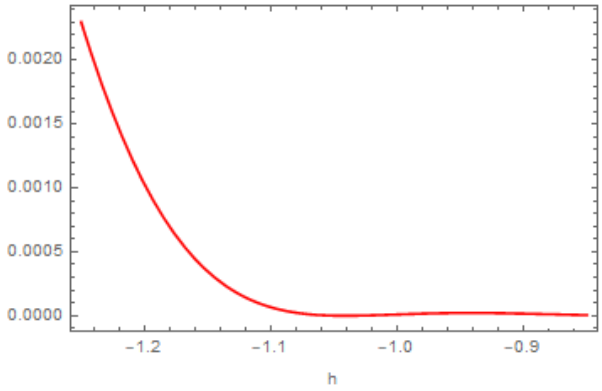


Figure 9: The optimum and Minimum value of C(t).

Table 3: The minimum values of $\bar{R}S\ (h_1^*)$, $\bar{R}I\ (h_2^*)$,$\bar{R}R\ (h_3^*)$ & $\bar{R}C\ (h_4^*)$ (see Figures 6-9)

	h^*	Minimum value
$\bar{R}S(h_1)$	-0.856097	8.38046×10^{-8}
$\bar{R}I(h_2)$	-1.0557	9.75647×10^{-8}
$\bar{R}R(h_3)$	-0.913549	2.79827×10^{-9}
$\bar{R}C(h_4)$	-1.04116	9.28625×10^{-9}

Table 4: The residual errors $E\bar{R}_1$, $E\bar{R}_2$, $E\bar{R}_3$ & $E\bar{R}_4$ for various $t \in (0,1)$.

t	$E\bar{R}_1(S,I,R,C;h_1^*)$	$E\bar{R}_2(S,I,R,C;h_2^*)$	$E\bar{R}_3(S,I,R,C;h_3^*)$	$E\bar{R}_4(S,I,R,C;h_4^*)$
0.0	9.05773×10^{-7}	9.58597×10^{-9}	9.14248×10^{-8}	2.30479×10^{-9}
0.1	1.52525×10^{-6}	5.94778×10^{-9}	1.69604×10^{-7}	8.5875×10^{-8}
0.2	7.34594×10^{-6}	6.4794×10^{-6}	5.70379×10^{-7}	3.30688×10^{-6}
0.3	3.06653×10^{-6}	3.89008×10^{-5}	2.02822×10^{-6}	1.80537×10^{-5}
0.4	5.53734×10^{-5}	1.19367×10^{-4}	1.20506×10^{-5}	5.20511×10^{-5}
0.5	1.66688×10^{-4}	2.54236×10^{-4}	3.24383×10^{-5}	1.03328×10^{-4}
0.6	3.26493×10^{-4}	4.1493×10^{-4}	6.11003×10^{-5}	1.53968×10^{-4}
0.7	4.74678×10^{-4}	5.24149×10^{-4}	8.70219×10^{-5}	1.71343×10^{-4}
0.8	4.78428×10^{-4}	4.52458×10^{-4}	8.63747×10^{-5}	1.20174×10^{-4}
0.9	1.09547×10^{-4}	2.87077×10^{-5}	1.88407×10^{-5}	1.26712×10^{-4}
1	9.76499×10^{-4}	9.33951×10^{-4}	1.75798×10^{-4}	1.80173×10^{-4}

5. Conclusion

In order to solve an epidemic SIRC model, the homotopy analysis method has been effectively developed and utilised in this study. The auxiliary parameter h, which is present in the HAM solution, provides an easy method for adjusting and controlling the convergence region of the resulting infinite series. The outcomes demonstrate that the HAM is an exact and effective method for determining the approximation.

Acknowledgements: The authors are grateful to anonymous referees for their excellent suggestions, which greatly improved the presentation of the paper.

Declarations

Competing interests The authors declare that there is no conflict of interests.

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