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A Heuristic Method to Improve the Robustness of Flight Schedules: Estimating Optimal Slack and Re-Timing Flights

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Abstract:
An investigation was conducted concerning the problem of flight delays to propose a solution to enhance the robustness of airline flight schedules. The robustness of flight schedules depended on the ability to reduce the effects of flight delays on daily operations. The approach to achieve this objective was optimizing flight schedules by allocating the optimal amount of slack to scheduled connections. Identifying appropriate slack between two successive flights involved modeling primary delay distributions at departure airports, considering both time intervals and delay lengths. A threshold approach was subsequently devised to determine the optimal slack based on this distribution. The ideal allocation of slack to schedules was achieved by modifying flight departure times, a technique known as re-timing. Recognizing that the optimal allocation of slack could make schedules infeasible, an optimization model was established for adjusting flight departure times. The proposed model produced new schedules that optimized all slack. The practical application of this model to the schedules of Indonesian airlines demonstrated the method's efficiency in improving the robustness of flight schedules.

Keywords: flight delay, slack, re-timing flight, robustness measure, robust schedules

INTRODUCTION

Multiple stages and a substantial allocation of airline resources comprise the highly complex process of airline schedule planning. This process can be divided into four primary sub-problems: schedule design, fleet assignment, aircraft routing, and crew scheduling (Ahmadbeygi et al., 2010; Dunbar et al., 2012). The optimization models for each of these sub-problems have been solved deterministically, assuming that flight schedules will be executed as intended. However, in the day-to-day operations of airlines, schedules often encounter disruptions, resulting in flight delays. These delays can lead to significant additional costs, such as increased fuel expenses, overtime payments for crew members, and the expenses associated with accommodating affected passengers (Dunbar et al., 2012).

To minimize operating costs, airlines should aim to reduce the occurrence of flight delays. When disruptions occur, a single flight delay can trigger a cascade of subsequent delays. Consequently, airlines need to develop optimal recovery strategies to restore their schedules (Abdelghany et al., 2004). These optimal recovery strategies are those that incur the lowest operating costs. In the recovery process, airlines have the flexibility to make decisions, such as re-routing aircraft, reassigning crews, or even delaying or canceling flights (Abdelghany et al., 2004). Due to the complexity of these recovery actions and the need for swift rescheduling, airlines often make these decisions manually, resulting in suboptimal outcomes (Abdelghany et al., 2004). Numerous authors have suggested algorithms, models, and strategies to determine the most effective recovery actions, and more details can be found in references (Ball et al., 2007; Barnhart, 2009; Clausen et al., 2010).

A practical method for addressing schedule disruptions is robust schedule planning. Although robust schedules may not necessarily minimize planned costs, they can significantly reduce airline expenses by mitigating additional costs during disruptions. Achieving robust schedules relies on insensitivity to minor changes. It can be improved by designing aircraft routes with extended connection times, scheduling extended crew rest periods, or planning flights with extended block times (Ahmadbeygi et al., 2010; Burke et al., 2010; Eggenberg, 2009; Lan et al., 2006; Schaefer et al., 2005; Yen & Birge, 2006; Ehrgott & Ryan, 2001). Furthermore, creating easily recoverable schedules can enhance the robustness of schedules (Aloulou et al., 2010). By maximizing flexibility in recovery, control over recovery solutions in scheduled operations was attained, comprising the optimization of aircraft swap opportunities (Ageeva, 2000) and increasing the availability of backup crews (Shebalov & Klabjan, 2006). Antunes et al (2019) designed a robust optimization model for the crew pairing problem to create crew schedules that are less susceptible to disruption. Sanjeevi & Venkatachalam (2021) analyzed a stochastic programming model aimed at anticipating uncertain future delays by incorporating buffers into flight turnaround times in a strategic manner. Teymouri et al (2023) put forward a strategy for making more effective decisions during disruptions to prevent flight delays or cancellations and recapture a significant portion of the lost demand. Additional methods for enhancing flexibility can be found in Gao et al (2009); Rosenberger et al (2004).

In the domain of airline schedules, every aircraft or crew is expected to complete specific flight sequences, with a required minimum connecting time between consecutive flights in a sequence. Time exceeding this minimum is termed "slack" (Chiraphadhanakul & Barnhart, 2013). Slack is a valuable tool for reducing flight delay, hence insufficient slack is capable of increasing the likelihood of delay propagating (Wu, 2005), while its excess may reduce aircraft and crew use efficiency. This means that robust schedules can be achieved due to optimal allocation of slack. Optimal slack is characterized as a period beyond the minimum turnaround time that adequately accommodates delays without significantly compromising utility.

Because flight delays are fundamentally stochastic events, calculating the optimal slack for airline planned connections is challenging. Most studies in this field determined optimal slack using historical delay data. Ahmadbeygi et al (2010) suggested allocating slack to scheduled connections by creating a discrete set of root delays that were randomly created based on the empirical distribution of one-year historical departure delay data. The overall delay propagation caused by these root delays was then computed using a propagation tree. The optimal slack was then established by shifting flight departure schedules earlier or later to reduce the overall propagated delay across all root delay scenarios. It was crucial to acknowledge that Ahmadbeygi et al (2010) acknowledged potential imprecision in their method for calculating propagation delay due to the absence of consideration for multiple flight delay in the network simultaneously (Ahmadbeygi et al., 2010). By adjusting flight departure or arrival times to minimize the total expected arrival delay across all flights and varied delay scenarios, Chiraphadhanakul & Barnhart (2013) determined optimal slack. Based on historical delay data, each scenario represented a one-day operation, resulting in independent arrival delays for all flight schedules. This method proved effective when each delay scenario was equally representative.

However, a substantial volume of historical delay data was necessitated for more accurate results, potentially resulting in computational inefficiency (Chiraphadhanakul & Barnhart, 2013).

In the previous studies, optimal slack was determined through the resolution of re-timing models. Due to the constraints imposed by time windows on both departure and arrival times, optimal slack obtained from these re-timing models cannot be assumed to be consistent with the actual optimal slack required for flight connections during operations. Consequently, both studies focused on redistributing existing slack more efficiently through re-timing models, aiming to enhance the capacity to absorb delay. A similar method consisting of the redistribution of slack through re-timing flight has been used in a study aiming to minimize the expected total propagated delay. These models also included the creation of new aircraft routings as part of re-timing process. Additionally, Lan et al (2006) created a model to optimize flight departure times, with the objective of reducing the expected number of disrupted passengers while maintaining the current fleet, aircraft routes, and passenger itineraries. In this scenario, re-timing models are considered an effective method of enhancing the robustness of integrated schedules. Lee (2007) introduced a multi-objective model that allows for the revision of flight departure times without affecting fleet assignments, aircraft routes, or crew pairings. A different model was created by Burke et al (2010) to simultaneously re-time flight and adjust aircraft routings while following the fixed fleet assignments. Novianingsih & Hadianti (2020) revised flight departure times after characterizing the types of perturbations in aircraft routings. Cacchiani & Salazar-González (2020) adjusted the flight's departure time by creating a limited set of options, while Wen et al (2020) considered the variations in flight durations. Novianingsih & Hadianti (2016) introduced a method for addressing flight retiming issues. The method involves creating many copies for each flight and utilizing a genetic algorithm to find the best solution.

OBJECTIVES

The objectives of this study are to determine the amount of optimal slack in scheduled connections and to adjust the flight schedule by revising the departure times of flights to allocate the estimated slack. This study represents the first instance of presenting a method for estimating optimal slack within an airline's scheduled connections, to the best of current knowledge. Airlines have several advantages in distributing the precise amount of optimal slack required for schedule connections. This method enables an analysis of whether the connections in existing schedules are adequate. Furthermore, optimal slack can be essential in constructing robust schedules, where all slack within schedules is optimized.

RESEARCH METHOD

Model of Departure Delay

A departure delay of flight f_i was defined as $d_i = \max\{\overline{dt}_i - dt_i, 0\}$, where dt_i and $\overline{dt}_i$ represented the planned and actual departure times of flight f_i, respectively. Lan et al (2006) provided a framework for decomposing delay into two categories: propagated delay and primary (non-propagated) delay. Propagated delay arose from waiting for incoming aircraft (or crew), while primary delay stemmed from other factors, including airport conditions such as weather and traffic. While circumstances at the departure airport were more strongly associated with primary delay, prior flights in a crew pairing or aircraft routing affected propagated delay. Using Lan's definition, a departure delay can be composed of a propagated and a primary delay. Mathematically,

$$d_i = pd_i + pr_i, \quad (1)$$

where pd_i and pr_i represented the propagated delay and the primary delay of flight f_i, respectively. Let F is a set of flight, and R denotes a set of aircraft routings. An aircraft route comprises a sequence of flights $(f_1, \dots, f_i, f_j, \dots, f_n)$ assigned for a specific aircraft. A set of aircraft connections A, comprised all pairs of consecutive flights flight (f_i, f_j) in r. The slack between two successive flights f_i and f_j in A, represented as s_{ij}, was defined as $s_{ij} = c_{ij} = m_{ij}$, where c_{ij} and m_{ij} indicate the connecting time and the minimum required connection time between flights between two consecutive flights (f_i, f_j) in r, respectively. The propagated delay of flight f_j caused by flight f_i in an aircraft connection is determined by

$$pd_1 = 0, pd_j = \max\{pd_i + pr_i - s_{ij}, 0\}. \quad (2)$$

assuming that all actual flight times are identical to the planned flight times (i.e., there is no delay). Hence, the departure delay could be formulated as

$$d_1 = pr_1, d_j = pr_j + \max\{pd_i + pr_i - s_{ij}, 0\}. \quad (3)$$

It was essential to recognize that propagated delays were not limited to aircraft connections; they could also occur during crew or passenger connections. This means the methods used to calculate propagated delays in aircraft connections could also be applied to passenger itineraries and crew pairings. Novianingsih & Hadianti (2016) provided a comprehensive explanation of the formula.

The following relationships were established:

$$pd_j = \max\{pd_j^A, pd_j^C, pd_j^P\},$$

and

$$pr_j = d_j - \max\{pd_j^A, pd_j^C, pd_j^P\}. \quad (4)$$

This was achieved by considering C and P as a set of crew and passenger connections, respectively, and pd_j^A, pd_j^C, pd_j^P as the propagated delay of flight f_j in the aircraft, crew, and passenger, respectively. If $pd_j^A = pd_j^C, = pd_j^P = 0$, then $pr_j = d_j$. The result indicates that to reduce propagated delay, the minimum slack duration allocated to the scheduled connection (f_i, f_j) must equal to the primary delay of flight f_i.

Bivariate Kernel Density Estimation

This Study focused on determining the optimal slack allocation in an airline schedule. Historical departure delays were used to model the distributions of flight delays. An analysis of the delay data identified a connection between departure delay and flight departure times. This observation led to developing a model that considered departure delay and their durations through bivariate probability distributions. These distributions played a crucial role in calculating optimal slack for flight connections. The analysis of historical delay data also enabled the identification of flights and airports with a higher likelihood of experiencing delay. Additionally, primary delay distributions at arrival airports were incorporated into schedules. The duration of the delay and the time of departure were the two main characteristics of each flight delay. The bivariate distributions for total departure delay and primary delay for each airport in the schedule were derived using the Kernel Density Estimation (KDE) method. These bivariate distributions allowed airport delay representation with a single distribution. When the focus was on delay at a specific departure time, the conditional distribution for that time could be extracted using the bivariate distribution. This method represented an advancement over prior studies, where flight delays were modeled using univariate distributions that depended solely on delay durations or delay times. For example, Lan (2006) applied a log-normal distribution to model arrival delay, while Schaefer et al (2005) used various distributions, including gamma, Erlang, and beta, to model flight and ground delay. Tu and Ball (2008) also used a mixture of normal distributions for modeling delay.

Consider n delay data $\mathbf{X}_1, \dots \mathbf{X}_n$, where $\mathbf{X}_i = (s_i, t_i)$, s_i and t_i were the departure time and delay duration of the i-th data, respectively. If $\mathbf{x} = (x_1, x_2)$, the bivariate KDE is defined to be

$$f_{\mathbf{H}}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} K_{\mathbf{H}}(\mathbf{x} - \mathbf{X}_i).$$

The bandwidth 2×2 matrix $\mathbf{H}$ was symmetric and positive definite, and $K_{\mathbf{H}}(\mathbf{x}) = |\mathbf{H}|^{-1/2} K\left(\mathbf{H}^{-1/2}\mathbf{x}\right)$. We used the bivariate normal kernel function $K(\mathbf{x}) = 2\pi^{-1/2}\exp\left(-\frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{x}\right)$, a choice made for its smoothness and symmetry. Therefore, we fixed $\mathbf{H} = \mathrm{diag}(h_1, h_2)$, where $\sqrt{h_i} = n^{-1/6}\sigma_i$, for i = 1,2 and σ_i was the standard deviation of the i-th variable.

The proposed method incorporated a threshold technique for identifying the ideal slack, using the total departure delay distributions and the primary delay distributions. This study utilized total departure delay distributions to identify flights likely to experience delays, emphasizing the importance of the initial threshold. For flights identified as having a higher likelihood of delay, the primary delay distributions were used to estimate optimal slack, with the second threshold coming into play. This threshold method represented a unique contribution to optimal slack estimation.

After optimal slack was calculated, it was allocated to scheduled connections by adjusting departure times within narrow time windows. However, it was important to acknowledge that allocating optimal slack could impact schedule feasibility. To address this, an optimization model was developed to rearrange flight departure times and create new schedules while ensuring that all slack within these schedules was optimized. The original aircraft routings and passenger itineraries were retained, as these routings from the original schedules served to minimize the total planned cost. The model was formulated as an integer non-linear programming problem, with the objective function representing the proportion of optimal slack that could be allocated within the new schedules. Consequently, the proposed model was implemented and solved using Lingo 20.

The methodology was implemented on an airline schedule, using one year of historical departure delay data from the airline to get both the departure delay distributions and the primary delay distributions. These distributions were used to determine optimal slack estimations using the threshold method. New

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flight schedules were then produced using the re-timing model. This effort resulted in a new schedule in which almost all optimal slack was allocated. A simulation was conducted to assess the performance of the new schedules under different conditions. This simulation included the introduction of primary delays based on the respective delay patterns. Subsequently, a comparison of the resilience between the original and the new schedules was carried out by calculating various robustness measures.

RESULTS AND DISCUSSION

Optimal Slack Calculation

The bivariate probability distribution of departure and primary delays, which included airports and flight departure times as variables, was employed to determine the optimal slack in this study. The first distribution was implemented to identify flights that may be experiencing delays. The appropriate slack allocation for each connection was determined in conjunction with the second distribution.

Let $f_B(x, y)$ be the probability density function (pdf) of a flight that departs from airport B with a planned departure time of *x* and experiences a *y*-minute delay. For flight *g*, the following formula was used to calculate the probability of the flight with a planned departure time of *t* being delayed by more than *a* minute:

$$\mathrm{P}(g \text{ delay} > a) = \int_{a}^{\infty} f_B(t, y) dy.$$

By setting a threshold probability b, a binary variable D was defined to identify whether flight g is delayed or not. We set $D_g^b = 1$ if $\mathrm{P}(g \text{ delay} > a) \geq b$ and $D_g^b = 0$ if otherwise.

Assume that $h_B(x, y)$ was a pdf of the primary delay in airport *B*, where *x* represented the time of day, and y represented the duration of the primary delay. The primary delay was estimated using this probability in conjunction with the prior ones. The estimated primary delay of flight g, which was scheduled to depart from *B* on *s*, was c minutes, if and only if $D_g^b = 1$ and $c = \max\{\alpha, \beta\}$, where $\beta = \max\{h_B(s, y) : y > 0\}$. Now, the estimated optimal slack was represented as a function of departure times.

The method for calculating optimal slack was straightforward to apply but, in practice, the values of *a* and *b* significantly influenced slack. Each value of *a* and *b* resulted in a discrete classifier, hence, result the best values for these thresholds was crucial for accurate predictions. Airlines could set their values for *a* and *b*, varying between carriers. Nevertheless, the subsequent approach was crucial in identifying appropriate values for a and b. The value of a was set at 15 minutes, based on the 15-minute On-Time Performance (15-OTP) standard. Under 15-OTP, a flight was considered delayed when it departed pass 15 minutes after the scheduled time. A value of b is a good predictor if it is the closest to the percentage of daily delay in historical delay data, as calculated by solving the optimization model below.

Let *S* represent a set of possible values for *b*, and *T* denotes a set of daily delay percentages obtained from historical delay data. The following was defined $x_i = 1$ if $i \in S$ was selected, and $x_i = 0$ if otherwise. The following optimization model was used to determine the optimal value of *b*.

Minimize:

$$\sum_{i \in S} |L_i - i| x_i$$

Subject to:

$$L_i = \frac{\left(\sum_{j \in F} D_j^i\right) x_i}{|F|}, \forall i \in S,$$

$$\sum_{i \in S} x_i = 1,$$

$$x_i \in \{0,1\}, \forall i \in S.$$

Re-timing Model

New flight schedules were developed to improve performance without increasing planned costs. This was accomplished by modifying the departure times of the planned flight schedules while maintaining the planned aircraft routings and passenger itineraries. To maintain the passenger projections in the scheduled design, departure time adjustments were implemented within limited periods.

New flight schedules were developed to improve performance without increasing planned costs. This was accomplished by modifying the departure times of the planned flight schedules while maintaining the planned aircraft routings and passenger itineraries. To maintain the passenger projections in the scheduled design, departure time adjustments were implemented within limited periods. In order to achieve this, the re-timing model was implemented to reschedule departing flights within the established schedules, provided that the minimum connecting time was maintained. Each delay in the scheduled connections was mapped to a scalar number using a non-decreasing function. S_{max} restricted the allocated slack for flight connections to prevent the addition of excess slack to flight connections. The optimal slack for a flight connection (f_i,f_j) was estimated as r_{ij}. The function v for slack v for slack s_{ij} was defined as a non-decreasing function:

$$v(s_{ij}) = \begin{cases} \frac{s_{ij}}{r_{ij}}, & s_{ij} \in [0, r_{ij}) \\ 1, & s_{ij} \in [r_{ij}, \infty) \end{cases}.$$

as the propagated delay in its connections will decrease as more slack is added to the scheduled connections.

By defining, w_{ij} as the weight of connection (f_i,f_j), the slack s_{ij} was measured by $w_{ij} v(s_{ij})$. The maximum departure changes of flight f_i to be moved earlier and later were limited by m_i and n_i, respectively.

The new slack for flight connection (f_i,f_j) and the change in departure time of f_i were determined by defining two decision variables y_i and s'_{ij}. Variable y_i received a negative value when the departure time of flight f_i was adjusted earlier and a positive value when it was delayed. The re-timing problem was formulated in the following model:

Minimize:

$$\sum_{(f_i,f_j)\in A\cup P} w_{ij} v(s'_{ij}), \tag{5}$$

Subject to:

$$s'_{ij} \geq s_{ij} - y_i + y_j, (f_i, f_j) \in A \cup P, \tag{6}$$

$$s'_{ij} \leq S_{max}, (f_i, f_j) \in A \cup P, \tag{7}$$

$$s'_{ij} \geq 0, (f_i, f_j) \in A \cup P, \tag{8}$$

$$y_i \in [m_i, n_i], f_i \in F, \tag{9}$$

$$y_i \in \mathbb{Z}, f_i \in F. \tag{10}$$

The objective function (5) aimed to maximize the overall slack measurement of all flight connections. Constraints (6) computed the revised slack between two flights after revising their departure schedules. The constraints (7) guaranteed that the change in the new slack wasn't higher than the maximum acceptable slack. Constraints (8) ensured the viability of flight connections within schedules. Constraints (9) limited for how much each flight's departure time could vary. Constraints (10) specified that the change in the departure time must be an integer value.

The Computational Results

The computational tests evaluated the method's effectiveness in calculating optimal slack and re-timing flights. The analysis utilized one-day schedules from an Indonesian airline, which included 292 flights, 93 aircraft routings, and 206 passenger itineraries. The airline generally allotted 15 minutes of slack for most scheduled connections. However, historical delay data indicated that delays frequently occurred at major airports. This suggested increased slack to minimize flight delays, particularly for flights departing from these airports. The primary objective was to compute the optimal slack for the airline's scheduled connections. This was achieved by modeling bivariate departure and primary delay distributions based on one year of historical departure delay data for each airport included in the schedules. The data were modeled using bivariate Kernel Density Estimation (KDE), and these distributions informed the determination of the optimal slack for planned connections. Furthermore, planned flights could be re-timed by 0.5, 10, or 15 minutes to enhance slack allocation. To preserve crew schedule feasibility, the departure times of the first flight in the crew’s duties were not changed earlier, and the departure times of the last flight in the crew’s duties were not changed later.

The airline's historical delay data documented each delayed flight's scheduled departure, planned arrival, and actual departure times. Consequently, the duration of the departure delay was determined by the difference between the scheduled and actual departure times. Their departure airports were used to categorize the departure delay data. The departure delay distribution of flights was modeled using bivariate KDE to derive flight delay probabilities and examine the relationship between their delay durations and departure times. The method of slack calculation outlined in Section 4.1 was then employed to determine the optimal slack.

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by the difference between the scheduled and actual departure times. Their departure airports were used to categorize the departure delay data. The departure delay distribution of flights was modeled using bivariate KDE to derive flight delay probabilities and examine the relationship between their delay durations and departure times. The method of slack calculation outlined in Section 4.1 was then employed to determine the optimal slack. The airline's primary delay distributions suggested that flight delays were only substantial at major airports. As a result, only adjustments were made to the departure times of flights departing from major airports. Figure 1 illustrates an illustration of the bivariate pdf of the primary delay (right) and the total departure delay (left) of flights departing from four main airports: A, B, C, and D. The curves' peaks corresponded to the periods of departure with the maximum number of flight delays. Delays were greater chances for flights departing during these times. The total departure delay distributions and primary delay distributions from airports A, C, and D were nearly identical in terms of both delay durations and departure times. Nevertheless, Airport B disclosed a different pattern of flight delays from 6:00 PM to 9:00 PM. This suggested that the delays experienced by flights departing from airports A, C, and D did not result from delay propagations. However, delays continued propagating for flights departing from Airport B during the specified time frame.

The method detailed in Section 3.2 was initially employed to determine the values of thresholds α and β to determine the optimal slack allocation for flight connections. This method set a minimum slack allocation of 15 minutes. Figure 2 illustrates the optimal slack values for flight connections to airports A, B, C, and D, which were determined using the bivariate distributions of primary delay presented in Figure 1. The analysis suggests that 15 minutes of slack was sufficient for flight connections to airport D. However, an increased amount of slack was required for flight connections to airports A, B, and C, particularly for flights departing from airport A between 01:00 PM and 07:00 PM, airport B between 03:00 PM and 09:00 PM, and airport C between 11:00 AM and 06:00 PM.

During the re-timing process, scheduled connections were classified into two categories based on flight frequency: those originating from major airports and those from other locations. Connections from major airports were assigned a weight value of 2, while connections from other locations received a weight value of 1. To solve the re-timing model, we utilized the optimization toolbox of MATLAB R2021b. Figure 3 illustrates the slack distribution in scheduled connections before (above) and after (below) re-timing. The optimal slack calculation was only matched by 5% of the slack in the scheduled connections. Approximately 40% of flights changed their departure times, leading to 39% of the new slack being equal in duration to the optimal slack. Before re-timing, the old slack deviated from the optimal slack by an average of 15 minutes; after re-timing, this deviation was reduced to 10 minutes. Table 1 summarizes the total slack in the original and new schedules. According to the table, the average slack was increased by only 2 minutes due to re-timing, changing from 15 to 17 minutes.

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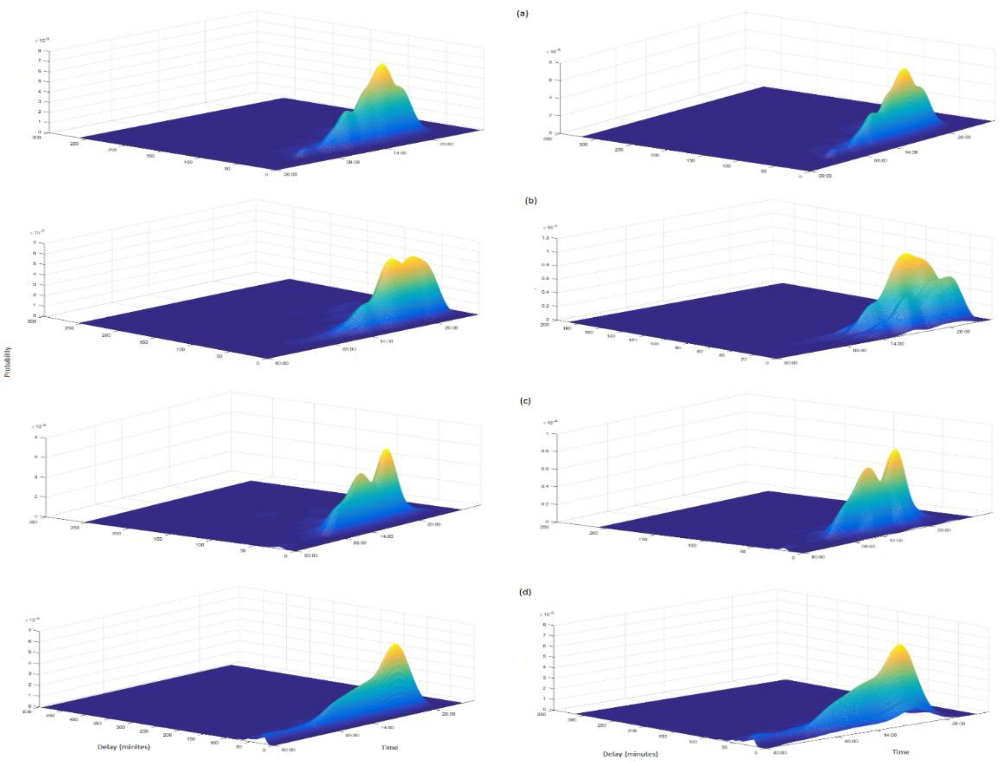

Figure 1. Bivariate pdf of departure delay (left) and primary delay (right) of flight departing from four major airports.

Table 1. The results before and after re-timing

Results (minutes)	Before re-timing	After re-timing
Total slack	3220	3475
The average of slack	15	17
The average of departure changes	-	5

Flight delay simulation was implemented in accordance with the methodology proposed by Novianingsih & Hadianti (2016) to evaluate the effectiveness of the new schedules. The historical delay data and the method explained by Novianingsih & Hadianti (2014) were employed to model the pdf of the primary delay for each airport. As a result, the primary delay's pdf was obtained as log-normal distributions, with parameter values of $\mu \in [2.8, 3.4]$ and $\sigma \in [0.4, 0.6]$. The average total departure delay, average propagated delay, average total flight delay, average total passenger delay, and the average number of passengers missing their connections were recorded after the simulation, as illustrated in Table 2. The results indicated that the new schedules substantially reduced the total propagated delay. Consequently, the total departure delay in the revised schedules decreased by an amount that was equivalent to the degradation in total propagated delay. A comparable pattern was observed in the number of flight delays, particularly when the 15-OTP was employed as the on-time performance metric, as illustrated in Table 3. The new schedules were able to decrease flight delays by over 20%, which suggests that slack allocation is effective in mitigating propagated delays. It was also discovered that disrupted passengers contributed only 5% to the average total passenger delay, while flight re-timing reduced over 10% of the total flight delay. These findings underscored the necessity for airlines to prioritize reducing flight delays over reducing passenger disruptions.

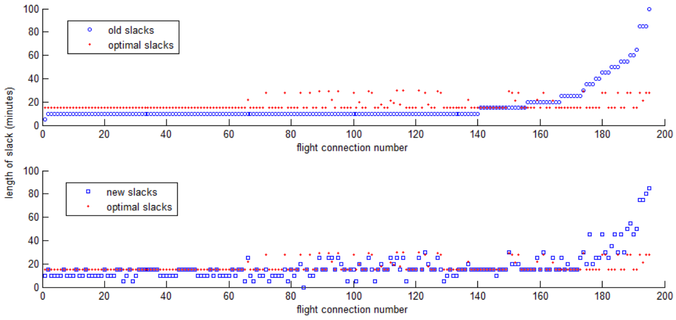

Figure 2. Slack distribution in scheduled connections before and after flight retiming

Table 2. A comparison of the robustness measures between the original and revised schedules

Measures	Original schedules	Revised schedules
Total propagated delay (minutes)	4403.2	2358.1
Total departure delay (minutes)	5870.3	3825.2
Delayed flight (%)	28	15
Passenger delay (minutes)	3251.2	2035.1
Passenger disruptions (%)	20	15

Table 3 The flight delay simulation results

Total departure delay (minutes)	[0,15)	[15,20)	[20,30)	[30,∞)
Original schedules (%)	30	15	22	35
Revised schedules (%)	54	34	5	7

CONCLUSION

This paper presents a method that uses slack-based distributions to enhance the robustness of flight schedules. The approach begins with estimating the optimal slack in scheduled connections by modeling bivariate historical departure delay distributions. A non-linear integer programming model was then developed to optimize the distribution of the estimated slack among flight connections by adjusting their departure times. The model's effectiveness was evaluated by comparing its results to the actual schedules of an Indonesian airline. The computational results demonstrated that the method successfully improved the robustness of the flight schedules.

Significantly, this method only adjusted flight departure times and did not affect aircraft routings, fleet assignments, or passenger itineraries. This allowed for improvements in schedule robustness without significantly increasing planning costs. Therefore, the method could seamlessly integrate into a post-traditional optimization step to create more robust schedules. While there may have been an increase in planned crew costs due to the adjustments in departure times, this impact remained feasible when considering crew schedules. The adjustments involved rescheduling flight departure times, excluding the first and last flights in duty cycles. Looking ahead, an additional area for future research could involve integrating crew costs into the re-timing model, as demonstrated in studies conducted by Teymouri et al (2023); Wen et al (2020).

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