Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 7s (2025)

Optimization of END -OF -LIFE strategies- Air conditioners

Jonnagiri Sowmya[1]*, Govada Rambabu[2]

[1] Senior lecturer at Govt. Polytechnic for Minorities, Kurnool, Part time PhD Scholar, Department of Mechanical Engineering, College of Engineering(A), Andhra University, Visakhapatnam, India.

[2] Associate Professor, Department of Mechanical Engineering, College of Engineering(A), Andhra University, Visakhapatnam, India.

* Email: sowmya.jonnagiri@gmail.com

Article History:

Received: *26-10-2024*

Revised:*10-11-2024*

Accepted:*18-12-2024*

Abstract:
Circular economy is a concept developed to seek sustainable development and resource usage optimization for sustaining the green environment.it is a deviation from the practice of manufacture, consume and dispose, reverse logistics is an activity that moves goods from customers back to sellers or manufacturers after their end-of life for repair and disposal.One common misconception about the circular economy is that it is an expensive and difficult endeavor. Its goal is to regenerate natural systems and ensure materials are reused rather than discarded, encompassing reverse logistics. Product recovery includes actions such as restoration, repair, refurbishment, manufacturing again, and recycling, all of which need an effective reverse logistics system.This network yields economic advantages by decreasing raw material acquisition, enhancing inventory management, and lowering the disposal of waste. A mixed-integer linear programming model for a multi-stage reverse logistics system is suggested to improve product recovery. optimization of refurbishment of air conditioners is considered in the paper.

Keywords: circular economy,air conditioning,environment,optimization.

1. Introduction

Society has traditionally followed a linear manufacturing and utilization paradigm termed the "take-make-dispose" strategy. This approach includes the extraction of raw materials, their transformation into items utilization by consumer, and their disposal into environment after post-utilization (Esposito et al., 2017). This method promoted industrial progress and the expansion of consumer markets, but it also resulted in considerable issues relating to environment and issues relating to society (Andrews, 2015). The inappropriate waste disposal and unsustainable exploitation of scarce resources, whether as raw materials or energy sources, highlight the constraints of this paradigm (Prieto-Sandoval et al., 2018). This linear approach leads to significant economic losses along the value chain, eventually compromising the competitiveness of enterprises (Schroeder et al., 2018).

The economy which is circular in nature aims to improve resource efficiency by reducing utilities available in nature and their exploitation. Thus avoiding the generation of non useful items ,which will result in increasing issues relating to society in different angles. The fundamental tenets of the circular economy, as articulated by several writers [MacArthur, E. (2013); Murray et al. (2017); Geng (2019); Velenturf et al. (2021)], highlight sustainable resource use, minimization of waste, and increase of value .

1. The notion of a circular economy places an emphasis on prolonging the usable life of items, which is intended to reduce the frequency with which replacements are required. The use of high-quality materials, modular designs, and an emphasis on repairability and upgradability are the main means by which this objective is accomplished.

2. Product lifespans may be greatly extended by the implementation of methods like re-use, refurbishment, remanufacturing, and repurposing, which in turn drastically decreases the need for new resources.
3. Recycling recovers materials from items at the final stage of their life cycle, enabling the creation of new items. Effective waste management, sorting, and reprocessing are critical components of the recycling process.
4. By optimizing utilization of fossil fuels, the circular economy minimizes waste generation and reduces environmental impacts.
5. The circular economy is inspired by nature's regenerative systems, offering several benefits:

a. Reduced extraction of new materials, preservation of earth's resources, and reduction of environmental effect are all goals of this strategy.
b. Reduce the amount of trash produced by recycling, refurbishing, and reusing items, which helps reduce the size of garbage which is ultimately useful in filling up low lying areas there by reducing the unnecessary damage to environment.
c. The transition to an economic system that is both regenerative and sustainable, has a beneficial influence not only on the economy but also on society and the environment.

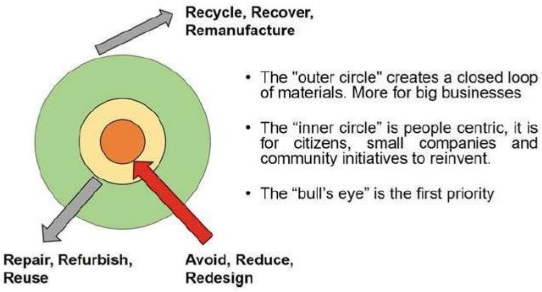


Fig. 1: Circular economy circles

The economy which is circular in nature is illustrated in the Fig. 1. The "outer circle" focuses on recycling, recovery, and remanufacturing to create a closed-loop system. For instance, recovering precious metals from electronic gadgets requires advanced technology, significant investment, and scale, which are typically managed by medium to large companies.

The "inner circle" prioritizes repair, refurbishment, and reuse, promoting a shift away from single-use, disposable culture. By extending product lifespans, the inner circle benefits individuals and supports small businesses. While both approaches are essential, the central focus should be on reducing, redesigning, and avoiding waste, aligning closely with sustainable consumption practices.

1.1 Air Cooling Cycle

Air cooling is a major energy-consuming process, as shown in Figure 2.

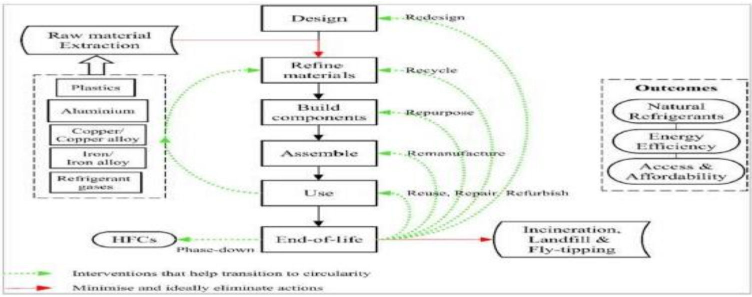


Fig. 2: Circular Production flow chart of cooling

– adapted from (Khosla et al., 2022)

1.2 motivation

The present study is motivated by the application of circular economy concepts, which hold significant.There is a substantial possibility of lowering the amount of carbon released and consumption of energy that occur during the production of new units.

2. Literature Review

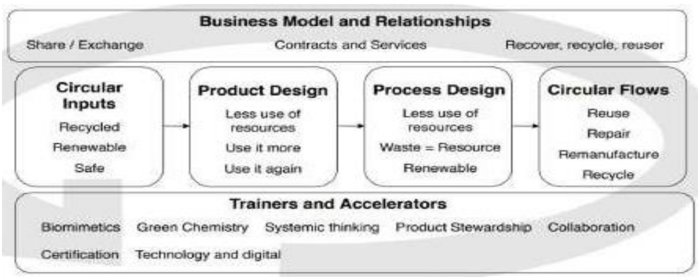
Marwan et al. (2023) explored energy optimization in air conditioning by modifying wall features, such as temperature and material composition, using materials like Styrofoam, soil, calcium carbonate, wood, and iron to design cost-effective buildings.

Li et al. introduced a technique for improving the efficiency of energy when it is used for HVAC systems via load forecasting and energy flexibility. Their methodology integrates a penalty coefficient attuned to thermal comfort, making it more efficient for energy conservation compared to static air temperature settings for comfort. A feedforward control approach is used to improve efficiency.

Baghoolizadeh et al. (2023) highlighted the threat of environmental air pollutants to human health and applied genetic algorithm optimization to improve occupant thermal comfort.

Chaturvedi et al. (2022) minimized yearly cooling energy use by determining optimum building envelope designs and air conditioner sizes using stochasticalgorithmsGA and PSO.

Akyuz et al. (2023) conducted research on a air conditioning systemwhich is solar-assisted, examining its performance in terms of energy consumption and climate during its whole life cycle. Their results highlight the need of developing effective ways to minimize the amount of energy that is used by such systems as well as the influence that they have on the environment while simultaneously enhancing their efficiency.



Source: Adapted from Wemba (2017)

The CE definition proposed by Sandoval et al. (2018), which emphasizes its close relationship with society's innovation processes and evolves on the flow of energy in circular path .The utilization of resources while attempting to reduce demand and utilization of waste which is generated when attempting to put the resources in the system as per with the framework offered by Wemba (2017). The information given by Wemba (2017) and Sandoval et al. (2018) is in agreement with the circular supply chain paradigm evolved by Barbosa et al. (2018). A circular supply chain was proposed by Barbosa et al. (2018). Using a genetic algorithm to determine the best possible configurations for each component, a design technique was introduced by Hapuwatte et al. (2022) that optimizes the closed-loop dynamic product sustainability performance. There is no real-time validity to its application. Degradation model for DC motor reliability evaluation by Yang et al., 2023

Return to manufacturer (RDM) was the subject of research by Zhang et al. (2021).

Literature gap:Identifying suitable end-of-life (EOL) destinations for abandoned items is an increasing difficulty, especially considering the implication f issues relating to environment and over filling of low lying areas.. To resolve these difficulties, the product design must be optimized to include an ecologically sustainable end-of-life scenario that adheres to economic and statutory limitations.(Marie 2012)

There has been abrupt increase of trend towards CE as compared with linear economy. This change represents an all-encompassing strategy for sustainability, as it affects components as well as products and processes. In order for CE to become standard practice, the end-of-life of the product has to be rethought to include visual health tool evaluation of each component state (Waqua et al., 2024).

End of life strategies are made during design stage of products which is evident in the literature2012 on wards. The concept of circular economy extending to component level is critically examined by Waqua et al (2024). Here an effort is made to bridge the component level suitability for assessing the end of life strategy to be adopted along with optimization of cost.

The objectives of the study

Minimisation of various costs and carbon emission in an air conditioning industry

3. Methodology

- Study of the system
- Formulation of model
- Computation of component level calculation
- Ascessing the composition of rejection, recycle, re manufacturing rates
- Computation of cost and carbon emissions.

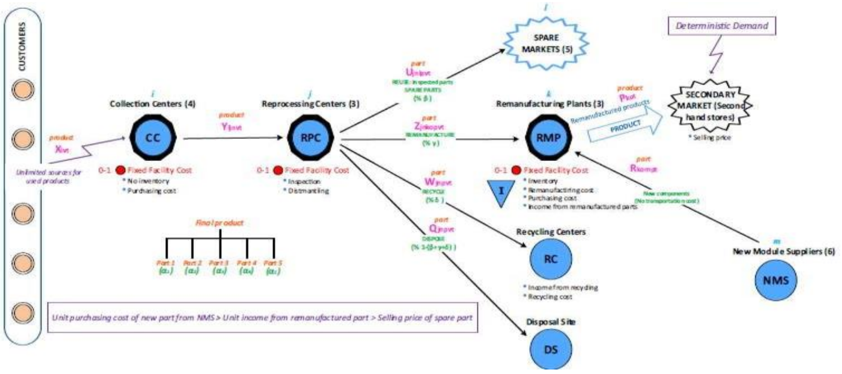


Fig 3.rpc

3.1 considered:

1. **Price and Quality Variance:** The variance in price and quality between both refurbished and new goods drives deterministic the need for remanufactured goods. Spare parts often hold a higher unit value. Holding costs are not considered in the collection center (CC). Dismantling operations are conducted at the Reprocessing Center (RPC).

2. **Transportation Costs:** Transportation costs are assumed to be based on full truckloads, calculated considering distance and overhead costs, including the cost of new products.

3. **Fixed Costs:** The CC, Remanufacturing Center (RMP), and Recycling Operation Center (ROC) incur fixed monthly costs. For the Reprocessing Center (RPC), a new setup operates on a fixed-cost rental sharing model.

4. **Disposition of Returned Products:**

- **x%** of returned products are disposed of.
- The rest of the rejected modules are allocated for remanufacturing in manufacturing plants or sent to the serviceable market.
- y% of articles undergo remanufacturing.
- Thirty percent of tested goods are delivered to the service component market.
- Zero percent of rejected modules are sent for recycling.

The values of x, y, and z are calculated at the component level.

3.1.1 fuzzy linear programming

The Linear Programming as proposed by Zimmermann is shown below (1-3):

$$\text{Minimize } Z = Cx \tag{1}$$

Subject to.

$$Ax \leq b \tag{2}$$

$$x \geq 0 \tag{3}$$

The equation modifies after fuzzification to (4-6).

$$\bar{C}x \lesssim Z\ominus \tag{4}$$

$$\bar{A}x \lesssim b \tag{5}$$

$$x \geq 0 \tag{6}$$

Here $\lesssim$ indicates "smaller than or equal to," allowing the model to attain a certain level of aspiration . Here $\bar{C}$ and $\bar{A}$ indicate the values which are fuzzy in nature. The fuzzy set A in X is defined as: $A = \{x, \mu_A(x)/x \in X\}$. Where $\mu_A(x) : x \rightarrow [0, 1]$ is termed as the function of membership of A and $\mu_A(x)$ indicates the degree of membership of x extending upto A.

The $\bar{Z}$òX which representing a Fuzzy Objective is a

fuzzy subset of X indicated by its membership function $\mu_A(x) : x \rightarrow [0, 1]$.

The membership functions which are linear in nature for minimization and maximization objectives were given as:

$$\mu z_j(x) = \begin{cases} 1 & \text{if } Z_j(x) \leq Z_j^{\min} \\ \dfrac{Z_j^{\max} - Z_j(x)}{Z_j^{\max} - Z_j^{\min}} & \text{if } Z_j^{\min} \leq Z_j(x) \leq Z_j^{\max} \\ 0 & \text{if } Z_j(x) \geq Z_j^{\max} \end{cases} \tag{7}$$

where $j = 1,2,\ldots,j$ (for maximization).

$$\mu z_j(x) = \begin{cases} 0 & \text{if } Z_j(x) \le Z_j^{\min} \\ \dfrac{Z_j(x) - Z_j^{\min}}{Z_j^{\max} - Z_j^{\min}} & \text{if } Z_j^{\min} \le Z_j(x) \le Z_j^{\max} \\ 1 & \text{if } Z_j(x) \ge Z_j^{\max} \end{cases} \quad (8)$$

where $j = 1,2,\ldots,j$ (for maximization).

In Eqs. (7)-(8), $Z_j^{\min}$ is $\min_j Z_j(x^*)$ and $Z_j^{\max}$ is $\max_i Z_j(x^*)$ and x^* is the maximum solution.

$$Z_j^{\min} \le Z_j \le Z_j^{\max} \text{ for all } j, \qquad j = 1,2,\ldots,J \quad (9)$$

$$\text{Maximize } \sum_{j}^{J} w_j \lambda_j \quad (10)$$

Subject to,

$$\lambda_j \le \mu_{zj} \text{ for all } j \qquad j = 1,2,\ldots,J \quad (11)$$

$$Ax \le b \qquad \text{constant which is deterministic in nature} \quad (12)$$

$$\sum_{j}^{J} w_j = 1 \quad (13)$$

$$w_j,\ x \ge 0 \text{ and integer, for all } j, \qquad j = 1,2,\ldots,J \quad (14)$$

$$0 \le \lambda \le 1. \quad (15)$$

here $w_j$represents the relative importance of fuzzy goals.

3.2 Best Worst Method (BWM)

The BWM is applied to ascertain the relative importance of factors and alternatives using fewer comparisons and achieving higher consistency. The steps include:

1. Defining the decision problem and its elements.
2. Identifying the most important (best) and least important (worst) elements.
3. Comparing the best element against all others using the Saaty scale (1–9).
4. Comparing all elements against the worst element using the same scale.
5. Checking consistency.
6. Calculating weight scores.

Using the Saaty scale, rank the components from 1 to 9 and choose the most essential one from the set (e_1, e_2,..., en) (1-9). As a result, the most essential element for the r vectors will be Ea = (e_{al}, e_{a2},..., e_{an}), with e_{aa} = 1 being the obvious collection center. Eb = (e_{1b}. e_{2b},...,e_{nb})T, on the other hand, would be the least significant element to other vectors if we used the same scale. Consistency is verified using the consistency ratio calculated as:

$$CR = \frac{\xi *}{\text{Consistency Index}} \quad (6)$$

The values of Consistency index are shown in Table-3 below

Table-3:

e_{ab}	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Consistency index (maxξ)	0.0	0.44	1.0	1.63	2.3	3.0	3.73	4.47	5.23

To determine the maximum weight for all elements, the highest absolute differences are $\left|\frac{w_a}{w_j}-e_{aj}\right|$ and $\left|\frac{w_j}{w_b}-e_{jb}\right|$, these differences are minimized across all criteria. The following is the formulation of the problem, which assumes that the total of the weights is positive.:

$$\min\max_j \left\{\left|\frac{w_a}{w_j}-e_{aj}\right|,\left|\frac{w_j}{w_b}-e_{jb}\right|\right\}$$

s.t. $\sum_j b_j = 1$ (7)

$b_j \geq 0$, for entire values of j.

There is thus the possibility of transforming the problem into the following optimization model for resolution:

min ξ

s.t. $\left|\frac{w_a}{w_j}-e_{aj}\right| \leq \xi,$ for all j

$\left|\frac{w_j}{w_b}-e_{jb}\right| \geq \xi,$ for all j

$\sum_j b_j = 1$ (8

$b_j \geq 0$, for all.

4. Mathematical Model Formulation

4.1 component level calculation

The fuzzy inference model, which is used to estimate the quality of items that have attained the End phase of life (EoL), comprises the following steps:

Step-1: Determine the parameters relating to input and output:

The parameter representing output is expressed in the form of number, Fuzzy Quality Level, ranging from 0 to 1, denoting the FQL of an EoL component. Three input parameters are essential for computing the FQL:

1. Usage Condition (UC):

This metric considers elements like use frequency, operational practices, staff proficiency, working conditions (e.g., moisture, humidity, and temperature cleanliness), and maintenance standards. Suboptimal consumption practices may result in degraded end-of-life situations. The UC is calculated as: $UC = \alpha_{UF}UF + \gamma_{EF}EF + \lambda_{ML}ML + \beta_{WS}WS$ (1)

where $\alpha_{UF}UF$, $\beta_{WS}WS$, $\gamma_{EF}EF$ + $\lambda_{ML}ML$re weights assigned by experts (on a scale of 1–10). Scores for sub-indices {*UF*, *EF*, *ML*, *WS*}are determined using:

$$X_i = \sum_{j=1}^{s} u(h_{ij})\square p(h_{ij}) \quad (2)$$

Here$p(h_{ij})$ is the statistical probability of a component belonging to partitionh_{ij}, and$u(h_{ij})$is the evaluation score (1–5).

2) **EoL component Reliability (ER)**:

By determining the chance that a component will perform its function once the product attains its end-of-life (EoL), reliability may be measured. It is expressed as:

$ER = P\{T \geq t\}$,

Here component's lifespan is represented by *T*is the and the mean usage age is represented by *t*is. Using the Weibull model. *ER* can be calculated

$$ER_k = \exp\left[-\left(\frac{t_k}{\theta_k}\right)^{\beta_k}\right] \quad (3)$$

Here mean usage age of *k*, is represented by t_k , h_k and b_k are Weibull parameters derived from failure data.

3) Component Performance (CP):

Since reliability alone cannot fully reflect EoL conditions, CP assesses component performance using lifecycle data:

$$CP = \alpha_{PD}(1 - PD_k) + \gamma_{PM}(1 - PM_k) \quad (4)$$

$$PD_k = \frac{\sum_{u=1}^{\varphi} P_{ku}(\text{defective} \mid \text{age}_u)}{\varphi} \quad (5)$$

Here the probability of a component being defective is represented by PD_k and . PM_k

is the probability of it absence.α_{PD} and γ_{PM} are weights($0 \leq \alpha_{PD}$, $\gamma_{PM} \leq 1$ and $\alpha_{PD} + \gamma_{PM} = 1$). Eq. (5) shows a technique to collective PD_k, Here size of the sample space *is represented by u* the, the actual usage life of product *u* is exhibited by age_u and finally P_{ku} is a conditional probability that provides the chance of component *k* being defective under a given usage life age_{uk}.

Step-2: Fuzzification:

This process results in the transformation of clear assessment values into fuzzy linguistic variables. For every input, there are three fuzzy sets that are used: low, medium, and high. On the other hand, the output (FQL) is specified with five levels, varying from very low to very high sensitivity. Figure 2 depicts the functions that are associated with membership.

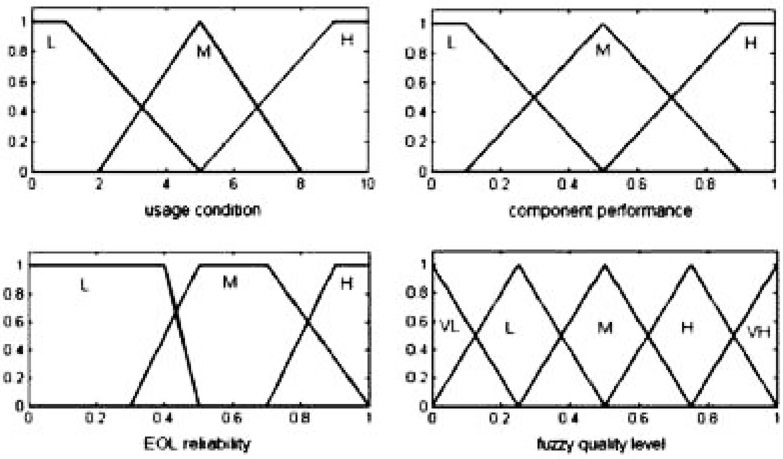


Fig. 2: A diagrammatic representation of the functions representing membership for both the input and the output

Step-3: Fuzzy inference:

. "IF-THEN" criteria delineate input premises and output conclusions, for instance, "If UC, ER, and CP are elevated, then FQL is significantly high." For three inputs and three fuzzy levels , a total of 27 criteria (3^3) is required. General rules include:

1. All inputs elevated (diminished): FQL is exceedingly high (remarkably low).

2. The condition that FQL is medium is possible only when l inputs are all medium:

3. The condition that FQL is medium(high) is possible only Two at high (medium) and one at medium (high);

4. The condition that FQL is low is possible only If both inputs are low

Certain circumstances are unfeasible. (e.g., poor UC and low ER with high CP) and marked as N/A. Rules can be refined through expert feedback.

Step-4: Defuzzification:

The centroid defuzzification method is used to transform fuzzy outputs into precise values:

$$FQL = \frac{\int_b^a \mu_z(x)x dx}{\int_b^a \mu_z(x) dx} \quad (6)$$

Here the membership function of the fuzzy set Z on the interval $[a,b]$ is represented by $\mu_z(x)$

]Recovery Decisions

Based on the calculated FQL, components are assigned recovery options:

1) Reuse or Resale: If $_{\text{FQLik}} \geq_{\text{Qh,k}}$, $EC_{ik} \in S_3$.

2) Remanufacture or Refurbish: If $Q_{ik} \leq FQL_{ik} < Q_{h,k}$, $EC_{ik} \in S_2$.

3) Recycle or Replace: If $FQL_{ik} < Q_{ik} EC_{ik} \in S3$.

Recovery options are categorized as:

- **S1:** Reuse

- **S2:** Remanufacture
- **S3:** Recycle

Step 5 :percentage calculation for remanufacturing = sum of no of componets/total products.similarly for all**(7)**

4.2 cost and emission minimisation

symbols

k Gathering Centers are represented by k and its values are ranged from 1 to K,

l Reprocessing Centers are represented by l and its values are varied from 1 to L

m Reproduction Plants are represented by m and its values are ranged from 1 to M

n Spare Marketplaces are represented by n and its values are varied from 1 to N

o fresh module dealers are represented by o and its values are ranged from 1 to O)

p Technologylevels of Rwprocessing center are represented by p and its values are varied from 1 to P

q Technologylevels of RpP are represented by q and its values are ranged from 1 to Q

r Set of parts are represented by r and its values are ranged from 1 to R

s Periods are represented by s and its values are ranged from 1 to S

t Shippingchoices are represented by t and its values are ranged from 1 to T

- **Parameters**

q_{jlp} . The price of selling of part *p* from Reprocessing*j* to SpareMarket *l* (Rs)

se_k . The price of selling remanufactured product from Reprocessing unit*k* to Secondary Market (Rs)

pp .The cost of purchasing of used products (Rs)

qc_{mp} . The cost of purchasing of new part *p* from supplier *m in* (Rs)

ic_{jnp} .The cost of inspection of part *p* for *n* technology-level Reprocessing C *j* (Rs)

dc_{jn} .The cost of dismantling of used products for *n* technology-level Reprocessing *j* (Rs)

wc_{ko} .The cost of remanufacturing of products for *o* technology-level RpP*k* (Rs)

wc_p . The cost of recycling of part *r*(Rs)

ac_p . The cost of disposal of part *r*(Rs)

hc_p .The cost of holding of part r*p* in RpP*k* for a period (Rs)

ta_{iv} The cost .carrying of product fromcustomers to COLLECTING CENTRE(CC)*i* with carrying option*v* (Rs/ton.km)

tb_{ijv} The cost of carrying of product from COLLECTING CENTRE(CC)*i* to REPROCESSING CENTRE(RPC)*j* with carrying option *v* (Rs/ton km)

tc_{jkpv} The cost of carrying of part *p* from REPROCESSING CENTRE(RPC)*j* to REMANUFACTURING CENTER((RMP)*k* with carrying option *v* (Rs/ton.km)

td_{jlpv} The cost of carrying of part p from REPROCESSING CENTRE(RPC) j to Spare Marketplaces l with carrying option v (Rs/ton)

we_{jpv} The cost of carrying of part p from REPROCESSING CENTRE(RPC)

j to RC with carrying option v (Rs/ton.km)

tf_{jpv} The cost of carrying of unit of part p from REPROCESSING CENTRE(RPC) j to DS with carrying option v (Rs/ton.km)

mfc_i The cost of fixed facility of COLLECTING CENTRE(CC)i (Rs)

nfr_{jn} The cost fixed facility of n technology-level REPROCESSING CENTRE(RPC)j (Rs)

fnm_{ko} The cost fixed facility of o technology-level REMANUFACTURING CENTER((RMP)k (Rs)

Cn_{CO2} Unit cost of CO_2 emissions (Rs/gr)

$Lrnpc_{jn}$ The quantity of CO_2 release into environment during the production of one unit product in n technology-level RCOLLECTING CENTRE(CC)j (g/ton)

nLRemanufacturing center((RMP)$_{ko}$ quantity of CO_2 emissions during manufacturing of one unit product in o technology-level REMANUFACTURING CENTER((RMP)k (g/ton)

$Lntv_v$ quantity of CO_2 release into environment of vehicle v per kilometer (gr/ton km)

dna_i Gap between customers and COLLECTING CENTRE(CC)i (km)

dnb_{ij} Gap between COLLECTING CENTRE(CC)i and RCOLLECTING CENTRE(CC)j (km)

dnc_{jk} Gap between RCOLLECTING CENTRE(CC)j and REMANUFACTURING CENTER((RMP)k (km)

dnd_{jl} Gap between RCOLLECTING CENTRE(CC)j and Spare Market$l$(km)

dne_j Gap between RCOLLECTING CENTRE(CC)j and RC (km)

dnf_j Gap between RCOLLECTING CENTRE(CC)j and DS (km)

dn_t the amount of remanufactured product existing Secondary Market in period t

cna_i storage size of COLLECTING CENTRE(CC)i (ton)

cb_{jn} storage size of n technology-level RCOLLECTING CENTRE(CC)j (ton)

collecting centre(CC)$_{kop}$, part p capacity of o technology-level REMANUFACTURING CENTER((RMP)k (ton)

cd_{lp} part p requirement of Spare Market l (ton)

ce_p part p holding size of RC (ton)

cf_p part p holding size of DS (ton)

cg_{mp} new part p holding size of NMS m (ton)

M A large number

α_p share of part p in a product percentage

βe share of parts to be reused (percrntage

γm share of parts to be remanufactured percentage

δe share of parts to be recycled (percentage)

- **Decision variables**

Xi_{ivt} .Quantity of unused products transported from customers to collection center (cc) *i* by vehicle *v* in during period*t* (ton)

yi_{ijnvt} . quantity of unused products sending from collecting centre(cc) *i* to *n* technology-level rcollecting centre(cc) *j* by vehicle *v* in period *t* (ton)

$zi_{jnkopvt}$. quantity of consumed part *p* sending from *n* technology-level reprocessing centre(rpc) *j* to technology- level remanufacturing center((rmp) *k* by vehicle *v* in period *t* (ton)

ui_{jnlpvt} . quantity of consumed part *p* sending from *n* technology-level reprocessing centre(rpc) *j* to spare market *l* by vehicle *v* in period *t* (ton)

wi_{jnpvt} . quantity of consumed part p sending from *n* technology-level reprocessing centre(rpc) *j* to rc by vehicle*v* in period *t* (ton)

qi_{jnpvt} . quantity of consumed part *p* sending from *n* technology-level reprocessing centre(rpc) *j* to ds by vehicle *v* in period *t* (ton)

ip_{kot} . quantity of reproduced product sending from *n* technology-level remanufacturing center (rmp) *k* to secondary market by vehicle *v* in period *t* (ton)

ri_{kompt} . quantity of new part *p* moving from nms*m*to *o* technology-level remanufacturing center((rmp) *k* by vehicle *v* in period *t* (ton)

i_{kopt} . the size of part *p* for o technology-level remanufacturing center((rmp) *k* in period *t* (ton)

ε_{it} = {1, if collecting centre(cc) *i* is opened/*o*. .else

ζ_{jnt} = {1, if *n* technology-level reprocessing centre(rpc) *j* is allowed for use /*o*, .else

η_{kot} = {1, if *o* technology-level remanufacturing center((rmp) *k* is allowed for use /*o*, .else

θ_{ivt} = {1, if vehicle *v* does transportation from customers to collecting centre(cc) *i*/*o*, .else

κ_{ijvt} = {1, if vehicle *v* does transportation from collecting centre(cc) *i* to reprocessing centre(rpc) *j*/*o*, .else

λ_{jkvt} = {1, if vehicle *v* does transportation from reprocessing centre(rpc) *j* to remanufacturing center((rmp) *k*/*o*, .else

μ_{jlvt} = {1, if vehicle *v* does transportation from reprocessing centre(rpc) *j* to spare market *l*/*o*, .else

ν_{jvt} = {1, if vehicle *v* does transportation from reprocessing centre(rpc) *j* to rc/*o*), .else

ξ_{jvt} = {1, if vehicle *v* does transportation from reprocessing centre(rpc) *j* to ds/*o*. .else

- **objective function**

The goal of objective function is to reduce the overall expenses to the possible extent and it can be expressed as stated below

Minimize Z = Total Cost – Total Income

Where:

- Total Cost comprises five key components, including:

- Total Transportation Cost (TTCost): Expenses associated with transporting goods across various stages of the network.
- Purchasing Cost (PCost): Costs incurred in acquiring raw materials and used products.
- Operating Cost (OCost): Expenditures related to the day-to-day operations of the system.
- Facility Cost (FCost): Fixed and variable costs of maintaining collection centers, reprocessing units, and remanufacturing facilities.
- Inventory Cost (ICost): Costs of holding and managing inventory at various locations.
- Environmental Cost (ECost): Expenses resulting from carbon emissions and environmental impacts.

TCOST = TTCOST + PCOST + OCOST + FCOST + ICOST + ECOST (18)

Transportation cost (TTC): Products travel through a series of stops throughout the transportation chain, beginning with the consumer and ending at the disposal location. This includes stops at centers which does the job of reprocessing, companies which does the job of re producing, markets which cater the need of spare parts, facilities which does the job of recycling , and collection centers. The following is the formula for determining the overall transportation expense:

$$\text{TTC cost}= \Bigg(\sum_i \sum_v \sum_t X_{ivt} ta_{iv} da_i + \sum_i \sum_j \sum_n \sum_v \sum_t Y_{ijnvbt} tb_{ijv} db_{ij} ;'$$

$$\text{llll} + \sum_j \sum_n \sum_k \sum_o \sum_p \sum_v \sum_t Z_{jnkopvt} tC_{jkpv} DC_{jk} + \sum_j \sum_l \sum_n \sum_p \sum_v \sum_t U_{jnlpvt} td_{jlpv} dd_{jt}$$

$$\text{ll} + \sum_j \sum_n \sum_p \sum_v \sum_t W_{jnpvt} te_{jpv} de_j + \sum_j \sum_n \sum_p \sum_v \sum_t Q_{jnpvt} tf_{jpv} df_j \Bigg) \text{l} \quad (8)$$

Cost of *Purchasing* (*PC*) is considered as the sum of two components:

- The expense associated with procuring pre-owned items from consumers.
- The cost of procuring new components from module suppliers to meet the demand of remanufacturing plants.

$$\text{P} = \left(\sum_i \sum_v \sum_t X_{ivt} pp + \sum_k \sum_o \sum_m \sum_p \sum_t R_{kompt} pc_{mp} \right) \quad (9)$$

Cost of *Operating* (*OP*) include:

- Reprocessing costs, which cover review and dismantling at reprocessing centers.
- Remanufacturing costs incurred at remanufacturing centers (RMP).
- Recycling costs at recycling centers (RC).
- Disposal costs at disposal centers (DC).

$$\text{OPcost} = \Bigg(\sum_i \sum_j \sum_n \sum_v \sum_t Y_{ijnvt} dc_{jn} + \sum_i \sum_j \sum_n \sum_p \sum_v \sum_t Y_{ijnpvt} \alpha_p ic_{jnp}$$

$$+ \sum_k \sum_o \sum_t P_{kot} rc_{ko} + \sum_j \sum_n \sum_p \sum_v \sum_t W_{jnpvt} ec_p + \sum_j \sum_n \sum_p \sum_v \sum_t Q_{jnpvt} ac_p \Bigg) \quad (10)$$

C*ost of Facility* (*FC*) are associated with fixed operational expenses of collection hubs, reprocessing hubs, and remanufacturing hubs within the network. The total facility cost is determined accordingly.

$$\mathrm{FC} = \left(\sum_{i}\sum_{t} fc_i \varepsilon_{it} + \sum_{j}\sum_{n}\sum_{t} fr_{jn}\zeta_{jnt} + \sum_{k}\sum_{o}\sum_{t} fm_{ko}\eta_{kot} \right) \ldots\ldots \tag{11}$$

C*ost Inventory* (*IC*): Inventory is only maintained by remanufacturing factories inside the network. The entire inventory cost is computed based on this information.

$$\text{Icost IC} = \left(\sum_{k}\sum_{o}\sum_{p}\sum_{t} I_{kopt} hc_p \right) \tag{12}$$

C*ost of Environmental issues* (*EC*):The expense associated with CO2 emissions from cars, as well as the cost of detrimental gasses emitted into the environment by operational reprocessing and reproduction centers..

$$\begin{aligned} \mathrm{EC} = C_{CO2} \Bigg[\sum_{v} Ltv_v \Bigg(& \sum_{i}\sum_{t} X_{ivt} da_i + \sum_{i}\sum_{j}\sum_{n}\sum_{t} Y_{ijnvt} db_{ij} \\ & + \sum_{j}\sum_{n}\sum_{k}\sum_{o}\sum_{p}\sum_{t} Z_{jnkopvt} dc_{jk} + \sum_{j}\sum_{l}\sum_{n}\sum_{p}\sum_{t} U_{jnlpvt} dd_{jk} \\ & + \sum_{j}\sum_{n}\sum_{p}\sum_{t} W_{jnpvt} de_j + \sum_{j}\sum_{n}\sum_{p}\sum_{t} Q_{jnpvt} df_j \Bigg) \\ & + \left(\sum_{i}\sum_{j}\sum_{n}\sum_{v}\sum_{t} Y_{ijnvt} Lrpc_{jn} + \sum_{k}\sum_{o}\sum_{t} P_{kot} Lrmp_{ko} \right) \Bigg] \end{aligned} \tag{13}$$

Total revenue (*TR*Total revenue is derived from two sources:

- Revenue generated from scrutinized parts sold to repair markets.
- Revenue from remanufactured goods sold to the subordinate market.

$$\mathrm{TR} = \left(\sum_{j}\sum_{l}\sum_{n}\sum_{p}\sum_{v}\sum_{t} U_{jnlpvt} + \sum_{k}\sum_{o}\sum_{t} P_{kot} se_k \right) \tag{14}$$

- **Constraints belonging to objective function**

$$\sum_{v} X_{ivt} - \sum_{j}\sum_{n}\sum_{v} Y_{ijnvt} = 0\ 1 \qquad \forall i,t \tag{15}$$

$$\sum_{i}\sum_{v} Y_{ijnvt} \alpha_p - \left(\sum_{k}\sum_{o}\sum_{t} Z_{jnkopvt} + \sum_{l}\sum_{v} U_{jnlpvt} + \sum_{v} W_{jnpvt} + \sum_{v} Q_{jnpvt} \right) = 0$$

$\forall j,n,p,t$ (16

1$\forall k,o,p,t$ (28)

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 7s (2025)

$$s\beta\left(\sum_i\sum_n\sum_v Y_{ijnvt}\alpha_p\right)-\sum_n\sum_l\sum_v U_{jnlpvt}=0\ \forall j,p, \quad (17)$$

$$s\quad\gamma\left(\sum_i\sum_n\sum_v Y_{ijnvt}\alpha_p\right)-\sum_n\sum_k\sum_o\sum_v Z_{jnkopvt}=0\ \forall j,p,t \quad (18)$$

s$\forall j,p,t$ (31)

$$[1-(\alpha+\beta+\theta)]\left(\sum_i\sum_n\sum_v Y_{ijnvt}\alpha_p\right)-\sum_n\sum_v Q_{jnpvt}=0\ \forall j,p,t \quad (19)$$

$$s\sum_k\sum_o P_{kot}=d_t \qquad \forall t \quad (20)$$

$$k\sum_v X_{ivt}\le ca_i\varepsilon_{it} \qquad \forall i,tk \quad (21)$$

$$\sum_i\sum_v Y_{ijnvt}\le cb_{jn}\zeta_{jnt} \qquad \forall j,n,t\text{k} \quad (22)$$

$$\sum_j\sum_n\sum_v Z_{jnkopvt}+\sum_m R_{kompt}+I_{kop(t-1)}\le cc_{kop}\eta_{kot}\ \forall k,o,kp,t \quad (23)$$

k $\forall lk,p,t$ (37)

k $\forall p,kt$ (38)

$$\sum_j\sum_n\sum_v Q_{jnpvt}\le cf_p \qquad \forall p,t \quad (24)$$

k $\forall m,p,t$ (40)

$$X_{ivt}\le X\theta_{ivt}\,\text{k} \qquad \forall i,v,t \quad (25)$$

$$\sum_n Y_{ijnvt}\le M\kappa_{ijvt}\,\text{k} \qquad \forall i,j,kv,t \quad (26)$$

$$\sum_n\sum_o\sum_p Z_{jnkopvt}\le M\lambda_{jkvt}\,\text{k} \qquad \forall j,k,v,kt \quad (27)$$

$$\sum_n\sum_p U_{jnlpvt}\le M\mu_{jlvtg}\,\text{k} \qquad \forall j,l,kv,t \quad (28)$$

$$\sum_n\sum_p W_{jnpvt}\le M\nu_{jvt} \qquad \forall j,v,t \quad (29)$$

$$\sum_n\sum_p Q_{jnpvt}\le M\xi_{jvt} \qquad \forall j,v,t \quad (30)$$

$\theta_{ivt}\le X_{ivtk}$ $\forall i,v,t$ (31

$$\kappa_{ijvt}\le\sum_n Y_{ijnvt}\,\text{k} \qquad \forall i,j,v,t \quad (32)$$

$$\lambda_{jkvt} \leq \sum_{n}\sum_{o}\sum_{p} Z_{jnkopvt}\ \text{k} \qquad \forall j,k,v,t \qquad (33)$$

$$\mu_{jlvt} \leq \sum_{n}\sum_{p} U_{jnlpvt} \qquad \forall j,ll,v,t \qquad 34$$

k $\forall j,kv,t$ (51)

$$\xi_{jvt} \leq \sum_{n}\sum_{p} Q_{jnpvt} \qquad \forall j,v,t \qquad (35)$$

$$1\sum_{v}\theta_{ivt} \leq 1 \qquad \forall i,lt \qquad (36)$$

$$1\sum_{v}\kappa_{ijvt} \leq 1 \qquad \forall i,jl,t \qquad (37)$$

$$1\sum_{v}\lambda_{jkvt} \leq 1 \qquad \forall j,k,lt \qquad (38)$$

$$1\sum_{v}\mu_{jlvt} \leq 1 \qquad \forall j,l,tl \qquad (39)$$

$$1\sum_{v}\nu_{jvt} \leq 1 \qquad \forall j,lt \qquad (40)$$

$$\sum_{v}\xi_{jvt} \leq 1 \qquad \forall j,t \qquad (41$$

$$X_{ivt}, Y_{ijnvt}, Z_{jnkopvt}, U_{jnlpvt}, W_{jnpvt}, Q_{jnpvt}, Pl_{kot}, R_{kompt} \geq 0 \qquad \forall i,j,k,l,m,n,p,t \qquad (59)$$

$$\varepsilon_{it}, \zeta_{jnt}, \eta_{knt}, \theta_{ivt}, \kappa_{ijvt}, \lambda_{jkvt}, \mu l_{jlvt}, \xi_{jvt} \in \{0, 1\} \qquad \forall i,j,k,l,n,v,t \qquad (60)$$

Constraints:

- Constraint 15: This ensures that the entire amount of goods that are delivered from collection centers to reprocessing facilities is equivalent to the total quantity of products that are collected at collection centers.
- Constraint 16: Maintains product flow balance across the network.
- Constraint 17: Balances the remanufactured product output and inventory in remanufacturing plants by equating it to the summation of incoming parts from recycling centers, novel parts from module suppliers, and previous inventory.
- Constraint 18: Indicating that a certain proportion of the components that are stored in reprocessing facilities are ultimately sold on the market.
- Constraints 19, 20, and 21: Define the percentages of parts allocated for remanufacturing, recycling, and disposal, respectively.
- Constraint 22: Serves as the demand constraint, ensuring product availability aligns with market demand.
- Constraints 23–29: Represent capacity constraints across various facilities.
- Constraints 30–41: Ensure no transportation occurs if facilities are closed and require facilities to be operational for transportation to take place.
- Constraints 42–47: Restrict transportation between facilities to a single type of vehicle.

Data considered

Table 1 : component level calculation for product

Product	Com.	UC	ER	CP	FQL_{ik}	ω_{ik}	PRO^b
Pr1	EP-11	6	0.71	0.65	0.57	0.25	rem
	EP-12	4	0.73	0.80	0.64	0.25	rem
	EP-13	7	0.99	0.75	0.81	0.20	reu/ref
	EP-14	8	0.89	0.70	0.66	0.10	rem
	EP-15	5	0.72	0.85	0.63	0.10	rem
Pr2	EP-21	5	0.871	0.701	0.631	0.25	rem
	EP-22	4	0.290	0.451	0.451	0.201	rem
	EP-23	6	0.73	0.60	0.58	0.25	rec/rep
	EP-24	4	0.211	0.221	0.331	0.20	rec/rep
	EP-25	5	0.361	0.381	0.421	0.25	rec/rep
Pr3	EP-31	6	0.77	0.65	0.61	0.05	rem
	EP-32	8	0.83	0.90	0.79	0.10	rem
	EP-33	7	0.790	0.75	0.55	0.10	rem
	EP-34	8	0.71	0.75	0.65	0.20	rem
	EP-35	8	0.95	0.50	0.70	0.22	rem
Pr4	EM-41	9	0.93	0.90	0.83	0.23	Reu/ref
	EM42	4	0.75	0.75	0.66	0.25	rem
	EM43	3	0.58	0.45	0.22	0.20	rec/rep
	EM44	8	0.70	.44	0.50	0.20	rem
	EM45	4	0.54	0.40	0.43	0.15	rec/rep
pr5	EP51	3	0.47	0.38	0.42	0.18	rec/rep
	EP52	6	0.79	0.62	0.64	0.18	rem
	EP53	S	0.78	0.50	0.57	0.18	rem
	EP54	3	0.94	0.20	0.64	0.18	rem
	EP55	6	0.45	0.45	0.45	0.18	rec/rep

Table 2.Data considered. For parameters group1

Parameters	Part1	Part2	Part3	
wc_p .	1.5	2	0.5	
ac_p	0.5	0.2	0.5	
hc_p	1.5	2.2	2.5	
wcp	0.11	0.22	0.11	
ce_p	410	520	500	
cf_p	13	21	4	
New module suppliers 1qc_{mp}	15	23	6.7	
2qc_{mp}	17	22	7	
3qc_{mp}	16	21.5	6.5	
Techonological-level1ic_{jnp}	3	2	0.75	
2ic_{jnp}	4	2.9	0.5	
ta_{iv} Vehicle1	1.1	2.3	3.6	
-Do -Vehicle 2	6.5	7	8	
Vehicle1. tb_{ijv}	0.2	1.3	2.4	
Vehicle 2 –do-	4.5	6	8	
Vehicle1tc_{jkpv}	0.55	1.1	1.8	
Vehicle 2 –do-	5.5	7.4	4.8	
Vehicle1td_{jlpv}	0.62	1.3	0.9	
Vehicle 2 –do-	5.3	5.1	7.9	

Table 3 :Data considered. For parameters group2

Collection centers			**1**	**2**	**3**
ta_{iv}	Vehicles	1	3	2.8	3
		2	4.5	4.3	4.5
tb_{ijv} (equal for all)	Vehicles	1	2	2	2
		2	5.7	5.7	5.7
Re-processing centers			**1**	**2**	**3**
se_k			160	165	170
dcollectingcentre(CC)$_{jn}$	Technologylevels	1	3	3	3

		2	3.5	3.5	.35
Re-manufacturing plants			**1**	**2**	**3**
rc_{ko}	Technology-levels	1	10	10	10
		2	10.8	10.8	10.8

Table4: Data considered. For parameters group3

Collection centers			**1**	**2**	**3**	**4**
fc_i			50	56	75	72
cd_i			400	550	500	350
da_i			50	56	75	20
db_o	Reprocating centers	1	60	50	80	58
		2	50	60	40	65
		3	70	80	20	70
Re-processing centers			**1**	**2**	**3**	
fr_{jn}	Technologylevels	1	2000	1000	1500	
		2	800	800	500	
LReprocessing centre(RPC)$_{jn}$	Technologylevels	1	35	28	32	
		2	50	48	49	
cb_{jn}	Technologylevels	1	100	300	280	
		2	600	500	205	
dc_{jk}	Remanufacturing plants	1	60	50	120	
		2	100	80	90	
		3	70	90	60	
dc_i			100	86	75	
df_j			180	250	200	
Remanufacturing plants			**1**	**2**	**3**	
frn_{kot}	Technologylevels	1	29,000	30,000	10,800	
		2	1000	9500	1050	
LRemanufacturingcenter((RMP)$_{kot}$	Technologylevels	1	50	58	50	
		2	70	76	75	
collecting centre(CC)$_{kop}$	Technology-levels	1	200	300	250	
(same for all *p*)		2	600	550	400	
dd_{ji}	Spare markets	1	60	50	120	

	2	100	80	40	
	3	70	90	60	
	4	100	80	40	
	5	70	90	60	
Spare markets	**1**	**2**	**3**	**4**	**5**
cd_{ip}(same for all p)	500	300	280	300	400
New module suppliers	**1**	**2**	**3**	**4**	**5**
cg_{mp}(same for all p)	500	300	280	300	500

Technology Level and Vehicle Analysis

In the planning concept, which encompasses three time periods, Technology Level 1 and Vehicle 2 are seen as ecologically sustainable alternatives. It is assumed that 500 used items are originally returned by consumers, each item consisting of five components. The unit acquisition cost of used items (pp) is established at 70 Rs, whereas the per unit cost of CO_2 emissions is 0.2 Rs/g. There are needs for 100, 300, and 150 units of the product for the first, second, and third periods, respectively. Vehicle 1 emits 22 g/ton.km of CO_2 every kilometer, whereas Vehicle 2 emits 3 g/ton.km. The order of the ratios are anticipated as follows:

- Reused (β\betaβ): 30%
- Remanufactured (γ\gammaγ): 45%
- Recycled (δ\deltaδ): 12%

Other key parameters are detailed in Tables 2, 3, and 4.

Solution Results

The whole income comes to 1,001,120 rupees, whereas the total expenditure is assessed to be 1,031,020.43 rupees. 930,908.43 rupees is the value of the optimum goal function, which minimizes the total amount of costs. A visual representation of the ideal flow for each and every period is shown in Figure 3, which illustrates the following operational choices:

- 1st and 3rd Periods:
  - Collection Centers: Only Collection center one is operational.
  - Reprocessing Centers: RPC3 with Technology Level 1 is used.
  - Remanufacturing Centers: RMP3 with Technology Level 2 is active.
- Second Period:
  - Collection Centers: Only CC1 is operational.
  - Reprocessing Centers: RPC3 with both Technology Levels are utilized.
  - Remanufacturing Centers: RMP3 with Technology Level 2 is active.
- Transportation:
  - Vehicle 2 is used to transport items from clients to collecting locations and back again to reprocessing plants.
  - All other transportation activities are carried out using Vehicle 1 across all periods.

Insights and Environmental Considerations

After transportation expenses, the environmental cost is the second largest part of the overall cost. Despite Vehicle 1 being less environmentally friendly, it was predominantly used due to its lower operational costs.

- For reprocessing centers, Technology Level 1, which prioritizes environmental sustainability, was preferred in every period.
- For remanufacturing centers, Technology Level 2 was chosen to achieve cost efficiency despite higher CO_2 emissions.

This strategy uses a variety of transportation alternatives and technological levels strategically at different points in the network to strike a good balance between reducing costs and protecting the environment.

Results and Sensitivity Analysis

The total cost, revenue, and expense minimization outcomes are as follows:

- Total Cost: 1,031,020.43 Rs
- Total Revenue: 1,001,120 Rs
- Minimized Expense: 930,908.43 Rs

A sensitivity analysis was directed to measure the impact of CO_2 emissions, with findings summarized in Table 6. Adjustments to CO_2 emission levels (from RPCs, RMPs, and vehicles) revealed:

- Purchasing Costs (PC): Unaffected by changes in CO_2 emissions.
- Operating Costs (OC): Showed a slight decrease as CO_2 emissions increased.
  - Low CO_2 emissions led to higher operating costs because of the preference for Technology Level 2 services.
  - High CO_2 emissions prompted the use of Technology Level 1 facilities, which emit less CO_2 but have slightly higher operating costs.

These results demonstrate the model's adaptability in balancing environmental impact with operational expenses, ensuring a sustainable yet cost-effective solution.

Table-5: The optimal objective function value

Objective function	Value (Rs)
TC	1,031,0.2043
TTC	540,14700
PC	72,47690
OC	12,17002
FC	8378.0
IC	57,
RC	397,79400
TR	100,11200
Z	930,90843

Table-6: The optimal objective function values acollectingcentre(CC)ording to CO_2 emission ratios

O.F	–50%	–40%	–30%	–20%	–10%	Value	+10%
TC	720,41160	784,27760	848,14360	912,00957	975,87557	1,031,02040	1,070,800
TIC	309,61500	309,61500	309,61500	309,61500	309,61500	540,14700	540,14700
PC	72,47690	72,47690	72,47690	72,47690	72,476.0	72,47690	72,47690
OC	13,27966	13,27966	13,27966	13,27966	13,27966	12,17002	12,17002
FC	561600	561600	561600	561600	561600	837500	837500
IC	9401	9401	9401	9401	9401	5751	57.51
EC	319,33000	383,19600	447,06200	510,92800	574,79400	397,79400	437,57340
TR	94,61200	94,61200	94,61200	94,61200	94,61200	100,112000	100,11200
Z	625,7996	689,6656	753,5316	817,39757	881,26370	930,90843	970,68780

- **Fuzzy-linear-programming-analysis**

A fuzzy multi-objective linear programming model has been formulated for the case study. The problem is modeled and solved using fuzzy set theory, where all objective functions are considered as fuzzy variables. Following the fuzzification process, the multi-objective model is transformed into a single-objective model, addressing one objective sequentially.

Solution Results

Tables 7 and 8 provide the solution outcomes for the seven goal functions, using the reward matrix dataset with the corresponding lower and higher limits. These results are derived following the Zimmerman approach, ensuring an optimized and balanced solution.

Integration with Weights Using BWM

The weights obtained through the Best-Worst Method (BWM) were incorporated into the fuzzy linear programming model to enhance its performance and applicability.

Application of BWM

Company administrators provided pairwise comparison vectors regarding the objective functions, as illustrated in Table 9. Using BWM, the importance weights for the objective functions were calculated, ensuring the model accurately reflects the decision-makers' priorities. The calculated weights serve to guide the optimization process effectively, aligning the solutions with organizational goals.

*w*1¼ 0:15*w*2¼ 0:09, *w*3¼ 0:079, *w*4¼ 0:062, *w*5 ¼ 0:034,

*w*6 ¼ 0:374, *w*7 ¼ 0:249 and *n* = 0.063.

Table-7: the solution resuts for various for z1,z2,z3

Objective functions	Z_1	Z_2	Z_3
*Z*1 (min)	**273,61500**	75,950.1800	12,803.3600
*Z*2 (min)	863,081.62	**72,476.90**	12,38498

*Z*3 (min)	792,12739	78,043.09	**11,51866**
*Z*4 (min)	610,37206	127,277.2	13,27966
*Z*5 (min)	691,27683	72,34370	12,37899
*Z*6 (min)	972,38681	78,23128	11,51866
*Z*7 (max)	3,730,879.1	384,85179	42,43932

Table-8: The data set for membership functions

Objective functions	μ = 0	μ = 1	μ = 0
Z_1	–	273,615	3,730,879.01
Z_2	–	72,476.90	384,851.79
Z_3	–	11,218.66	42,449.32
Z_4	–	4712	240,309
Z_5	–	0	9578.49
Z_6	–	109,266.40	2,992,329.55
Z_7	94,612	129,204.80	–

Table-9: bwm method

Objective functions		(BWM)		
		Weights	λ	Value (Rs)
Z_1	TTC	0.146	0.8	930.61300
Z_2	PC	0.088	1	72,49278
Z_3	OC	0.073	0.98	13,22947
Z_4	FC	0.073	0.98	837500
Z_5	IC	0.055	1	1271
Z_6	EC	0.37	1	118474700
Z_7	TR	0.319	0.16	100,112

$$\text{Maximize } 0.192 * \lambda_1 + 0.133 * \lambda_2 + 0.075 * \lambda_3 + 0.043 * \lambda_4 + 0.016 * \lambda_5 + 0.313 * \lambda_1 + 0.228 * \lambda_7$$

Subject to

$$\lambda_2 \leq \frac{3854851.79-(PC)}{312374.89}$$

$$\lambda_4 \leq \frac{240309-(FC)}{235597}$$

$$\lambda_5 \leq \frac{9578.49-(IC)}{9578.49}$$

$$\lambda_6 \leq \frac{2992329.55-(EC)}{2882863.15}$$

$$\lambda_7 \leq \frac{(TR)*94612}{34592.8}$$

and all other constraints.

The proposed model identifies the best locations for infrastructural facilities, concentrating on both environmental advantages and cost savings. It encompasses all 5kinds of costs, prioritizing the reduction of environmental expenses. The model's first component addresses CO2 emissions from vehicles, while the second focuses on emissions related to processing centers.

The study incorporates various facilities: disposal sites, recycling facilities, reprocessing facilities, remanufacturing factories, secondary markets, spare parts marketplaces, and new spare providers. It evaluates returns through reuse, reproduction, recycling, and disposal. Reproduced products can include both new and used parts, accommodating missing components as needed. Manufacturers aim to recover each returned product through all feasible means while supplying necessary new parts.

The proposed formulation optimizes total costs while prioritizing resource conservation and carbon emission reduction. A mixed-integer linear programming (MILP) model is utilized to encounter real-world constraints. The integration of weights derived through the Best-Worst Method (BWM) into a fuzzy linear programming model further refines the outcomes. This approach allows enterprises to choose operational and transportation strategies that balance carbon emission reductions with profit maximization, as highlighted in Table 6.

Conclusions

The model successfully addresses the challenges of carbon emission reduction and cost optimization in supply chain networks. By incorporating options for reuse, remanufacturing, recycling, and disposal, it provides a holistic approach to sustainable operations. It also allows enterprises to select appropriate strategies for transportation and facility management to achieve environmental and economic goals.

Future Scope

The current model can be extended in the following directions:

- • Integrating stochastic parameters and fuzzy constraints, including variations in capacity and demand.
- Expanding the model to accommodate multiple products.
- Creating an all-encompassing supply chain model by combining forward and reverse logistics.

These extensions would further enhance the model's applicability and effectiveness in real-world scenarios.

References

[1] Andrews, D., 2015. The circular economy, design thinking and education for sustainability. Local Econ. 30 (3), 305–315.

[2] Esposito, M., Tse, T., Soufani, K., 2017. Is the circular economy a new fast-expanding market? Thunderbird Int. Bus. Rev. 59 (1), 9–14.

[3] Prieto-Sandoval, V., Jaca, C., Ormazabal, M., 2018. Towards a consensus on the circular economy. J. Clean Prod. 179, 605–615.

[4] Schroeder, P., Dewick, P., Kusi-Sarpong, S., Hofstetter, J.S., 2018. Circular economy and power relations in global value chains: tensions and trade-offs for lower income countries. Res., Conserv. Recycl. 136, 77–78.

[5] Li, W., Gong, G., Ren, Z., Ouyang, Q., Peng, P., Chun, L., & Fang, X. (2022). A method for energy consumption optimization of air conditioning systems based on load prediction and energy flexibility. Energy, 243, 123111

[6] Baghoolizadeh, M., Rostamzadeh-Renani, M., Hakimazari, M., & Rostamzadeh-Renani, R. (2023). Improving CO2 concentration, CO2 pollutant and ocollectingcentre(CC)upants' thermal comfort in a residential building using genetic algorithm optimization. Energy and Buildings, 291, 113109

[7] Chaturvedi, S., Bhatt, N., Gujar, R., & Patel, D. (2022). Application of PSO and GA stochastic algorithms to select optimum building envelope and air conditioner size-A case of a residential building prototype. Materials Today: Proceedings, 57, 49-56.

[8] Akyüz, A., Yıldırım, R., Gungor, A., & Tuncer, A. D. (2023). Experimental investigation of a solarassisted air conditioning system: Energy and life cycle climate performance analysis. Thermal Science and Engineering Progress, 101960.

[9] Marwan, M., Jamal, J., Hamid, A., Nasir, N., La Nafie, N. A., Gunawan, A., ... &Musaruddin, M. (2023). Optimize electrical energy cost of air conditioning considering to different wall characteristics. Results in Engineering, 17, 100990.

[10] MacArthur, E. (2013). Towards the circular economy. *Journal of Industrial Ecology, 2*(1), 23-44.Murray, A., Skene, K., & Haynes, K. (2017). The circular economy: an interdisciplinary exploration of the concept and application in a global context. *Journal of business ethics*, *140*, 369-380.Geng, Y., Sarkis, J., &Bleischwitz, R. (2019). How to globalize the circular economy. *Nature*, *565*(7738), 153-155.

[11] Velenturf, A. P., & Purnell, P. (2021). Principles for a sustainable circular economy. *Sustainable Production and Consumption*, *27*, 1437-1457.

[12] Tautenhain, C. P., Barbosa-Povoa, A. P., & Nascimento, M. C.(2018). A multi-objective matheuristic for designing and planning sustainable supply chains. Computers & Industrial Engineering,135, 1203–1223.

[13] https://doi.org/10.1016/j.cie.2018.12.062.

[14] Dubey, R., Gunasekaran, A., Childe, S. J., Papadopoulos, T., & Fosso Wamba, S. (2017). World class sustainable supply chain

[15] Zhen, L., Huang, L., & Wang, W. (2019). Green and sustainable closed-loop supply chain network design under uncertainty. Journal of Cleaner Production, 227, 1195–1209.

[16] Ramos, T. R. P., Gomes, M. I., & Barbosa-Povoa, A. P. (2014).Planning a sustainable reverse logistics system: Balancing costs with environmental and social concerns. Omega, 48, 60–74.

[17] Hapuwatte, B.M., Badurdeen, F., Bagh, A., Jawahir, I.S., 2022. Optimizing sustainability performance through component commonality for multi-generational products. Resour. Conserv. Recycl. 180, 105999.

[18] Yang, T., Li, S., Duan, S., Tao, Y., Han, X., 2023. Performance degradation model and reliability evaluation of brush DC motor for the intelligent on–off valve. J. Electr. Eng. Technol. 18 (3), 1909–1918.

[19] Zhang, X., Zhang, S., Zhang, L., Qiu, L., 2021. Reverse design for remanufacture based on failure feedback and polychromatic sets. J. Clean. Prod. 295, 126355.

[20] **Marie Remery ,Christian Mascle&Bruno Agar ,** A new method for evaluating the best product end-of-life strategy during the early design phaseJournal of Engineering Design 23(6):419-441,2012

[21] 20 . Waqas Ahmed , Vanajah Siva , Jenny Backstrand , Niklas Sarius , Hans-Åke Sundberg ,Circular economy: Extending end-of-life strategies ,Sustainable Production and ConsumptionVolume 51, November 2024, Pages 67-78