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New General Complex Integral Transform on Time Scales

Dipali Kaklij[1], Dinkar Patil[2]
[1]K. R. T. Art's, B. H. Commerce and A. M. Science College, Nashik, Maharashtra, India.
[2]Art's and Commerce College, Wadala, Nashik, Maharashtra, India.

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Abstract:

Introduction: We begin by defining the New General Complex Integral Transform [13], and then we present a New General Complex Integral Transform on Time Scales $\mathbb{T}$. To solve a variety of dynamic equations with beginning values or boundary conditions that are represented by integral equations, integral transform methods are frequently employed. In order to solve dynamic equations, the New General Complex Integral transform on Time Scale is presented in this article.

Objectives: Within the Laplace Transform class, we provide the New General Complex Integral transform on Time Scales in this study. We examine this transform's characteristics. An initial value problem with a dynamic form of the equation is the primary focus of this research.

Methods: Differential equations of any order and the integral of a function can both be solved using the New General Complex Integral Transform on Time Scales. By establishing the convolution theorem, the idea of convolution is examined in further detail.

Results: This integral transform is used for solving higher order initial value problems and integral equations.

Keywords: Time scales, new general integral transform, dynamic equation.

1. Introduction

An arbitrary nonempty closed subset of real numbers is called a time scale $\mathbb{T}$. Due in part to transform methods for solving differential equations, transforms are essential in analysis. Many significant changes have been introduced over the past 20 years, including Kamal [2], Shehu [6], Soham [7], Sumudu [10], Sawi [12], Kushare [14], Elzaki [17] and others. Additionally, a few time-scale integral transforms are previously introduced. In 2007, John M. Devis et al. examined the Laplace transform on time scales [8]. In 2012, Hassan Ahmed Agwa introduced the Sumudu transform on time scales [1]. The α-Laplace transform on time scales [16] was introduced by T.G. Thange et al. in 2023. He also presented a new general integral transform on time scales [15] in 2024.

Noting that it is an extension of the new general complex integral transform, we define the new general complex integral transform on time scales. In addition to discussing situations when results might not be generalized from the real example to time scales, we provide features of this transform. This transform is used in examples to solve dynamic equations. The integral equations are also solved using the transform. The prospects for a general complex integral transform theory based on time scales are finally discussed.

2. Basic Results

2.1 Forward jump operator [4]: $\sigma: \mathbb{T} \to \mathbb{T}$ is defined by $\sigma(t) = \inf\{s \in \mathbb{T} \mid s > t\}$.

2.2 Backward jump operator [4]: $\rho: \mathbb{T} \to \mathbb{T}$ is defined by $\rho(t) = \sup\{s \in \mathbb{T} \mid s < t\}$.

2.3 Forward graininess function [4]: $\mu: \mathbb{T} \to [0, \infty)$ is defined by $\mu(t) = \sigma(t) - t$.

2.4 Definition 1 [5] A function f is called regulated provided its right-sided limits exist at all right dense points in $\mathbb{T}$ and left sided limits exists at all left dense points in $\mathbb{T}$.

2.5 Definition 2 [5] A function f is called rd-continuous provided it is continuous at right dense points in $\mathbb{T}$ and its left-sided limits exists at left dense points in $\mathbb{T}$. We denote the set of rd-continuous functions by C_{rd}.

2.6 Definition 3 [5] A function f:$\mathbb{T}\to \mathbb{C}$ is called regressive if $1 + \mu(t)f(t) \neq 0\ \forall t \in \mathbb{T}$. Here $\mathcal{R}$ denotes the set of regressive functions.

2.7 Definition 4 [5**]** The function $f: \mathbb{T} \to \mathbb{R}$ is said to be of exponential type-I if there exists constants $M, c > 0$ such that $|f(t)| \leq Me^{ct}$. Furthermore, f is said to be of exponential type-II if there exists constants $M, c > 0$ such that $|f(t)| \leq Me_c(t, 0)$.

2.8 Definition 5 [5] For $f \in \mathcal{R}$ the time scale exponential function is defined as $e_f(t,s) = exp\left(\int_s^t \xi_{\mu(\tau)} f(\tau)\Delta\tau\right)$ for $s, t \in \mathbb{T}$ and $\xi_{\mu(t)}$ is a cylinder transformation.

2.9 Definition 6 [4] We say that a function $f: \mathbb{T} \to \mathbb{R}$ is delta differentiable at $t \in \mathbb{T}^{\kappa}$ if there exists a number $f^{\Delta}(t)$ such that for all $\epsilon > 0$ there exists a neighbourhood U of t such that $\left|f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s)\right| \leq \epsilon|\sigma(t) - s|$ for all $s \in U$. ($\mathbb{T}^{\kappa} \coloneqq \mathbb{T}\backslash\{\sup \mathbb{T}\}$)

2.10 If $\mathbb{T} = \mathbb{R}$, then $f: \mathbb{R} \to \mathbb{R}$ is delta differentiable at $t \in \mathbb{R}$ if and only if f is differentiable in the ordinary sense at t. That is $f^{\Delta}(t) = \frac{df}{dt}$. [9]

2.11 Laplace transform on time scales [5]: Assume that $x: \mathbb{T} \to \mathbb{R}$ is regulated. Then the Laplace transform of x is defined by $\boldsymbol{\mathcal{L}}\{x\}(z) = \int_0^{\infty} e^{\sigma}_{\ominus z}(t, 0)x(t)\Delta t$ for $z \in \mathcal{D}\{x\}$ where $\mathcal{D}\{x\}$ consists of all complex numbers $z \in \mathcal{R}$ for which improper integral exists.

2.12 Sumudu transform on time scales [1]: Assume that $f: \mathbb{T} \to \mathbb{R}$ is rd-continuous function, then the Sumudu transform of f is $S\{f\}(u) = \frac{1}{u}\int_{t_0}^{\infty} e^{\sigma}_{\ominus\frac{1}{u}}(t, t_0)f(t)\Delta t$ For $u \in \mathcal{D}\{f\}$ where $\mathcal{D}\{f\}$ consists of all complex numbers $u \in \mathcal{R}$ for which improper integral exists.

2.13 New General Integral Transform [11]: Let $f(t)$ be an integrable function defined for

$t \geq 0, p(s) \neq 0$ and $q(s)$ are positive real functions then define New General Integral Transform $\mathcal{T}(s)$ of $f(t)$ by the formula $T\{f(t), s\} = \mathcal{T}(s) = p(s)\int_0^{\infty} f(t)e^{-q(s)t}dt$. Provided that the integral exists for some $q(s)$.

2.14 New General Integral Transform on Time Scales [15]: Let $g: \mathbb{T} \to \mathbb{C}$ is an rd-continuous function with $p_1(z), p_2(z): \mathbb{R} \to \mathbb{C}$ are positively regressive functions. Define the new general integral transform on time scale $\mathcal{G}(z)$ for the function $g(t)$ by the formula

$\mathrm{N}(g(t))(z) = \mathcal{G}(z) = p_1(z)\int_{t_0}^{\infty} e^{\sigma}_{\ominus p_2(z)}(t, t_0)g(t)\Delta t$ Provided that the integral exists for some $p_2(z)$ and $p_1(z) \neq 0$.

3. Results

3.1 Theorem 1

The set of all regressive functions $\mathcal{R}$ form an abelian group under the operation $\oplus$ defined by

$f \oplus g = f + g + \mu(t)fg$. The additive inverse of f in this group given by $\ominus f = -\frac{f}{1+\mu f}$

3.2 Lemma 1

If $q: \mathbb{T} \to \mathbb{R}$ is regressive then

$$e^{\sigma}_{\ominus iq(s)}(t,0) = \frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)} = -\frac{\ominus iq(s)}{q(s)} e_{\ominus iq(s)}(t,0) = \frac{i\ominus iq(s)}{q(s)} e_{\ominus iq(s)}(t,0)$$

Proof: We have the result $e^{\sigma}_{p}(t,s) = e_p(t,s) + \mu(t)e^{\Delta}_{p}(t,s)$ and $e^{\Delta}_{p}(t,0) = p(t)e_p(t,0)$

$$e^{\sigma}_{\ominus iq(s)}(t,0) = e_{\ominus iq(s)}(t,0) + \mu(t)e^{\Delta}_{\ominus iq(s)}(t,0) = e_{\ominus iq(s)}(t,0) + \mu(t)(\ominus\, iq(s))e_{\ominus iq(s)}(t,0)$$

$$\therefore e^{\sigma}_{\ominus iq(s)}(t,0) = e_{\ominus iq(s)}(t,0)\left(1 + \mu(t)\left(\frac{-iq(s)}{1 + i\mu(t)q(s)}\right)\right)$$

$$\therefore e^{\sigma}_{\ominus iq(s)}(t,0) = e_{\ominus iq(s)}(t,0)\left(\frac{1+i\mu(t)q(s)-i\mu(t)q(s)}{1+i\mu(t)q(s)}\right) = \frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)} \quad (1)$$

$$e^{\sigma}_{\ominus iq(s)}(t,0) = \frac{e_{\ominus iq(s)}(t,0)}{-iq(s)}\left(\frac{-iq(s)}{1 + i\mu(t)q(s)}\right) = -\frac{e_{\ominus iq(s)}(t,0)}{iq(s)}(\ominus\, iq(s))$$

$$\therefore e^{\sigma}_{\ominus iq(s)} = -\frac{\ominus iq(s)}{iq(s)} e_{\ominus iq(s)}(t,0) \quad (2)$$

Also $e^{\sigma}_{\ominus iq(s)} = \frac{i\ominus iq(s)}{q(s)} e_{\ominus iq(s)}(t,0)$ (3)

From equations (1), (2) and (3)

$$e^{\sigma}_{\ominus iq(s)}(t,0) = \frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)} = -\frac{\ominus iq(s)}{q(s)} e_{\ominus iq(s)}(t,0) = \frac{i\ominus iq(s)}{q(s)} e_{\ominus iq(s)}(t,0)$$

3.3 New General Complex Integral Transform on Time Scales $\mathbb{T}$

In 2022, Jinan A. Jasim, Sadiq A. Mehdi and Emad A. Kuffi presented a novel general complex integral transform [13]. For an integrable function $f(t)$ defined for $t \geq 0,\ p(s) \neq 0$ and $q(s)$ are real functions that are positive, i is the complex number then the transform $T^c_g(s)$ of $f(t)$ is given by

$T^c_g\{f(t), s\} = F^c_g(s) = p(s)\int_0^\infty e^{-iq(s)t} f(t)dt$ if the integral exists for some $q(s)$.

In this section we present a new general complex integral transform on time scales $\mathbb{T}$.

Definition 7 Let $f(t)$ be an integrable function, $p(s) \neq 0, \forall s \in \mathbb{C}$ and $(s) \in \mathcal{D}\{f\}$. Where $\mathcal{D}\{f\}$ consists of all complex numbers for which the improper integral exists. Then we define the new general complex integral transform on time scales $\mathbb{T}$ by the formula

$\mathcal{T}_g^c(f(t), s) = \mathcal{F}_g^c p(s) = \int_0^\infty e^\sigma_{\ominus iq(s)}(t,0)\, f(t)\, \Delta t.$

3.3.1 Linearity Property: Assume that $\mathcal{T}_g^c\{f\}$ and $\mathcal{T}_g^c\{g\}$ exists for $q(s) \in \mathcal{D}\{f\}$ and $\mathcal{D}\{g\}$, where f and g are rd-continuous functions on $\mathbb{T}$ and $\alpha,\ \beta \in \mathbb{R}$ are constants. Then

$\mathcal{T}_g^c\{\alpha f + \beta g\}(s) = \alpha \mathcal{T}_g^c\{f\}(s) + \beta \mathcal{T}_g^c\{g\}(s).\qquad \because q(s) \in \mathcal{D}\{f\} \cap \mathcal{D}\{g\}$

Proof: $\mathcal{T}_g^c\{(\alpha f + \beta g)(t)\} = p(s) \int_0^\infty (\alpha f + \beta g)(t)\, e^\sigma_{\ominus iq(s)}(t,0)\Delta t$

$= p(s) \int_0^\infty \left(\alpha f(t) e^\sigma_{\ominus iq(s)}(t,0) + \beta g(t) e^\sigma_{\ominus iq(s)}\right)\Delta t$

$= p(s) \int_0^\infty \alpha f(t) e^\sigma_{\ominus iq(s)} \Delta t + p(s) \int_0^\infty \beta g(t) e^\sigma_{\ominus iq(s)} \Delta t$

$$= \alpha\left(p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)} \Delta t\right) + \beta\left(p(s) \int_0^\infty g(t) e^\sigma_{\ominus iq(s)} \Delta t\right)$$

$\therefore \mathcal{T}_g^c\{(\alpha f + \beta g)(t)\} = \alpha \mathcal{T}_g^c\{f(t)\}(s) + \beta \mathcal{T}_g^c\{g(t)\}(s).$

3.3.2 Theorem 2 (Convergence Theorem)

The integral $p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t$ converges absolutely for $q(s) \in \mathcal{D}$ if $f(t)$ is of exponential type II with exponential constant k.

Proof:

Consider $\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le |p(s)| \int_0^\infty \left|e^\sigma_{\ominus iq(s)}(t,0) f(t) \Delta t\right|$

But $f(t)$ is of exponential type II with exponential constant k.

$$\therefore |f(t)| \le M e_k(t,0),\ \ M, k > 0$$

Hence above equation gives

$$\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le |p(s)| \int_0^\infty \left|M e_k(t,0) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right|$$

$$\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le M|p(s)| \int_0^\infty \left|e_k(t,0) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right|$$

$\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le M|p(s)| \int_0^\infty \left|e_k(t,0) \frac{e_{\ominus iq(s)}(t,0)}{1 + i\mu(t) q(s)} \Delta t\right| \qquad \because$ by using lemma (1).

$$\therefore \left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le M|p(s)| \int_0^\infty \left|\frac{1}{1 + i\mu(t) q(s)}\, e_{k \ominus iq(s)}(t,0)\Delta t\right|$$

($\because$ by property $e_p(t,s) e_q(t,s) = e_{p \oplus q}(t,s)$)

$\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le \frac{M|p(s)|}{k - iq(s)} \int_0^\infty \left|\frac{k - iq(s)}{1 + i\mu(t) q(s)}\, e_{k \ominus iq(s)}(t,0)\Delta t\right|$

$\left|p(s) \int_0^\infty f(t) e^\sigma_{\ominus iq(s)}(t,0)\Delta t\right| \le \frac{M|p(s)|}{k - iq(s)} \int_0^\infty \left|k \ominus iq(s) e_{k \ominus iq(s)}(t,0)\Delta t\right|$

$$\left|p(s)\int_0^\infty f(t)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t\right| \le \frac{M|p(s)|}{k-iq(s)}\int_0^\infty e^{\Delta}_{k\ominus iq(s)}(t,0)$$

$\left|p(s)\int_0^\infty f(t)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t\right| \le \frac{M|p(s)|}{k-iq(s)}\left[e_{k\ominus iq(s)}(t,0)\right]_0^\infty = \frac{M|p(s)|}{k-iq(s)}$

Since $q(s) \in \mathbb{C}_{\mu_*}(k)$ and $|p(s)|$ is a real number. Hence the integral converges if $f(t)$ is of exponential type II.

3.4 New General Complex Integral Transform on Time Scales of some functions.

3.4.1 If $f(t) = 1$ then $\mathcal{T}_g^c\{f(t)\}(s) = \frac{p(s)}{iq(s)}$.

$$\mathcal{T}_g^c\{1\}(s) = p(s)\int_0^\infty e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t = p(s)\int_0^\infty -\frac{\ominus iq(s)}{iq(s)}e_{\ominus iq(s)}(t,0)\Delta t$$

$$= -\frac{p(s)}{iq(s)}\int_0^\infty \ominus iq(s)e_{\ominus iq(s)}(t,0)\Delta t = -\frac{p(s)}{iq(s)}\int_0^\infty \left(e_{\ominus iq(s)}(t,0)\right)^{\Delta}\Delta t$$

$\mathcal{T}_g^c\{1\}(s) = -\frac{p(s)}{iq(s)}\left(e_{\ominus iq(s)}(t,0)\right)_{t=0}^{\infty} = \frac{p(s)}{iq(s)}$

3.4.2 If $f(t) = e_\alpha(t,0)$ then $\mathcal{T}_g^c\{f(t)\}(s) = \frac{p(s)}{iq(s)-\alpha} = -p(s)\left(\frac{\alpha}{\alpha^2+(q(s))^2} + i\frac{q(s)}{\alpha^2+(q(s))^2}\right)$.

$$\mathcal{T}_g^c\{e_\alpha(t,0)\}(s) = p(s)\int_0^\infty e_\alpha(t,0)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t = p(s)\int_0^\infty e_\alpha(t,0)\left(\frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)}\right)\Delta t$$

$$= \frac{p(s)}{\alpha - iq(s)}\int_0^\infty e_\alpha(t,0)e_{\ominus iq(s)}(t,0)\left(\frac{\alpha - iq(s)}{1+i\mu(t)q(s)}\right)\Delta t$$

$$= \frac{p(s)}{\alpha - iq(s)}\int_0^\infty e_{\alpha\ominus iq(s)}(t,0)\left(\frac{\alpha - iq(s)}{1+i\mu(t)q(s)}\right)\Delta t$$

But $\alpha \ominus q(s) = \frac{\alpha - q(s)}{1+i\mu(t)q(s)}$

$$\therefore \mathcal{T}_g^c\{e_\alpha(t,0)\}(s) = \frac{p(s)}{\alpha - iq(s)}\int_0^\infty \left(\alpha \ominus iq(s)\right)e_{\alpha\ominus iq(s)}(t,0)\Delta t$$

$$= \frac{p(s)}{\alpha - iq(s)}\int_0^\infty \left(e_{\alpha\ominus iq(s)}(t,0)\right)^{\Delta}\Delta t = \frac{p(s)}{iq(s)-\alpha} = \frac{p(s)}{\alpha^2+(q(s))^2}(-\alpha - iq(s))$$

$\therefore \mathcal{T}_g^c T\{e_\alpha(t,0)\}(s) = \frac{p(s)}{iq(s)-\alpha} = -p(s)\left(\frac{\alpha}{\alpha^2+(q(s))^2} + i\frac{q(s)}{\alpha^2+(q(s))^2}\right)$.

3.4.3 If $f(t) = cos_\alpha(t,0)$ then $\mathcal{T}_g^c\{f(t)\}(s) = \frac{-ip(s)q(s)}{(q(s))^2-\alpha^2}$ where $|q(s)| > |\alpha|$

Let $cos_\alpha(t,0) = \frac{e_{i\alpha}(t,0)+e_{-i\alpha}(t,0)}{2}$

$\therefore \mathcal{T}_g^c\{cos_\alpha(t,0)\}(s) = \mathcal{T}_g^c\left\{\frac{e_{i\alpha}(t,0)+e_{-i\alpha}(t,0)}{2}\right\} = \frac{1}{2}\mathcal{T}_g^c\{e_{i\alpha}(t,0)\} + \frac{1}{2}\mathcal{T}_g^c\{e_{-i\alpha}(t,0)\}$

$$=\frac{1}{2}\left(\frac{p(s)}{iq(s)-i\alpha}\right)+\frac{1}{2}\left(\frac{p(s)}{iq(s)+i\alpha}\right)=\frac{p(s)}{2i}\left(\frac{1}{q(s)-\alpha}+\frac{1}{q(s)+\alpha}\right)=\frac{p(s)q(s)}{i((q(s))^2-\alpha^2)}$$

$\therefore \mathcal{T}_g^c\{cos_\alpha(t,0)\}(s)=\frac{ip(s)q(s)}{(q(s))^2-\alpha^2}$.

3.4.4 If $f(t)=cosh_\alpha(t,0)$ then $\mathcal{T}_g^c\{f(t)\}(s)=\frac{ip(s)q(s)}{(q(s))^2+\alpha^2}$ where $q(s)>0$

Let $cosh_\alpha(t,0)=\frac{e_\alpha(t,0)+e_{-\alpha}(t,0)}{2}$

$\therefore \mathcal{T}_g^c\{cosh_\alpha(t,0)\}(s)=\mathcal{T}_g^c\left\{\frac{e_\alpha(t,0)+e_{-\alpha}(t,0)}{2}\right\}=\frac{1}{2}\mathcal{T}_g^c\{e_\alpha(t,0)\}+\frac{1}{2}\mathcal{T}_g^c\{e_{-\alpha}(t,0)\}$

$$=\frac{1}{2}\left(\frac{p(s)}{iq(s)-\alpha}\right)+\frac{1}{2}\left(\frac{p(s)}{iq(s)+\alpha}\right)=\frac{p(s)}{2}\left(\frac{1}{iq(s)-\alpha}+\frac{1}{iq(s)+\alpha}\right)$$

$\therefore \mathcal{T}_g^c\{cosh_\alpha(t,0)\}(s)=\frac{-ip(s)q(s)}{(q(s))^2+\alpha^2}$.

3.4.5 If $f(t)=sin_\alpha(t,0)$ then $\mathcal{T}_g^c\{f(t)\}(s)=\frac{-\alpha p(s)}{(q(s))^2-\alpha^2}$.

Let $sin_\alpha(t,0)=\frac{e_{i\alpha}(t,0)-e_{-i\alpha}(t,0)}{2i}$

$\therefore \mathcal{T}_g^c\{sin_\alpha(t,0)\}(s)=\mathcal{T}_g^c\left\{\frac{e_{i\alpha}(t,0)-e_{-i\alpha}(t,0)}{2i}\right\}=\frac{1}{2i}\mathcal{T}_g^c\{e_{i\alpha}(t,0)\}-\frac{1}{2i}\mathcal{T}_g^c\{e_{-i\alpha}(t,0)\}$

$$=\frac{1}{2i}\left(\frac{p(s)}{iq(s)-i\alpha}\right)-\frac{1}{2i}\left(\frac{p(s)}{iq(s)+i\alpha}\right)=\frac{p(s)}{-2}\left(\frac{1}{q(s)-\alpha}-\frac{1}{q(s)+\alpha}\right)$$

$\therefore \mathcal{T}_g^c\{sin_\alpha(t,0)\}(s)=\frac{-\alpha p(s)}{(q(s))^2-\alpha^2}$.

3.4.6 If $f(t)=sinh_\alpha(t,0)$ then $\mathcal{T}_g^c\{f(t)\}(s)=\frac{-\alpha p(s)}{(q(s))^2+\alpha^2}$ where $q(s)>0$

Let $sinh_\alpha(t,0)=\frac{e_\alpha(t,0)-e_{-\alpha}(t,0)}{2}$

$\therefore \mathcal{T}_g^c\{sinh_\alpha(t,0)\}(s)=\mathcal{T}_g^c\left\{\frac{e_\alpha(t,0)-e_{-\alpha}(t,0)}{2}\right\}=\frac{1}{2}\mathcal{T}_g^c\{e_\alpha(t,0)\}-\frac{1}{2}\mathcal{T}_g^c\{e_{-\alpha}(t,0)\}$

$$=\frac{1}{2}\left(\frac{p(s)}{iq(s)-\alpha}\right)-\frac{1}{2}\left(\frac{p(s)}{iq(s)+\alpha}\right)=\frac{p(s)}{2}\left(\frac{1}{iq(s)-\alpha}-\frac{1}{iq(s)+\alpha}\right)$$

$\therefore \mathcal{T}_g^c\{sinh_\alpha(t,0)\}(s)=\frac{-\alpha p(s)}{(q(s))^2+\alpha^2}$.

Now we introduce the New General Integral Transform and New General Complex Integral Transform on Time Scales for some basic functions in the following table.

Functions $f(t)$	$\mathcal{N}(f(t))(z)=\mathcal{F}(z)$ New general integral transform on time scales	$\mathcal{T}_g^c\{f(t)\}=\mathcal{F}_g^c(s)$ New general complex integral transform on time scales
1	$\frac{p(s)}{q(s)}$	$\frac{p(s)}{iq(s)}$
$e_\alpha(t,0)$	$\frac{p(s)}{q(s)-\alpha}$, $\lvert q(s)\rvert>\lvert\alpha\rvert$	$\frac{p(s)}{iq(s)-\alpha}$, $\lvert q(s)\rvert>\lvert\alpha\rvert$

$cos_\alpha(t,0)$	$\frac{p(s)q(s)}{(q(s))^2+\alpha^2}$	$\frac{ip(s)q(s)}{(q(s))^2-\alpha^2}, \lvert q(s)\rvert > \lvert\alpha\rvert$
$sin_\alpha(t,0)$	$\frac{\alpha p(s)}{(q(s))^2+\alpha^2}$	$\frac{-\alpha p(s)}{(q(s))^2-\alpha^2}, \lvert q(s)\rvert > \lvert\alpha\rvert$
$cosh_\alpha(t,0)$	$\frac{p(s)q(s)}{(q(s))^2-\alpha^2}, \lvert q(s)\rvert > \lvert\alpha\rvert$	$\frac{-ip(s)q(s)}{(q(s))^2+\alpha^2}, \lvert q(s)\rvert > \lvert\alpha\rvert$
$sinh_\alpha(t,0)$	$\frac{\alpha p(s)}{(q(s))^2-\alpha^2}, \lvert q(s)\rvert > \lvert\alpha\rvert$	$\frac{-\alpha p(s)}{(q(s))^2+\alpha^2}$

Table 1

3.5 Theorem 3 Let $\omega \in \mathbb{T}, \omega > 0$ and $u_v(t)$ is the unit step function the the new general complex integral transform on time scales $\mathbb{T}$ of the function $u_v(t)f(t)$ is $e_{\ominus iq(s)}(v,0)\mathcal{T}_g^c\{f(t)\}$ where

$$u_v(t) = \begin{cases} 0, if\ t \in \mathbb{T} \cap (-\infty, v) \\ 1, if\ t \in \mathbb{T} \cap [v, \infty) \end{cases}$$

Proof: $\mathcal{T}_g^c\{u_v(t)f(t)\}(s) = p(s)\int_0^\infty e_{\ominus iq(s)}^\sigma(t,0)\, u_v(t)f(t)\,\Delta t$

$$= p(s)\int_v^\infty e_{\ominus iq(s)}^\sigma(t,0)\, f(t)\,\Delta t = p(s)\int_v^\infty \frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)}\, f(t)\,\Delta t$$

$$= p(s)\int_v^\infty \frac{e_{\ominus iq(s)}(t,v)e_{\ominus iq(s)}(v,0)}{1+i\mu(t)q(s)}\, f(t)\,\Delta t$$

$$= p(s)e_{\ominus iq(s)}(v,0)\int_v^\infty \frac{e_{\ominus iq(s)}(t,v)}{1+i\mu(t)q(s)}\, f(t)\,\Delta t$$

$$= e_{\ominus iq(s)}(v,0)\left(p(s)\int_v^\infty e_{\ominus iq(s)}^\sigma(t,v)f(t)\Delta t\right) = e_{\ominus iq(s)}(v,0)\mathcal{T}_g^c\{f(t)\}$$

3.6 Definition 8 [3] Convolution of two functions.

If $f: \mathbb{T} \to \mathbb{C}$ and $g \in C_{prd-e_2}(\mathbb{T}, \mathbb{C})$ then the convolution of two functions f and g is denoted by $f * g$ and is given by $(f * g)(t) = \int_0^t f(\tau)g\big(t, \sigma(\tau)\big)\Delta\tau$ where $C_{prd-e_2}(\mathbb{T}, \mathbb{C})$ denotes the space of piecewise right dese continuous functions of exponential type-II.

3.6.1 Theorem 4 Convolution theorem

Let $f: \mathbb{T} \to \mathbb{C}$ and $g: \mathbb{C} \to \mathbb{C}$ have new general complex integral transforms on time scales $\mathbb{T}$ are $\mathcal{F}_g^c(s)$ and $\mathcal{G}_g^c(s)$ respectively. Then the new general complex integral transform on time scales for the convolution of these functions is $\frac{1}{p(s)}\mathcal{F}_g^c(s)\mathcal{G}_g^c(s)$.

Proof: Let $(f * g)(t) = \int_0^t f(\tau)g\big(t, \sigma(\tau)\big)\Delta\tau$.

Applying the new general complex integral transform to both sides

$$\mathcal{T}_g^c\{\,(f * g)(t)\} = \mathcal{T}_g^c\left\{\int_0^t f(\tau)g\big(t,\sigma(\tau)\big)\Delta\tau\right\} = p(s)\int_0^\infty e_{\ominus iq(s)}^\sigma(t,0)\left(\int_0^t f(\tau)g\big(t,\sigma(\tau)\big)\Delta\tau\right)\Delta t$$

$$= p(s)\int_0^\infty f(\tau)\left(\int_{\sigma(\tau)}^\infty u_{\sigma(\tau)}(t)g\big(t,\sigma(\tau)\big)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t\right)\Delta\tau$$

$$= \int_0^\infty f(\tau)\,\mathcal{T}_g^c\{u_{\sigma(\tau)}(t)g\big(t,\sigma(\tau)\big)\}\Delta\tau$$

But $\mathcal{T}_g^c\{u_{\sigma(\tau)}(t)g\big(t,\sigma(\tau)\big)\} = \mathcal{G}_g^c(s)e^{\sigma}_{\ominus iq(s)}(\tau,0)$ hence the above equation gives

$$\mathcal{T}_g^c\{\,(f*g)(t)\} = \frac{p(s)}{p(s)}\int_0^\infty f(\tau)\,\mathcal{G}_g^c(s)e^{\sigma}_{\ominus iq(s)}(\tau,0)\Delta\tau$$

$$\mathcal{T}_g^c\{\,(f*g)(t)\} = \frac{\mathcal{G}_g^c(s)}{p(s)}\left(p(s)\int_0^\infty f(\tau)\,e^{\sigma}_{\ominus iq(s)}(\tau,0)\Delta\tau\right) = \frac{1}{p(s)}\mathcal{F}_g^c(s)\mathcal{G}_g^c(s)$$

As f and g are of exponential type II with constants k_f and k_g respectively, we have

$$|(f*g)(t)| = \left|\int_0^t f(\tau)g\big(t,\sigma(\tau)\big)\Delta\tau\right| \le \int_0^t |f(\tau)|\big|g\big(t,\sigma(\tau)\big)\big|\Delta\tau$$

$\therefore\ |(f*g)(t)| \le \int_0^t M_1e_{k_f}(\tau,0)M_2e_{k_g}\big(t,\sigma(\tau)\big)\Delta\tau = \int_0^\infty Me_{k_f}(\tau,0)e_{k_g}(t,0)e_{k_g}\big(0,\sigma(\tau)\big)\Delta\tau$

where $|f(\tau)| \le M_1e_{k_f}(\tau,0)$, $\big|g\big(t,\sigma(\tau)\big)\big| \le M_2e_{k_g}\big(t,\sigma(\tau)\big)$ and $M = M_1M_2$

$$\therefore\ |(f*g)(t)| \le Me_{k_g}(t,0)\int_0^\infty e_{k_f}(\tau,0)e_{k_g}\big(0,\sigma(\tau)\big)\Delta\tau$$

$$|(f*g)(t)| \le Me_{k_g}(t,0)\int_0^\infty e_{k_f}(\tau,0)e_{\ominus k_g}(\tau,0)\Delta\tau$$

$$|(f*g)(t)| \le Me_{k_g}(t,0)\int_0^\infty e_{k_f\ominus k_g}(\tau,0)\Delta\tau$$

Hence $|(f*g)(t)| \le \frac{M}{|k_f-k_g|}e_{k_g}(t,0)\left(e_{k_f\ominus k_g}(t,0)-1\right) \le \frac{M}{|k_f-k_g|}\left(e_{k_f}(t,0)+e_{k_g}(t,0)\right)$

$$|(f*g)(t)| \le \frac{2M}{|k_f-k_g|}e_{\hat{k}}(t,0)$$

Hence $f*g$ is of exponential type II with exponential constant $\hat{k}$.

4. Discussion

4.1 Theorem 5 Assume that $f:\mathbb{T}\to\mathbb{C}$ is such that f^{Δ} and $f^{\Delta\Delta}$ are regulated. Then

i) $\mathcal{T}_g^c\{f^{\Delta}(t)\}(s) = iq(s)\mathcal{T}_g^c\{f(t)\} - p(s)f(0)$.

ii) $\mathcal{T}_g^c\{f^{\Delta\Delta}(t)\}(s) = (iq(s))^2\mathcal{T}_g^c\{f(t)\} - iq(s)p(s) - p(s)f^{\Delta}(0)$

For those regressive $q(s)\in\mathbb{C}$ satisfying $\lim_{t\to\infty} f(t)e_{\ominus iq(s)}(t,0) = 0$ and $\lim_{t\to\infty} f^{\Delta}(t)e_{\ominus iq(s)}(t,0) = 0$

Proof: i) $\mathcal{T}_g^c\{f^{\Delta}(t)\}(s) = p(s)\int_0^\infty f^{\Delta}(t)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t$

$$= p(s)\left(\left(f(t)e_{\ominus iq(s)}(t,0)\right)_{t=0}^{t=\infty} - \int_0^\infty f(t)e^{\Delta}_{\ominus iq(s)}(t,0)\,\Delta t\right)$$

∵ by rule for integration by parts.

$\mathcal{T}_g^c\{f^\Delta(t)\}(s) = p(s)\big((0-f(0)) - \int_0^\infty f(t)\big(\ominus iq(s)\big)\, e_{\ominus iq(s)}(t,0)\Delta t\big)$

$$= p(s)\left(-f(0) - iq(s)\int_0^\infty f(t)\left(\frac{\ominus iq(s)}{iq(s)}\right)e_{\ominus iq(s)}(t,0)\Delta t\right)$$

$= -p(s)f(0) + ip(s)q(s)\int_0^\infty f(t)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t$ ∵ by using lemma (1)

$$= -p(s)f(0) + iq(s)\left(p(s)\int_0^\infty f(t)e^{\sigma}_{\ominus iq(s)}(t,0)\Delta t\right)$$

$$\therefore \mathcal{T}_g^c\{f^\Delta(t)\}(s) = iq(s)\mathcal{T}_g^c\{f(t)\} - p(s)f(0)$$

ii) $\mathcal{T}_g^c\{f^{\Delta\Delta}(t)\}(s) = \mathcal{T}_g^c\left\{\left(f^\Delta(t)\right)^\Delta\right\} = iq(s)\mathcal{T}_g^c\{f^\Delta(t)\} - p(s)f^\Delta(0)$

$$= iq(s)\left(iq(s)\mathcal{T}_g^c\{f(t)\} - p(s)f(0)\right) - p(s)f^\Delta(0)$$

$$\therefore \mathcal{T}_g^c\{f^{\Delta\Delta}(t)\}(s) = (iq(s))^2\mathcal{T}_g^c\{f(t)\} - iq(s)p(s)f(0) - p(s)f^\Delta(0)$$

More generally we obtain

$\mathcal{T}_g^c\{(f^\Delta)^n(t)\}(s) = (iq(s))^n\mathcal{T}_g^c\{f(t)\} - \sum_{k=1}^n p(s)(iq(s))^{k-1}(f^\Delta)^{n-k}(0)$ for any integer $n \geq 2$.

4.2 Theorem 6

Assume that $f(t)$ is a regulated function with $F(t) = \int_0^t f(s)\Delta s$ then

$\mathcal{T}_g^c\{F(t)\}(s) = \frac{1}{iq(s)}\mathcal{T}_g^c\{f(t)\}(s)$ for all regressive functions $q(s) \neq 0$ satisfying $\lim_{t\to\infty} e_{\ominus iq(s)}(t,0)\int_0^t f(s)\Delta s = 0$

Proof: $\mathcal{T}_g^c(F(t),s) = p(s)\int_0^\infty e^{\sigma}_{\ominus iq(s)}(t,0)\,F(t)\,\Delta t = p(s)\int_0^\infty \left(\frac{e_{\ominus iq(s)}(t,0)}{1+i\mu(t)q(s)}\right)F(t)\Delta t$

$$= \frac{-p(s)}{iq(s)}\int_0^\infty \left(\frac{-iq(s)}{1+i\mu(t)q(s)}\right)e_{\ominus iq(s)}(t,0)F(t)\Delta t = \frac{-p(s)}{iq(s)}\int_0^\infty e^{\Delta}_{\ominus iq(s)}(t,0)F(t)\Delta t$$

$$= \frac{-p(s)}{iq(s)}\int_0^\infty \left[\left(e_{\ominus iq(s)}(t,0)F(t)\right)^\Delta - e^{\sigma}_{\ominus iq(s)}(t,0)F^\Delta(t)\right]\Delta t$$

$$= -\frac{p(s)}{iq(s)}\left[\left(e_{\ominus iq(s)}(t,0)F(t)\right)_0^\infty - \int_0^\infty e^{\sigma}_{\ominus iq(s)}(t,0)F^\Delta(t)\Delta t\right]$$

$$= \frac{p(s)F(0)}{iq(s)} + \frac{1}{iq(s)}\left(p(s)\int_0^\infty e^{\sigma}_{\ominus iq(s)}(t,0)f(t)\Delta t\right) = \frac{1}{iq(s)}\mathcal{T}_g^c\{f(t)\}(s)$$

4.3 Applications

Example 1 Consider the following initial value problem.

$$y^{\Delta\Delta}(t) - 6y^{\Delta}(t) + 8y(t) = e_3(t,0), y(0) = 1, y^{\Delta}(0) = 0$$

Applying new general complex integral transform to both sides of the dynamic equation.

$$\mathcal{T}_g^c\{y^{\Delta\Delta}(t) - 6y^{\Delta}(t) + 8y(t)\}(s) = \mathcal{T}_g^c\{e_3(t,0)\}(s)$$

$$\mathcal{T}_g^c\{y^{\Delta\Delta}(t)\} - 6\mathcal{T}_g^c\{y^{\Delta}(t)\} + 8\mathcal{T}_g^c\{y(t)\} = \mathcal{T}_g^c\{e_3(t,0)\}$$

$$(iq(s))^2\mathcal{T}_g^c\{y(t)\} - iq(s)p(s)y(0) - p(s)y^{\Delta}(0) - 6\left(iq(s)\mathcal{T}_g^c\{y(t)\} - p(s)y(0)\right) + 8\mathcal{T}_g^c\{y(t)\} = \mathcal{T}_g^c\{e_3(t,0)\}$$

$$\Rightarrow \mathcal{T}_g^c\{y(t)\}((iq(s))^2 - 6iq(s) + 8) = \frac{p(s)}{iq(s)-3} + p(s)\big(iq(s)\big) - 6p(s)$$

$$\Rightarrow \mathcal{T}_g^c\{y(t)\} = p(s)\left(\frac{(iq(s))^2 - 9(iq(s) + 19}{(iq(s)-3)(iq(s)-4)(iq(s)-2)}\right)$$

$$\Rightarrow \mathcal{T}_g^c\{y(t)\} = -\left(\frac{p(s)}{iq(s)-3}\right) - \frac{1}{2}\left(\frac{p(s)}{iq(s)-4}\right) + \frac{5}{2}\left(\frac{p(s)}{iq(s)-2}\right)$$

Hence from the table (1) $y(t) = -e_3(t,0) - \frac{1}{2}e_4(t,0) + \frac{5}{2}e_2(t,0)$.

Example 2 Consider the following third order dynamic equation

$$y^{\Delta\Delta\Delta} + y^{\Delta} = e_1(t,0), y(0) = y^{\Delta} = y^{\Delta\Delta} = 0$$

Applying new general complex integral transform on time scale to both sides

$$\mathcal{T}_g^c\{y^{\Delta\Delta\Delta}(t) + y^{\Delta}(t)\} = \mathcal{T}_g^c\{e_1(t,0)\}(s)$$

$$\left(\big(iq(s)\big)^3\mathcal{T}_g^c\{y(t)\} - \big(iq(s)\big)^2 p(s)y(0) - iq(s)p(s)y^{\Delta}(0) - p(s)y^{\Delta\Delta}(0)\right) + \left(iq(s)\mathcal{T}_g^c\{y(t)\} - p(s)y(0)\right) = \frac{p(s)}{iq(s)-1}$$

Using given initial conditions we obtain

$$\mathcal{T}_g^c\{y(t)\}((iq(s))^3 + iq(s)) = \frac{p(s)}{iq(s)-1}$$

$$\Rightarrow \mathcal{T}_g^c\{y(t)\} = \frac{p(s)}{iq(s)(iq(s)-1)((iq(s))^2+1)}$$

$$\mathcal{T}_g^c\{y(t)\} = \frac{-p(s)}{iq(s)} + \frac{1}{2}\left(\frac{p(s)}{iq(s)-1}\right) + \frac{1}{2}\left(\frac{-iq(s)p(s)}{\big(q(s)\big)^2-1}\right) - \frac{1}{2}\left(\frac{-p(s)}{\big(q(s)\big)^2-1}\right)$$

Using the table 1 we get $y(t) = -1 + \frac{1}{2}e_1(t,0) + \frac{1}{2}cos_1(t,0) - \frac{1}{2}sin_1(t,0)$

Example 3 Consider the volterra integral equation $y(t) = e_2(t,0) + 4\int_0^t y(\tau)\Delta\tau$

Applying the new general complex integral transform on time scales to the equation

$$\mathcal{T}_g^c\{y(t)\} = \mathcal{T}_g^c\{e_2(t,0)\} + 4\mathcal{T}_g^c\left\{\int_0^t y(\tau)\Delta\tau\right\}$$

$$\mathcal{T}_g^c\{y(t)\} = \frac{p(s)}{iq(s)-2} + 4\left(\frac{1}{iq(s)}\mathcal{T}_g^c\{y(t)\}\right)$$

$$\Rightarrow \mathcal{T}_g^c\{y(t)\} = \left(\frac{p(s)}{iq(s)-2}\right)\left(\frac{iq(s)}{iq(s)-4}\right) = -\left(\frac{p(s)}{iq(s)-2}\right) + 2\left(\frac{p(s)}{iq(s)-4}\right)$$

$$\therefore y(t) = -e_2(t,0) + 2e_4(t,0)$$

Example 4 We consider the problem from the field of pharmacokinetics to find the concentration of drug in the blood at any given time t during continuous intravenous injection of drug and find its solution in this problem for physical explanation of the present method. The following is the first order ordinary differential equation with constant coefficients that can be used to solve this problem.

$\frac{dg(t)}{dt} + \xi g(t) = \frac{\rho}{vol}$, where $t > 0$ (1)

with $g(0) = 0$.

Here $g(t)$ is the amount of a drug in the blood at any given time t, ξ: elimination at a fixed speed, ρ: the rate of infusion(in mg/min.), vol: the total amount of medication distributed. By using the result (2.10) for the equation (1) we get $g^{\Delta}(t) + \xi g(t) = \frac{\rho}{vol}$ applying the new general complex integral transform on time scales to this equations we get

$\mathcal{T}_g^c\{g^{\Delta}(t)\} + \xi\mathcal{T}_g^c\{g(t)\} = \frac{\rho}{vol}\mathcal{T}_g^c\{1\}$

$$iq(s)\mathcal{T}_g^c\{g(t)\} - p(s)g(0) + \xi\mathcal{T}_g^c\{g(t)\} = \frac{\rho}{vol}\frac{p(s)}{iq(s)}$$

$$\Rightarrow \mathcal{T}_g^c\{g(t)\}(iq(s) + \xi) = \frac{\rho}{vol}\frac{p(s)}{iq(s)}$$

$$\Rightarrow \mathcal{T}_g^c\{g(t)\} = \frac{\rho}{vol}\left(\frac{p(s)}{iq(s)(iq(s)+\xi)}\right) = \frac{\rho}{\xi vol}\left(\frac{p(s)}{iq(s)} - \frac{p(s)}{iq(s)+\xi}\right)$$

Hence $g(t) = \frac{\rho}{\xi vol}\left(1 - e_{-\xi}(t,0)\right)$.

Therefore continuous intravenous drug administration requires a certain concentration of drug in the blood at all the times.

Conclusion

The novel general complex integral transform on time scales $\mathbb{T}$ for solving dynamic equations of any given order and integral equations has been proven in terms of definition and applications. Few examples in real life problems such as pharmacokinetics.

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