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Binomial Labeling of Intersection Mirror Lagoon Step Graph and Intersection Opposite Lagoon Step Graph with Python Implementation

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Abstract:

Assume G = {V, E} be the graph with n vertices and e edges. The binomial labeling classification has been used for Intersection Mirror Lagoon Step Graph IM L_nS_m and Intersection Opposite Lagoon Step Graph IO L_n S_m in this paper. The concepts stated below in the paper have been implemented using Python coding.

Keywords: Intersection Mirror Lagoon Step Graph, Intersection Opposite Lagoon Step Graph, Binomial labeling.

1. Introduction

Rosa [1] first proposed the idea of graph labeling in 1967. Gallian [2] updates a dynamic overview of graph labeling techniques on a regular basis, and the Electronic Journal of Combinatory publishes it. We adhere to Bondy and Murthy's [3] notation and nomenclature for several aspects of graph theory. Binomial labeling introduced by Chandrakala and Sekar [4] et.al. The Binomial labeling for Intersection Mirror Lagoon Step Graph IM L_n S_m and the Intersection Opposite Lagoon Step Graph IO L_n S_m are constructed here using Python code [5][6].

2. Preliminaries

Definition 1 :A Lagoon Step graph L_n S_m is obtained from Lagoon L_n (L Shape) graph with vertices n (n≥3, must be an odd) joining with step graph S_m (m≥1,m is number of steps with m =(n-1)/2 , the graph consists of n+2m-1 vertices and n+2m-1 edges and is described below

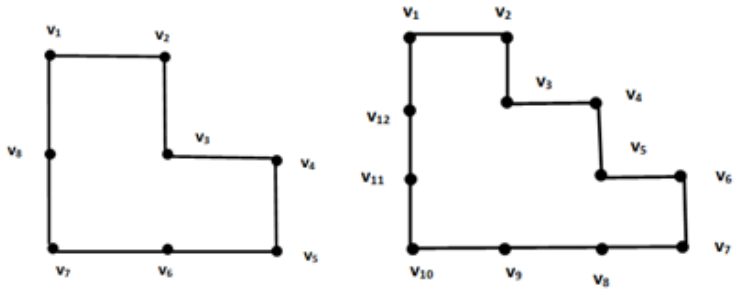


Figure 1: L_5S_2 & L_7S_3

Definition 2 : Intersection Mirror Lagoon Step graph IM L_n S_m obtained by attaching Lagoon step graph with Mirror image Lagoon step graph consists of 3n+m-4 vertices and 3n+m-3 edges (with n ≥ 3 must be an odd and m = (n-1)/2, m ≥ 1) described below

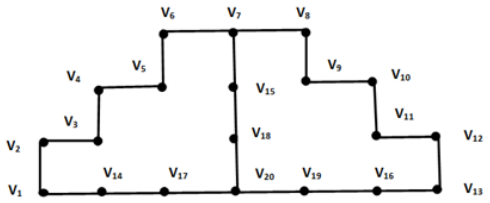


Figure 2: IM L_7S_3

Definition 3: Intersection Opposite Lagoon Step graph IO L_nS_m obtained by attaching Lagoon step graph with Opposite direction of Lagoon step graph consists of vertices 2n+4m-4 vertices and edges 2n+4m-3(with $n \geq 3$ must be an odd and $m = (n-1)/2$, $m \geq 1$) described below

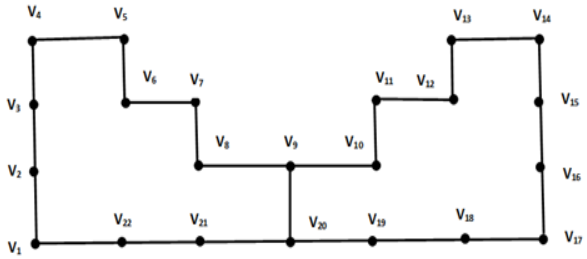


Figure 3: IO L_7S_3

Definition 4 : For positive integers n and r such that $0 \leq r \leq n$, then Binomial coefficient ${}^nC_r = n!/(r!(n-r)!)$ is always an integer.

Definition 5: Consider G = (p, q) be a graph. If the graph is an injective map f: V(G) → {1, 2, 3, … q+1} where the induced f*: E(G) → N is provided by, then G has a Binomial Labeling by f* (u v) = ${}^M C_m = M!/(m!(M-m)!)$ where M = max {f(u), f(v)}, m=min {f(u), f(v)}assigns distinct labels for the edges.

Definition 6: The term "binomial graph" refers to a graph that has the Binomial Labeling.

3. Main Results

3. 1 Theorem: Intersection Mirror Lagoon step graph IM $L_n S_m$ is a Binomial graph.

Proof: Consider the graph G be the intersection Mirror Lagoon step graph IM $L_n S_m$ with 3n+m-4 vertices and 3n+m-3 edges when n = 3, 5, 7, 9… then m = 1, 2, 3… respectively.

Case i : When n = 5, 7, 9… then m = 2, 3,4… respectively.

Let v_i be the outer vertices starts from left step corner to right step corner, w_i be the vertical inner vertices and xi be the base vertices. Classify f: V (G) → {1, 2, 3… 3n+m-2} is given below

$f(v_i) = i$, $i = 1, 2 \ldots n+2m-2$, when n = 5, 7, 9…, then m = 2, 3, 4… respectively,

$f(w_i) = 2n+2m+j-1$, which can be expressed as

i	n	m	j
1	5	2	0
1, 2	7	3	0, 1
1, 2, 3	9	4	0, 1, 2

. . .	. . .	. . .	. . .

respectively.

$f(x_i) = n+2m+i-2$ which can be expressed as

n	m	i
5	2	1, 2, 3, 4, 5
7	3	1, 2, 3, 4, 5, 6, 7
9	4	1, 2, 3, 4, 5, 6, 7, 8, 9
. . .	. . .	. . .

respectively.

The equation $f^*(e=uv) = {}^{R}C_r = R!/(r!(R-r)!)$ yields $f^*: E(G) \rightarrow N$, wherein $R = \max\{f(u), f(v)\}$ and $r = \min\{f(u), f(v)\}$ for every $uv \in E(G)$ .Every edge in this case has a unique label.

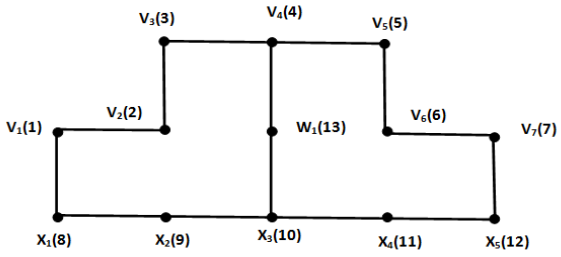


Figure 4: Binomial graph IML_5S_2

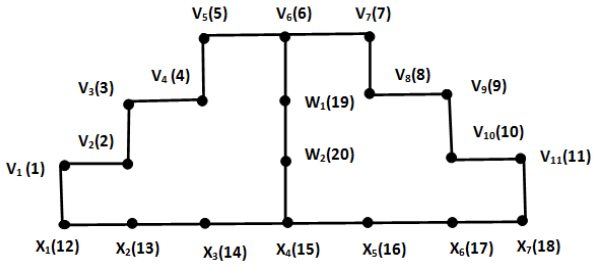


Figure 5: Binomial graph IM L_7S_3

Special case IM L_3S_1: Consider v_1, v_2, v_3, v_4, v_5 and v_6 be the six vertices of IM L_3S_1. Vertex labeling are $f(v_i) = i$, $i = 1, 2, 3, 4, 5$ and $f(v_i) = i+1$, $i = 6$.

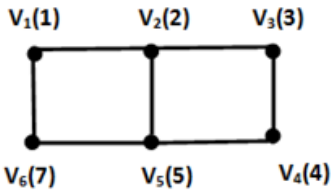


Figure 6: Binomial graph IML_3S_1

Every edge in this case has a unique label. IML_nS_m is a binomial graph as a result.

3. 2 Theorem : Intersection Opposite Lagoon step graph IO L_nS_m is a Binomial graph.

Proof :Consider the graph G be the Intersection Opposite Lagoon Step graph with 2n+4m-4 vertices and 2n+4m-3 edges when n = 3, 5, 7…, then m = 1, 2, 3, 4… respectively.

Case i: When n = 5, 7, 9… then m = 2, 3, 4… respectively.

Let v_i be the outer vertices except last vertex v_{i1} where v_{i1} be the last vertex.

Classify f: V(G) → {1, 2,3… 2n+4m-2} as follows

$f(v_i) = i$, i = 1, 2,3…2n+4m-5 when n = 5, 7, 9… then m = 2, 3, 4… respectively,

$f(v_{i1})) = i + 1$, where i = 2n+4m-4 when n = 5, 7, 9… then m = 2, 3, 4… respectively.

f*: E(G) → N is given by f* (e=uv) = $^{R}C_r$ = R!/(r!(R-r)!) for all uv ϵ E(G) where R = max {f(u), f(v)} and r = min {f(u), f(v)}.Here all the edges have distinct labels.

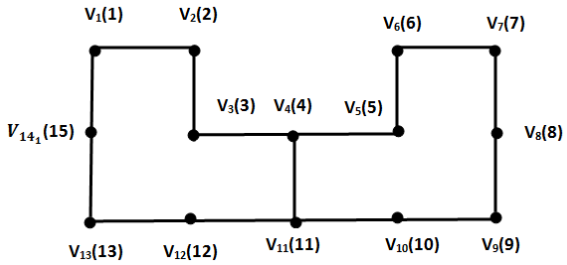


Figure 7: Binomial graph IOL_5S_2

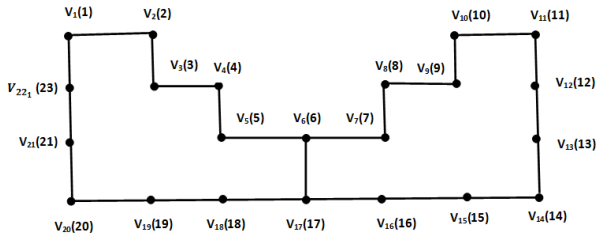


Figure 8: Binomial graph IOL_7S_3

Special case IO L_3S_1: Let v_1, v_2, v_3, v_4, v_5 and v_6 be the six vertices of IO L_3S_1. Vertex labelings are $f(v_i) = i$, i = 1, 2, 3, 4, 5 and $f(v_i) = i+1$, i = 6.

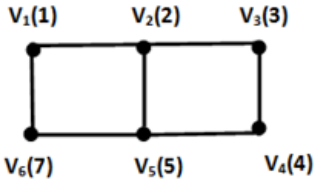


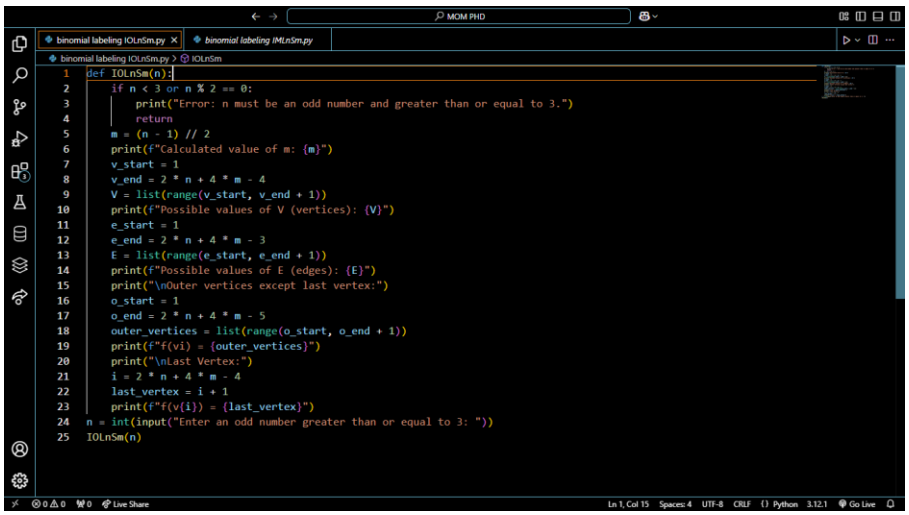
Figure 9: Binomial graph IOL_3S_1

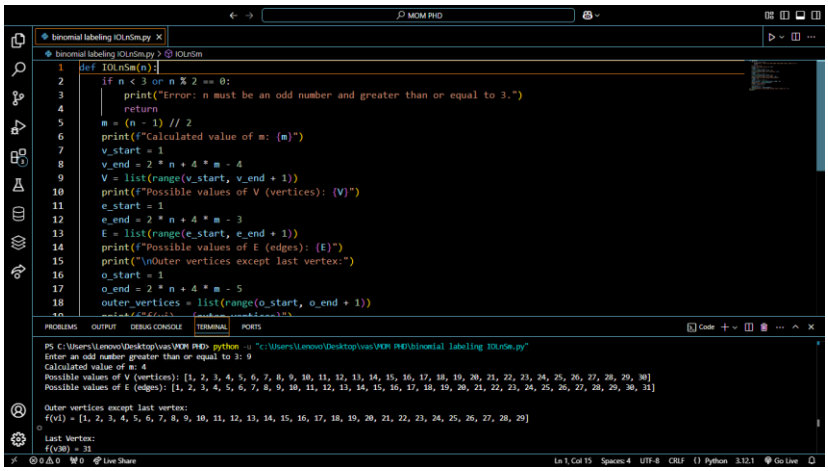
Every edge in this case has a unique label. IOL_nS_m is a binomial graph as a result.

4. Verification Using Python Coding

4. 1 Verification of the Binomial of IM L_9 S_4 and IM L_{11} S_5 using Python Implementation

4. 2 Verification of the Binomial of IO L_9 S_4 and IO L_{11} S_5 using Python Implementation





5. Conclusion

The demonstration of graphs like Intersection Mirror Lagoon step graph and Intersection Opposite Lagoon step graph exist when it comes to Binomial labeling. This paper also discusses the limitations and possible uses of this type of labeling. Python coding for Binomial labeling has been implemented for the above graphs. In the near future, more research under the Lagoon Step graph using various labeling strategies is needed.

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