

## $\theta$ -separation Axioms on Fuzzy Hypersoft Topological Spaces

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### Abstract:

In this article, the concept of fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-separation axioms in fuzzy hypersoft topological spaces are introduced by developing fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-neighbourhood with respect to fuzzy hypersoft points. Also, the properties and relations between fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_i$ -spaces ( $i = 0,1,2,3,4$ ) are discussed.

**Keywords:** FHS  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-neighbourhood, FHS  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-separation axioms, FHS  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_i$ -spaces ( $i = 0,1,2,3,4$ ).

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## 1 Introduction

The real-world decision-making problems in medical diagnosis, engineering, economics, management computer science, artificial intelligence, social sciences, environmental science and sociology contain more uncertain and inadequate data. Traditional mathematical methods cannot deal with these kinds of problems due to imprecise data. To deal with the problems with uncertainty, Zadeh [29] introduced the fuzzy set in 1965 which contains the membership value in  $[0,1]$ . A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called the membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang [9] as fuzzy topological space. Molodstov [12] introduced a new mathematical tool, soft set theory in 1999 to deal with uncertainties in which a soft set is a collection of approximate descriptions of an object. A soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory has several applications in different fields such as decision-making, optimization, forecasting, data analysis etc. Shabir and Naz [23] presented soft topological spaces.

Smarandache [24] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing a function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in decision-making issues involving a different kinds of attributes.

Saeed et al. [20, 21] studied the fundamentals of hypersoft set theory by introducing aggregate operators, relations, functions, matrices and operations on hypersoft matrices. Abbas et al. [2] defined the basic operations on hypersoft sets and hypersoft point in the fuzzy, intuitionistic and neutrosophic environments. Ajay and Charisma [3] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology.

Saha [22] defined  $\delta$ -open sets and continuous maps in fuzzy topological spaces. Aranganayagi et al., Revathi et al., Surendra et al. and Vadivel et al. [4, 5, 14, 15, 16, 18, 25, 26, 27] introduced  $\delta$ -open sets,  $e$ -open sets in neutrosophic, neutrosophic soft, fuzzy hypersoft, neutrosophic hypersoft topological spaces and studied its maps, separation axioms and compact spaces. In 2023, Revathi et al. [17] developed contra  $e$ -continuous maps in neutrosophic soft topological spaces. In 2019, the separation axioms on neutrosophic soft topological spaces were studied by Aras et al. [6]. The soft  $b$ -separation axioms were introduced by Khattak et al. [11] and pre-separation axioms were developed by Acikgoz et al. [1] in neutrosophic soft topological spaces. Gunduz et al. [10] and Ozturk [13] introduced separation axioms in neutrosophic hypersoft and fuzzy hypersoft topological spaces.

The class of sets namely,  $\theta$  open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. In 1968 Velicko [28] defined  $\theta$  open set in  $H$ -closed Topological Spaces. In [7, 8], Caldas et al. studied various kinds of  $\theta$  open sets and their properties in topological spaces. Revathi et al. [19] introduced  $\theta$  open sets and studied its maps in fuzzy hypersoft topological spaces.

The goal of this paper is to define the notions of fuzzy hypersoft  $\theta$  (resp. semi, pre,  $\theta$  semi &  $\theta$  pre)-neighbourhood and fuzzy hypersoft  $\theta$  (resp. semi, pre,  $\theta$  semi &  $\theta$  pre)-separation axioms in fuzzy hypersoft topological spaces using fuzzy hypersoft points. In addition, the characteristics of fuzzy hypersoft  $\theta$  (resp. semi, pre,  $\theta$  semi &  $\theta$  pre)- $T_i$ - spaces ( $i = 0, 1, 2, 3, 4$ ) and relations between them are studied.

## Preliminaries

**Definition 2.1** [29] Let  $\mathfrak{M}$  be an initial universe. A function  $\lambda$  from  $\mathfrak{M}$  into the unit interval  $I$  is called a fuzzy set in  $\mathfrak{M}$ . For every  $m \in \mathfrak{M}$ ,  $\lambda(m) \in I$  is called the grade of membership of  $m$  in  $\lambda$ . Some authors say that  $\lambda$  is a fuzzy subset of  $\mathfrak{M}$  instead of saying that  $\lambda$  is a fuzzy set in  $\mathfrak{M}$ . The class of all fuzzy sets from  $\mathfrak{M}$  into the closed unit interval  $I$  will be denoted by  $I^{\mathfrak{M}}$ .

**Definition 2.2** [12] Let  $\mathfrak{M}$  be an initial universe,  $Q$  be a set of parameters and  $\mathcal{P}(\mathfrak{M})$  be the power set of  $\mathfrak{M}$ . A pair  $(\tilde{\Phi}, \Lambda)$  is called the a soft set over  $\mathfrak{M}$  where  $\tilde{\Phi}$  is a mapping  $\tilde{\Phi}: Q \rightarrow \mathcal{P}(\mathfrak{M})$ . In other words, the soft set is a parametrized family of subsets of the set  $\mathfrak{M}$ .

**Definition 2.3** [24] Let  $\mathfrak{M}$  be an initial universe and  $\mathcal{P}(\mathfrak{M})$  be the power set of  $\mathfrak{M}$ . Consider  $q_1, q_2, q_3, \dots, q_n$  for  $n \geq 1$ , be  $n$  distinct attributes, whose corresponding attribute values are respectively the sets  $Q_1, Q_2, \dots, Q_n$  with  $Q_i \cap Q_j = \emptyset$ , for  $i \neq j$  and  $i, j \in \{1, 2, \dots, n\}$ . Then the

pair  $(\tilde{\Phi}, Q_1 \times Q_2 \times \dots \times Q_n)$  where  $\tilde{\Phi}: Q_1 \times Q_2 \times \dots \times Q_n \rightarrow \mathcal{P}(\mathfrak{M})$  is called a hypersoft set over  $\mathfrak{M}$ .

**Definition 2.4** [2] Let  $\mathfrak{M}$  be an initial universal set and  $Q_1, Q_2, \dots, Q_n$  be pairwise disjoint sets of parameters. Let  $\mathcal{P}(\mathfrak{M})$  be the set of all fuzzy sets of  $\mathfrak{M}$ . Let  $E_i$  be the nonempty subset of the pair  $Q_i$  for each  $i = 1, 2, \dots, n$ . A fuzzy hypersoft set (briefly,  $FH_{ySs}$ ) over  $\mathfrak{M}$  is defined as the pair  $(\tilde{\Phi}, E_1 \times E_2 \times \dots \times E_n)$  where  $\tilde{\Phi}: E_1 \times E_2 \times \dots \times E_n \rightarrow \mathcal{P}(\mathfrak{M})$  and  $\tilde{\Phi}(E_1 \times E_2 \times \dots \times E_n) = \{(q, \langle m, \mu_{\tilde{\Phi}(q)}(m) \rangle) : m \in \mathfrak{M}\} : q \in E_1 \times E_2 \times \dots \times E_n \subseteq Q_1 \times Q_2 \times \dots \times Q_n\}$  where  $\mu_{\tilde{\Phi}(q)}(m)$  is the membership value such that  $\mu_{\tilde{\Phi}(q)}(m) \in [0, 1]$ .

**Definition 2.5** [2] Let  $(\tilde{\Phi}, \Lambda_1)$  and  $(\tilde{\Psi}, \Lambda_2)$  be two  $FH_{ySs}$ 's over  $\mathfrak{M}$ . Then  $(\tilde{\Phi}, \Lambda_1)$  is the fuzzy hypersoft subset of  $(\tilde{\Psi}, \Lambda_2)$  if  $\mu_{\tilde{\Phi}(q)}(m) \leq \mu_{\tilde{\Psi}(q)}(m)$ .

It is denoted by  $(\tilde{\Phi}, \Lambda_1) \subseteq (\tilde{\Psi}, \Lambda_2)$ .

**Definition 2.6** [2] Let  $(\tilde{\Phi}, \Lambda_1)$  and  $(\tilde{\Psi}, \Lambda_2)$  be  $FH_{ySs}$ 's over  $\mathfrak{M}$ .  $(\tilde{\Phi}, \Lambda_1)$  is equal to  $(\tilde{\Psi}, \Lambda_2)$  if  $\mu_{\tilde{\Phi}(q)}(m) = \mu_{\tilde{\Psi}(q)}(m)$ .

**Definition 2.7** [2] A  $FH_{ySs}$   $(\tilde{\Phi}, \Lambda)$  over  $\mathfrak{M}$  is called null fuzzy hypersoft set if  $\mu_{\tilde{\Phi}(q)}(m) = 0, \forall q \in \Lambda$  and  $m \in \mathfrak{M}$ . It is denoted by  $\tilde{0}_{(\mathfrak{M}, Q)}$ .

A  $FH_{ySs}$   $(\tilde{\Psi}, \Lambda)$  over  $\mathfrak{M}$  is called absolute fuzzy hypersoft set if  $\mu_{\tilde{\Psi}(q)}(m) = 1 \forall q \in \Lambda$  and  $m \in \mathfrak{M}$ . It is denoted by  $\tilde{1}_{(\mathfrak{M}, Q)}$ .

Clearly,  $\tilde{0}_{(\mathfrak{M}, Q)}^c = \tilde{1}_{(\mathfrak{M}, Q)}$  and  $\tilde{1}_{(\mathfrak{M}, Q)}^c = \tilde{0}_{(\mathfrak{M}, Q)}$ .

**Definition 2.8** [2] Let  $(\tilde{\Phi}, \Lambda)$  be  $FH_{ySs}$  over  $\mathfrak{M}$ .  $(\tilde{\Phi}, \Lambda)^c$  is the complement of  $(\tilde{\Phi}, \Lambda)$  if  $\mu_{\tilde{\Phi}(q)}^c(m) = \tilde{1}_{(\mathfrak{M}, Q)} - \mu_{\tilde{\Phi}(q)}(m)$  where  $\forall q \in \Lambda$  and  $\forall m \in \mathfrak{M}$ . It is clear that  $((\tilde{\Phi}, \Lambda)^c)^c = (\tilde{\Phi}, \Lambda)$ .

**Definition 2.9** [2] Let  $(\tilde{\Phi}, \Lambda_1)$  and  $(\tilde{\Psi}, \Lambda_2)$  be  $FH_{ySs}$ 's over  $\mathfrak{M}$ . Extended union  $(\tilde{\Phi}, \Lambda_1) \cup (\tilde{\Psi}, \Lambda_2)$  is defined as

$$M((\tilde{\Phi}, \Lambda_1) \cup (\tilde{\Psi}, \Lambda_2)) = \begin{cases} \mu_{\tilde{\Phi}(q)}(m) & \text{if } q \in \Lambda_1 - \Lambda_2 \\ \mu_{\tilde{\Psi}(q)}(m) & \text{if } q \in \Lambda_2 - \Lambda_1 \\ \max\{\mu_{\tilde{\Phi}(q)}(m), \mu_{\tilde{\Psi}(q)}(m)\} & \text{if } q \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

**Definition 2.10** [2, 3] Let  $(\tilde{\Phi}, \Lambda_1)$  and  $(\tilde{\Psi}, \Lambda_2)$  be  $FH_{ySs}$ 's over  $\mathfrak{M}$ . Extended intersection  $(\tilde{\Phi}, \Lambda_1) \cap (\tilde{\Psi}, \Lambda_2)$  is defined as

$$\mu((\tilde{\Phi}, \Lambda_1) \cap (\tilde{\Psi}, \Lambda_2)) = \begin{cases} \mu_{\tilde{\Phi}(q)}(m) & \text{if } q \in \Lambda_1 - \Lambda_2 \\ \mu_{\tilde{\Psi}(q)}(m) & \text{if } q \in \Lambda_2 - \Lambda_1 \\ \min\{\mu_{\tilde{\Phi}(q)}(m), \mu_{\tilde{\Psi}(q)}(m)\} & \text{if } q \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

**Definition 2.11** [3] Let  $(\mathfrak{M}, Q)$  be the family of all  $FH_{ySs}$ 's over  $\mathfrak{M}$  and  $\tilde{\tau} \subseteq FH_{ySs}(\mathfrak{M}, Q)$ . Then  $\tilde{\tau}$  is said to be a fuzzy hypersoft topology (briefly,  $FH_{ySt}$ ) on  $\mathfrak{M}$  if

- (i).  $\tilde{0}_{(\mathfrak{M},Q)}$  and  $\tilde{1}_{(\mathfrak{M},Q)}$  belongs to  $\tilde{\tau}$
- (ii). the union of any number of  $FH_{\mathfrak{Y}}Ss$ 's in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$
- (iii). the intersection of finite number of  $FH_{\mathfrak{Y}}Ss$ 's in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .

Then  $(\mathfrak{M}, Q, \tilde{\tau})$  is known as a fuzzy hypersoft topological space (briefly,  $FH_{\mathfrak{Y}}Sts$ ) over  $\mathfrak{M}$ . Each member of  $\tilde{\tau}$  is said to be fuzzy hypersoft open set (briefly,  $FH_{\mathfrak{Y}}Sos$ ). A  $FH_{\mathfrak{Y}}Ss$   $(\tilde{\Phi}, \Lambda)$  is called a fuzzy hypersoft closed set (briefly,  $FH_{\mathfrak{Y}}Scs$ ) if its complement  $(\tilde{\Phi}, \Lambda)^c$  is  $FH_{\mathfrak{Y}}Sos$ .

**Definition 2.12** [3] Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a  $FH_{\mathfrak{Y}}Sts$  over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be a  $FH_{\mathfrak{Y}}Ss$  in  $\mathfrak{M}$ . Then,

(i). the fuzzy hypersoft interior (briefly,  $FHSint$ ) of  $(\tilde{\Phi}, \Lambda)$  is defined as  $FHSint(\tilde{\Phi}, \Lambda) = \cup \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda) \text{ where } (\tilde{\Psi}, \Lambda) \text{ is } FHSos\}$ .

(ii). the fuzzy hypersoft closure (briefly,  $FHScl$ ) of  $(\tilde{\Phi}, \Lambda)$  is defined as  $FHScl(\tilde{\Phi}, \Lambda) = \cap \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \supseteq (\tilde{\Phi}, \Lambda) \text{ where } (\tilde{\Psi}, \Lambda) \text{ is } FHScs\}$ .

**Definition 2.13** [2] Let  $FHS$ 's  $(\tilde{\Phi}, \Lambda)$  be the family of all  $FHS$ 's over  $\mathfrak{M}$  and let  $m \in \mathfrak{M}$ ,  $0 \leq \varphi \leq 1$ ,  $q \in Q$ . Then the  $FHSs$   $m_{\varphi}^q$  is called a fuzzy hypersoft point (briefly,  $FHSp$ ) and is defined as follows: For each  $n \in \mathfrak{M}$ ,

$$m_{\varphi}^q(q')(n) = \begin{cases} \varphi \text{ if } q' = q \text{ and } n = m \\ 0 \text{ if } q' \neq q \text{ or } n \neq m. \end{cases}$$

**Definition 2.14** [13] Let  $m_{\varphi}^q$  and  $n_{\varphi'}^{q'}$  be two  $FHSp$ 's. For the  $FHSp$ 's  $m_{\varphi}^q$  and  $n_{\varphi'}^{q'}$  over a common universe  $\mathfrak{M}$ , we say that  $FHSp$ 's are distinct points, if  $m_{\varphi}^q \cap n_{\varphi'}^{q'} = 0_{(\mathfrak{M},Q)}$ . It is clear that  $m_{\varphi}^q$  and  $n_{\varphi'}^{q'}$  are distinct  $FHSp$ 's iff  $m \neq n$  and  $q' \neq q$ .

**Definition 2.15** [13] Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be  $FHSts$  over  $\mathfrak{M}$ . A  $FHS$ 's  $(\tilde{\Phi}, \Lambda)$  in  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft neighbourhood (briefly,  $FHSnbd$ ) of the  $FHSp$   $m_{\varphi}^q \in (\tilde{\Phi}, \Lambda)$ , if there exists a  $FHSos$   $(\tilde{\Psi}, \Lambda)$  such that  $m_{\varphi}^q \in (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ .

**Definition 2.16** [13] Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a  $FHSts$  over  $\mathfrak{M}$ . Let  $(\tilde{\Phi}, \Lambda)$  be a  $FHSs$  over  $\mathfrak{M}$  and  $m_{\varphi}^q$  be a  $FHSp$  over  $\mathfrak{M}$ . [(i)]

(i).  $m_{\varphi}^q$  is a fuzzy hypersoft interior point of  $(\tilde{\Phi}, \Lambda)$ , if  $(\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$  for some  $(\tilde{\Psi}, \Lambda) \in FHSnbd$  of the  $FHSp$   $m_{\varphi}^q$ .

(ii).  $m_{\varphi}^q$  is a fuzzy hypersoft adherent point of  $(\tilde{\Phi}, \Lambda)$ , if  $(\tilde{\Psi}, \Lambda) \cap (\tilde{\Phi}, \Lambda) \notin 0_{(\mathfrak{M},Q)}$  for any  $(\tilde{\Psi}, \Lambda) \in FHSnbd$  of the  $FHSp$   $m_{\varphi}^q$ .

**Theorem 2.1** [13] Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a  $FHSts$  over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be a  $FHSs$  over  $\mathfrak{M}$ . Then

1.  $FHSint(\tilde{\Phi}, \Lambda) = \cup \{m_{\varphi}^q : m_{\varphi}^q \text{ is a fuzzy hypersoft interior point of } (\tilde{\Phi}, \Lambda)\}$ .

2.  $FHScl(\tilde{\Phi}, \Lambda) = \cup \{m_\phi^q : m_\phi^q \text{ is a fuzzy hypersoft adherent point of } (\tilde{\Phi}, \Lambda)\}.$

**Definition 2.17** [13] Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be an arbitrary FHS's. Then  $\tilde{\tau}_{(\tilde{\Phi}, \Lambda)} = \{(\tilde{\Phi}, \Lambda) \cap (\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \in \tilde{\tau}\}$  is called FHSt on  $(\tilde{\Phi}, \Lambda)$  and  $((\tilde{\Phi}, \Lambda), \tilde{\tau}_{(\tilde{\Phi}, \Lambda)}, Q)$  is known as a fuzzy hypersoft topological subspace (briefly, FHStss) of  $(\mathfrak{M}, Q, \tilde{\tau})$ .

### 3 Fuzzy hypersoft $\theta$ -separation axioms

**Definition 3.1** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a  $FH_y$ Sts over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be a  $FH_y$ Ss on  $\mathfrak{M}$ . Then the fuzzy hypersoft

(i).  $\theta$ -interior (briefly, FHSint) of  $(\tilde{\Phi}, \Lambda)$  is defined by  $FHS\theta int(\tilde{\Phi}, \Lambda) = \cup \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda) \text{ and } (\tilde{\Psi}, \Lambda) \text{ is a FHScs in } \mathfrak{M}\}$

(ii).  $\theta$ -closure (briefly, FHScl) of  $(\tilde{\Phi}, \Lambda)$  is defined by  $FHS\theta cl(\tilde{\Phi}, \Lambda) = \cap \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \supseteq (\tilde{\Phi}, \Lambda) \text{ and } (\tilde{\Psi}, \Lambda) \text{ is a FHSos in } \mathfrak{M}\}$

**Definition 3.2** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . An FHSs  $(\tilde{\Phi}, \Lambda)$  is said to be a fuzzy hypersoft

(i).  $\theta$ -open set (briefly, FHS $\theta$ os) if  $(\tilde{\Phi}, \Lambda) = FHS\theta int(\tilde{\Phi}, \Lambda)$

(ii).  $\theta$ -pre open set (briefly, FHS $\theta$ Pos) if  $(\tilde{\Phi}, \Lambda) \subseteq FHSint(FHS\theta cl(\tilde{\Phi}, \Lambda))$

(iii).  $\theta$ -semi open set (briefly, FHS $\theta$ Sos) if  $(\tilde{\Phi}, \Lambda) \subseteq FHScl(FHS\theta int(\tilde{\Phi}, \Lambda))$

The complement of FHS $\theta$ os (resp. FHS $\theta$ Pos & FHS $\theta$ Sos) is called a FHS $\theta$  (resp. FHS $\theta$  pre & FHS $\theta$  semi) closed set (briefly, FHS $\theta$ cs (resp. FHS $\theta$ Pcs & FHS $\theta$ Scs)) in  $\mathfrak{M}$ .

The family of all FHS $\theta$ os (resp. FHS $\theta$ cs, FHS $\theta$ Pos, FHS $\theta$ Pcs, FHS $\theta$ Sos & FHS $\theta$ Scs) of  $\mathfrak{M}$  is denoted by FHS $\theta$ OS( $\mathfrak{M}$ ) (resp. FHS $\theta$ CS( $\mathfrak{M}$ ), FHS $\theta$ POS( $\mathfrak{M}$ ), FHS $\theta$ Pcs( $\mathfrak{M}$ ), FHS $\theta$ SOS( $\mathfrak{M}$ ), FHS $\theta$ Pcs( $\mathfrak{M}$ ), FHS $\theta$ SOS( $\mathfrak{M}$ ) & FHS $\theta$ SCS( $\mathfrak{M}$ )).

**Definition 3.3** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be a FHSs on  $\mathfrak{M}$ . Then the fuzzy hypersoft

(i).  $\theta$ -pre (resp.  $\theta$ -semi) interior (briefly, FHS $\theta$ Pint (resp. FHS $\theta$ Sint)) of  $(\tilde{\Phi}, \Lambda)$  is defined by  $FHS\theta Pint(\tilde{\Phi}, \Lambda) = \cup \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda) \text{ and } (\tilde{\Psi}, \Lambda) \text{ is a FHS}\theta Pos \text{ (resp. FHS}\theta Sos) \text{ in } \mathfrak{M}\}$

(ii).  $\theta$ -pre (resp.  $\theta$ -semi) closure (briefly, FHS $\theta$ Pcl (resp. FHS $\theta$ Scl)) of  $(\tilde{\Phi}, \Lambda)$  is defined by  $FHS\theta Pcl(\tilde{\Phi}, \Lambda) = \cap \{(\tilde{\Psi}, \Lambda) : (\tilde{\Psi}, \Lambda) \supseteq (\tilde{\Phi}, \Lambda) \text{ and } (\tilde{\Psi}, \Lambda) \text{ is a FHS}\theta Pcs \text{ (resp. FHS}\theta Scs) \text{ in } \mathfrak{M}\}$

**Definition 3.4** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ . A FHS's  $(\tilde{\Phi}, \Lambda)$  in  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-neighbourhood (briefly, FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd) of the FHS $\theta$   $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ , if there exists a FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ .

**Theorem 3.1** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$  and  $(\tilde{\Phi}, \Lambda)$  be a FHS's on  $\mathfrak{M}$ . Then  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos) iff  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$ - (resp.  $\theta$  semi &  $\theta$  pre)nbd of its FHS $\theta$ 's.

**Proof.** Let  $(\tilde{\Phi}, \Lambda)$  be a FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os) and  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ . Then,  $m_\phi^q \in (\tilde{\Phi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ . Thus  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd of  $m_\phi^q$ .

Conversely, let  $(\tilde{\Phi}, \Lambda)$  be a FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd of its FHS $\mathcal{P}$ 's. Let  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ . Since  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd of the FHS $\mathcal{P}$   $m_\phi^q$ , there exists  $(\tilde{\Psi}, \Lambda) \in \tilde{\tau}$  such that  $m_\phi^q \in (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ . Since  $(\tilde{\Phi}, \Lambda) = \cup \{m_\phi^q : m_\phi^q \in (\tilde{\Phi}, \Lambda)\}$ , it follows that  $(\tilde{\Phi}, \Lambda)$  is a union of FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os)'s. Then  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os).

The FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd system of a FHS $\mathcal{P}$   $m_\phi^q$  denoted by  $\cup (m_\phi^q, Q)$ , is the family of all its FHS $\theta$  (resp.  $\delta$  semi &  $\theta$  pre)-nbd's.

**Theorem 3.2** The FH $_y$ S $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-nbd system  $\cup (m_\phi^q, Q)$  at  $m_\phi^q$  in a FHSts  $(\mathfrak{M}, Q, \tilde{\tau})$  has the following properties:

- (i). If  $(\tilde{\Phi}, \Lambda) \in \cup (m_\phi^q, Q)$ , then  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ .
- (ii). If  $(\tilde{\Phi}, \Lambda) \in \cup (m_\phi^q, Q)$  and  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{\Omega}, \Lambda)$ , then  $(\tilde{\Omega}, \Lambda) \in \cup (m_\phi^q, Q)$ .
- (iii).  $(\tilde{\Phi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda) \in \cup (m_\phi^q, Q)$ , then  $(\tilde{\Phi}, \Lambda) \cap (\tilde{\Psi}, \Lambda) \in \cup (m_\phi^q, Q)$ .
- (iv). If  $(\tilde{\Phi}, \Lambda) \in \cup (m_\phi^q, Q)$ , then there exists a  $(\tilde{\Psi}, \Lambda) \in \cup (m_\phi^q, Q)$  such that  $(\tilde{\Psi}, \Lambda) \in \cup (n_{\phi'}^{q'}, Q)$  for each  $n_{\phi'}^{q'} \in (\tilde{\Psi}, \Lambda)$ .

**Proof.** The proofs of (i), (ii) and (iii) directly follow from the Definition 3.4.

(iv) Suppose  $(\tilde{\Phi}, \Lambda) \in \cup (m_\phi^q, Q)$ . Then there exists a FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os)  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ . Then by Theorem 3.1,  $(\tilde{\Psi}, \Lambda) \in \cup (m_\phi^q, Q)$ . So for each  $n_{\phi'}^{q'} \in (\tilde{\Psi}, \Lambda)$ ,  $(\tilde{\Psi}, \Lambda) \in (n_{\phi'}^{q'}, Q)$ .

**Definition 3.5** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ . Let  $m_\phi^q$  and  $n_{\phi'}^{q'}$  be distinct FHS $\mathcal{P}$ 's. If there exist FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os)'s  $(\tilde{\Phi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$  and  $m_\phi^q \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$  or  $n_{\phi'}^{q'} \in (\tilde{\Psi}, \Lambda)$  and  $n_{\phi'}^{q'} \cap (\tilde{\Phi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ , then  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_0$ - space (briefly, FHS $\theta$  (resp. FHS $\theta$  $\delta$ , & FHS $\theta$  $\mathcal{P}$ )-  $T_0$ -space).

**Definition 3.6** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ . Let  $m_\phi^q$  and  $n_{\phi'}^{q'}$  be distinct FHS $\mathcal{P}$ 's. If there exist FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FH $_y$ S $\theta$  $\mathcal{P}$ os)'s  $(\tilde{\Phi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ ,  $m_\phi^q \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$  and  $n_{\phi'}^{q'} \in (\tilde{\Psi}, \Lambda)$ ,  $n_{\phi'}^{q'} \cap (\tilde{\Phi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ , then  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_1$ - space (briefly, FHS $\theta$  (resp. FHS $\theta$  $\delta$  & FHS $\theta$  $\mathcal{P}$ )-  $T_1$ -space).

**Definition 3.7** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ . Let  $m_\phi^q$  and  $n_{\phi'}^{q'}$  be distinct FHS $\mathcal{P}$ 's. If there exist FHS $\theta$ os (resp. FHS $\theta$  $\delta$ os & FHS $\theta$  $\mathcal{P}$ os)'s  $(\tilde{\Phi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Phi}, \Lambda)$ ,  $n_{\phi'}^{q'} \in$

$(\tilde{\Psi}, \Lambda)$  and  $(\tilde{\Phi}, \Lambda) \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ , then  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_2$ - space (briefly, FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )-  $T_2$ - space).

**Example 3.1** Let  $\mathfrak{M} = \{m_1, m_2\}$  be the FH $_y$ S initial universe and the attribute be  $Q = Q_1 \times Q_2$ .

The attribute is given as:

$$Q_1 = \{a_1, a_2\} \ \& \ Q_2 = \{b_1\} \ \text{and}$$

$$\Lambda = \{q_1 = (a_1, b_1) \ \& \ q_2 = (a_2, b_1)\}.$$

Let  $m_{1(0.8)}^{q_1}$ ,  $m_{1(0.7)}^{q_2}$ ,  $m_{2(0.9)}^{q_1}$  and  $m_{2(0.5)}^{q_2}$  be FHSp's. Let  $(\mathfrak{M}, Q)$  be the class of FH $_y$ S sets. Let the FH $_y$ Ss's  $(\tilde{\Phi}_1, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda)$ ,  $(\tilde{\Phi}_3, \Lambda)$ ,  $(\tilde{\Phi}_4, \Lambda)$ ,  $(\tilde{\Phi}_5, \Lambda)$  over the universe  $\mathfrak{M}$  be

$$(\tilde{\Phi}_1, \Lambda) = \left\{ \left\langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0} \right\} \right\rangle, \left\langle (a_2, b_1), \left\{ \frac{m_1}{0}, \frac{m_2}{0} \right\} \right\rangle \right\}$$

$$(\tilde{\Phi}_2, \Lambda) = \left\{ \left\langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0} \right\} \right\rangle, \left\langle (a_2, b_1), \left\{ \frac{m_1}{0}, \frac{m_2}{0} \right\} \right\rangle \right\}$$

$$(\tilde{\Phi}_3, \Lambda) = \left\{ \left\langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.9} \right\} \right\rangle, \left\langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.5} \right\} \right\rangle \right\}$$

$$(\tilde{\Phi}_4, \Lambda) = \left\{ \left\langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0.1} \right\} \right\rangle, \left\langle (a_2, b_1), \left\{ \frac{m_1}{0.3}, \frac{m_2}{0.5} \right\} \right\rangle \right\}$$

$$(\tilde{\Phi}_5, \Lambda) = \left\{ \left\langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.1} \right\} \right\rangle, \left\langle (a_2, b_1), \left\{ \frac{m_1}{0.3}, \frac{m_2}{0.5} \right\} \right\rangle \right\}$$

$\tilde{\tau} = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{\Phi}_1, \Lambda), (\tilde{\Phi}_2, \Lambda), (\tilde{\Phi}_3, \Lambda), (\tilde{\Phi}_4, \Lambda), (\tilde{\Phi}_5, \Lambda)\}$  is FH $_y$ Sts. Hence,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHSts over  $\mathfrak{M}$ . Here,  $(\tilde{\Phi}_3, \Lambda)$  and  $(\tilde{\Phi}_4, \Lambda)$  are FHS  $\theta$ os's. Also,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$ -  $T_0$ - space but not a FHS $\theta$ -  $T_1$ - space because for FH $_y$ Sp's  $m_{1(0.8)}^{q_1}$  and  $m_{2(0.5)}^{q_2}$ ,  $(\mathfrak{M}, Q, \tilde{\tau})$  is not a FHS $\theta$ -  $T_1$ - space.

**Example 3.2** Consider a set of natural numbers  $\mathfrak{M} = \mathbb{N}$  and a parameter set  $Q = \{\Lambda\}$ . Let the FHSp's be  $n_{\varphi_n}^q$ . Now we can take  $\varphi_n$  appropriate values and the FHSp's  $n_{\varphi_n}^q$ ,  $m_{\varphi_m}^q$  are distinct FHSp's iff  $n \neq m$ . It is obvious that there is one-to-one compatibility between the set of natural numbers and the set of FHSp's  $N^q = \{n_{\varphi_n}^q\}$ . Here we define cofinite topology on this set. Then FHSs's  $(\tilde{\Phi}, \Lambda)$  is a FHS $\theta$ os iff the finite FHSp's are discarded from  $N^q$ . Hence,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$ -  $T_1$ - space but not a FHS $\theta$ -  $T_2$ - space.

**Example 3.3** Let  $\mathfrak{M} = \{m_1, m_2\}$  be the FH $_y$ S initial universe and the attribute be  $Q = Q_1 \times Q_2$ .

The attribute is given as:

$$Q_1 = \{a_1, a_2\} \ \& \ Q_2 = \{b_1\} \ \text{and}$$

$$\Lambda = \{q_1 = (a_1, b_1) \ \& \ q_2 = (a_2, b_1)\}.$$

Let  $m_{1(0.8)}^{q_1}$ ,  $m_{1(0.3)}^{q_2}$ ,  $m_{2(0.9)}^{q_1}$  and  $m_{2(0.4)}^{q_2}$  be FHS $\mathcal{P}$ 's. Let  $(\mathfrak{M}, Q)$  be the class of FH $\gamma$ S sets. Let the FH $\gamma$ Ss's  $(\tilde{\Phi}_1, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda)$ ,  $(\tilde{\Phi}_3, \Lambda)$ ,  $(\tilde{\Phi}_4, \Lambda)$ ,  $(\tilde{\Phi}_5, \Lambda)$  over the universe  $\mathfrak{M}$  be

$$(\tilde{\Phi}_1, \Lambda) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0}, \frac{m_2}{0} \right\} \rangle \right\}$$

$$(\tilde{\Phi}_2, \Lambda) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0}, \frac{m_2}{0} \right\} \rangle \right\}$$

$$(\tilde{\Phi}_3, \Lambda) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.9} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.7}, \frac{m_2}{0.6} \right\} \rangle \right\}$$

$$(\tilde{\Phi}_4, \Lambda) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.2}, \frac{m_2}{0.1} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.3}, \frac{m_2}{0.4} \right\} \rangle \right\}$$

$$(\tilde{\Phi}_5, \Lambda) = \left\{ \langle (a_1, b_1), \left\{ \frac{m_1}{0.8}, \frac{m_2}{0.1} \right\} \rangle, \langle (a_2, b_1), \left\{ \frac{m_1}{0.3}, \frac{m_2}{0.4} \right\} \rangle \right\}$$

$\tilde{\tau} = \{\tilde{0}_{(\mathfrak{M}, Q)}, \tilde{1}_{(\mathfrak{M}, Q)}, (\tilde{\Phi}_1, \Lambda), (\tilde{\Phi}_2, \Lambda), (\tilde{\Phi}_3, \Lambda), (\tilde{\Phi}_4, \Lambda), (\tilde{\Phi}_5, \Lambda)\}$  is FH $\gamma$ Sts. Hence,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHSts over  $\mathfrak{M}$ . Here,  $(\tilde{\Phi}_3, \Lambda)$  and  $(\tilde{\Phi}_4, \Lambda)$  are FHS  $\theta$ os's. Also,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$ -  $T_2$ - space.

**Theorem 3.3** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . Then  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )-  $T_1$ - space iff each FHS $\mathcal{P}$  is a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs).

**Proof.** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )-  $T_1$ - space and  $m_\varphi^q$  be an arbitrary FHS $\mathcal{P}$ . Let  $n_{\varphi'}^{q'} \in (m_\varphi^q)^c$ . Then  $m_\varphi^q$  and  $n_{\varphi'}^{q'}$  are distinct FHS $\mathcal{P}$ 's. Thus  $m \neq n$  or  $q' \neq q$ . Since  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )-  $T_1$ - space, there exists a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)  $(\tilde{\Psi}, \Lambda)$  such that  $n_{(\alpha, \beta, \gamma)}^{q'} \in (\tilde{\Psi}, \Lambda)$  and  $m_\varphi^q \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . Since  $m_\varphi^q \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ , we have  $n_{\varphi'}^{q'} \in (\tilde{\Psi}, \Lambda) \subseteq (m_\varphi^q)^c$ . Thus  $(m_\varphi^q)^c$  is a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os), ie,  $m_\varphi^q$  is a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs).

Conversely, suppose that each FHS $\mathcal{P}$   $m_\varphi^q$  is a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs). Then  $(m_\varphi^q)^c$  is a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os). Let  $m_\varphi^q \cap n_{\varphi'}^{q'} = 0_{(\mathfrak{M}, Q)}$ . Thus,  $n_{\varphi'}^{q'} \in (m_\varphi^q)^c$  and  $m_\varphi^q \cap (m_\varphi^q)^c = 0_{(\mathfrak{M}, Q)}$ . So  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )-  $T_1$ - space on  $\mathfrak{M}$ .

**Theorem 3.4** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . Then  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )-  $T_2$ - space iff for distinct FHS $\mathcal{P}$ 's  $m_\varphi^q$  and  $n_{\varphi'}^{q'}$ , there exists a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)  $(\tilde{\Phi}, \Lambda)$  containing  $m_\varphi^q$  but not  $n_{\varphi'}^{q'}$ , such that  $n_{\varphi'}^{q'}$  does not belong to FHScl $(\tilde{\Phi}, \Lambda)$ .

**Proof.** Let  $m_\varphi^q$  and  $n_{\varphi'}^{q'}$  be two FHS $\mathcal{P}$ 's in FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )-  $T_2$ - space  $(\mathfrak{M}, Q, \tilde{\tau})$ . Then there exist disjoint FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s  $(\tilde{\Phi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda)$  such that

$m_\phi^q \in (\tilde{\Phi}, \Lambda)$ ,  $n_{\phi'}^{q'} \in (\tilde{\Psi}, \Lambda)$ . Since  $m_\phi^q \cap n_{\phi'}^{q'} = 0_{(\mathfrak{M}, Q)}$  and  $(\tilde{\Phi}, \Lambda) \cap (\tilde{\Psi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ ,  $n_{\phi'}^{q'}$  does not belong to  $(\tilde{\Phi}, \Lambda)$ . It implies that  $n_{\phi'}^{q'}$  does not belong to  $\text{FHScl}(\tilde{\Phi}, \Lambda)$ .

Conversely suppose that, for distinct FHS $\mathcal{P}$ 's  $m_\phi^q, n_{\phi'}^{q'}$ , there exists a FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)  $(\tilde{\Phi}, \Lambda)$  containing  $m_\phi^q$  but not  $n_{\phi'}^{q'}$  such that  $n_{(\alpha, \beta, \gamma)'}^{q'}$  does not belong to  $\text{FHScl}(\tilde{\Phi}, \Lambda)$ . Then  $n_{\phi'}^{q'} \in (\text{FHScl}(\tilde{\Phi}, \Lambda))^c$ , i. e.,  $(\tilde{\Phi}, \Lambda)$  and  $(\text{FHScl}(\tilde{\Phi}, \Lambda))^c$  are disjoint FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)'s containing  $m_\phi^q, n_{\phi'}^{q'}$  respectively

**Theorem 3.5** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_1$ - space for every FHS $\mathcal{P}$   $m_\phi^q \in (\tilde{\Phi}, \Lambda) \in \tilde{\tau}$ . If there exists a FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)  $(\tilde{\Psi}, \Lambda)$  such that  $m_\phi^q \in (\tilde{\Psi}, \Lambda) \subseteq \text{FHScl}(\tilde{\Psi}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ , then  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_2$ - space.

**Proof.** Suppose that  $m_\phi^q \cap n_{\phi'}^{q'} = 0_{(\mathfrak{M}, Q)}$ . Since  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta$ S, & FHS $\theta$ P)-  $T_1$ - space,  $m_\phi^q$  and  $n_{\phi'}^{q'}$  are FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs)'s in  $\tilde{\tau}$ . Then  $m_\phi^q \in (n_{\phi'}^{q'})^c \in \tilde{\tau}$ . Thus there exists a FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)  $(\tilde{\Psi}, \Lambda)$  in  $\tilde{\tau}$  such that  $m_\phi^q \in (\tilde{\Psi}, \Lambda) \subseteq \text{FHScl}(\tilde{\Psi}, \Lambda) \subseteq (n_{\phi'}^{q'})^c$ . So, we have  $n_{\phi'}^{q'} \in (\text{FHScl}(\tilde{\Psi}, \Lambda))^c$ ,  $m_\phi^q \in (\tilde{\Psi}, \Lambda)$  and  $(\tilde{\Psi}, \Lambda) \cap (\text{FHScl}(\tilde{\Psi}, \Lambda))^c = 0_{(\mathfrak{M}, Q)}$ , i. e.,  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\delta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_2$ - space

**Remark 3.1** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_i$ - space for  $i = 0, 1, 2$ . For each  $m \neq n$ , FHS $\mathcal{P}$ 's  $m_\phi^q$  and  $n_{\phi'}$  have neighbourhoods satisfying conditions of  $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_i$ - space in FHSts  $(\mathfrak{M}, \tilde{\tau}^q)$  for each  $q \in Q$  because  $m_\phi^q$  and  $n_{\phi'}^{q'}$  are distinct FHS $\mathcal{P}$ 's.

**Definition 3.8** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ . Let  $(\tilde{\Phi}, \Lambda)$  be a FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs) and  $m_\phi^q \cap (\tilde{\Phi}, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . If there exist FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)'s  $(\tilde{Y}_1, \Lambda)$  and  $(\tilde{Y}_2, \Lambda)$  such that  $m_\phi^q \in (\tilde{Y}_1, \Lambda)$ ,  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)$  and  $(\tilde{Y}_1, \Lambda) \cap (\tilde{Y}_2, \Lambda) = 0_{(\mathfrak{M}, Q)}$ , then  $(\mathfrak{M}, Q, \tilde{\tau})$  is called a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)- regular (briefly, FHS $\theta$  (resp. FHS $\theta$ S, & FHS $\theta$ P)-regular) space.  $(\mathfrak{M}, Q, \tilde{\tau})$  is said to be a fuzzy hypersoft  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_3$ - space (briefly, FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_3$ - space) if it is both a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-regular and FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_1$ -space.

**Theorem 3.6** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be FHSts over  $\mathfrak{M}$ .  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_3$ -space iff for every  $m_\phi^q \in (\tilde{\Phi}, \Lambda) \in \tilde{\tau}$ , there exists  $(\tilde{Y}, \Lambda) \in \tilde{\tau}$  such that  $m_\phi^q \in (\tilde{Y}, \Lambda) \subseteq \text{FHScl}(\tilde{Y}, \Lambda) \subseteq (\tilde{\Phi}, \Lambda)$ .

**Proof.** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$ -  $T_3$ -space and  $m_\phi^q \in (\tilde{\Phi}, \Lambda) \in \tilde{\tau}$ . Since  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_3$ - space for the FHS $\mathcal{P}$   $m_\phi^q$  and FHS $\theta$ cs  $(\tilde{\Phi}, \Lambda)^c$ , there exist  $(\tilde{Y}_1, \Lambda)$ ,  $(\tilde{Y}_2, \Lambda) \in \tilde{\tau}$  such that  $m_\phi^q \in (\tilde{Y}_1, \Lambda)$ ,  $(\tilde{\Phi}, \Lambda)^c \subseteq (\tilde{Y}_2, \Lambda)$  and  $(\tilde{Y}_1, \Lambda) \cap (\tilde{Y}_2, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . Then we have  $m_\phi^q \in (\tilde{Y}_1, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)^c \subseteq (\tilde{\Phi}, \Lambda)$ . Since  $(\tilde{Y}_2, \Lambda)^c$  is a FHS $\delta$ cs (resp. FHS $\delta$ Scs & FHS $\delta$ Pcs),  $\text{FHScl}(\tilde{Y}_1, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)^c$ .

Conversely, let  $m_\phi^q \cap (\tilde{\Omega}, \Lambda) = 0_{(\mathfrak{M}, Q)}$  and  $(\tilde{\Omega}, \Lambda)$  be a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs). Then  $m_\phi^q \in (\tilde{\Omega}, \Lambda)^c$  and from the condition of the theorem, we have  $m_\phi^q \in (\tilde{Y}, \Lambda) \subseteq \text{FHScl}(\tilde{Y}, \Lambda) \subseteq (\tilde{\Omega}, \Lambda)^c$ . Thus  $m_\phi^q \in (\tilde{Y}, \Lambda)$ ,  $(\tilde{\Omega}, \Lambda) \subseteq (\text{FHScl}(\tilde{Y}, \Lambda))^c$  and  $(\tilde{Y}, \Lambda) \cap (\text{FHScl}(\tilde{Y}, \Lambda))^c = 0_{(\mathfrak{M}, Q)}$ . So  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\delta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_3$ -space

**Definition 3.9** A FHSts  $(\mathfrak{M}, Q, \tilde{\tau})$  over  $\mathfrak{M}$  is called a FHS  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-normal (briefly, FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )-normal) space, if for every pair of disjoint FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs)'s  $(\tilde{\Phi}_1, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda)$ , there exist disjoint FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s  $(\tilde{\Omega}_1, \Lambda)$ ,  $(\tilde{\Omega}_2, \Lambda)$  such that  $(\tilde{\Phi}_1, \Lambda) \subseteq (\tilde{\Omega}_1, \Lambda)$  and  $(\tilde{\Phi}_2, \Lambda) \subseteq (\tilde{\Omega}_2, \Lambda)$ .  $(\mathfrak{M}, Q, \tilde{\tau})$  is said to be a FHS  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)- $T_4$ -space (briefly, FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_4$ -space) if it is both a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )-normal and FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_1$ -space.

**Theorem 3.7** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . Then  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )- $T_4$ -space iff for each FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs)  $(\tilde{\Phi}, \Lambda)$  and FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)  $(\tilde{\Omega}, \Lambda)$  with  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{\Omega}, \Lambda)$ , there exists a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)  $(\tilde{Y}, \Lambda)$  such that  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{Y}, \Lambda) \subseteq \text{FHScl}(\tilde{Y}, \Lambda) \subseteq (\tilde{\Omega}, \Lambda)$ .

**Proof.** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )- $T_4$ -space. Let  $(\tilde{\Phi}, \Lambda)$  be a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs) and let  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{\Omega}, \Lambda) \in \tilde{\tau}$ . Then  $(\tilde{\Omega}, \Lambda)^c$  is a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs) and  $(\tilde{\Phi}, \Lambda) \cap (\tilde{\Omega}, \Lambda)^c = 0_{(\mathfrak{M}, Q)}$ . Since  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_4$ -space, there exist FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os, FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s  $(\tilde{Y}_1, \Lambda)$  and  $(\tilde{Y}_2, \Lambda)$  such that  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{Y}_1, \Lambda)$ ,  $(\tilde{\Omega}, \Lambda)^c \subseteq (\tilde{Y}_2, \Lambda)$  and  $(\tilde{Y}_1, \Lambda) \cap (\tilde{Y}_2, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . Thus  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{Y}_1, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)^c \subseteq (\tilde{\Omega}, \Lambda)$ ,  $(\tilde{Y}_2, \Lambda)^c$  is a FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs) and  $(\tilde{Y}_1, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)^c$ . So,  $(\tilde{\Phi}, \Lambda) \subseteq (\tilde{Y}_1, \Lambda) \subseteq \text{FHScl}(\tilde{Y}_1, \Lambda) \subseteq (\tilde{\Omega}, \Lambda)$ .

Conversely, let  $(\tilde{\Phi}_1, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda)$  be two disjoint FHS $\theta$ cs (resp. FHS $\theta\mathcal{S}$ cs & FHS $\theta\mathcal{P}$ cs)'s. Then  $(\tilde{\Phi}_1, \Lambda) \subseteq (\tilde{\Phi}_2, \Lambda)^c$ . From the condition of theorem, there exists a FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)  $(\tilde{Y}, \Lambda)$  such that  $(\tilde{\Phi}_1, \Lambda) \subseteq (\tilde{Y}, \Lambda) \subseteq \text{FHScl}(\tilde{Y}, \Lambda) \subseteq (\tilde{\Phi}_2, \Lambda)^c$ . Thus  $(\tilde{Y}, \Lambda)$ ,  $(\text{FHScl}(\tilde{Y}, \Lambda))^c$  are FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s and  $(\tilde{\Phi}_1, \Lambda) \subseteq (\tilde{Y}, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda) \subseteq (\text{FHScl}(\tilde{Y}, \Lambda))^c$  and  $(\tilde{Y}, \Lambda) \cap (\text{FHScl}(\tilde{Y}, \Lambda))^c = 0_{(\mathfrak{M}, Q)}$ . So  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$ , & FHS $\theta\mathcal{P}$ )- $T_4$ -space

**Theorem 3.8** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . If  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_1$ -space, then the FHSts  $((\tilde{\Phi}, \Lambda), \tilde{\tau}_{(\tilde{\Phi}, \Lambda)}, Q)$  is a FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_i$ -space for  $i = 0, 1, 2, 3$ .

**Proof.** Let  $m_\phi^q, n_\phi^{q'} \in ((\tilde{\Phi}, \Lambda), \tilde{\tau}_{(\tilde{\Phi}, \Lambda)}, Q)$  such that  $m_\phi^q \cap n_\phi^{q'} = 0_{(\mathfrak{M}, Q)}$ . Then there exist FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s  $(\tilde{\Phi}_1, \Lambda)$  and  $(\tilde{\Phi}_2, \Lambda)$  satisfying the conditions of FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_1$ -space such that  $m_\phi^q \in (\tilde{\Phi}_1, \Lambda)$ ,  $n_\phi^{q'} \in (\tilde{\Phi}_2, \Lambda)$ . Thus,  $m_\phi^q \in (\tilde{\Phi}_1, \Lambda) \cap (\tilde{\Phi}, \Lambda)$  and  $n_\phi^{q'} \in (\tilde{\Phi}_2, \Lambda) \cap (\tilde{\Phi}, \Lambda)$ . Also, the FHS $\theta$ os (resp. FHS $\theta\mathcal{S}$ os & FHS $\theta\mathcal{P}$ os)'s  $(\tilde{\Phi}_1, \Lambda) \cap (\tilde{\Phi}, \Lambda)$ ,  $(\tilde{\Phi}_2, \Lambda) \cap (\tilde{\Phi}, \Lambda)$  in  $\tilde{\tau}_{(\tilde{\Phi}, \Lambda)}$  satisfy the conditions of FHS $\theta$  (resp. FHS $\theta\mathcal{S}$  & FHS $\theta\mathcal{P}$ )- $T_i$ -space for  $i = 0, 1, 2, 3$ .

**Theorem 3.9** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHSts over  $\mathfrak{M}$ . If  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FH $\gamma$ S $\theta$ S & FHS $\theta$ P)-  $T_4$ -space and  $(\tilde{\Omega}, \Lambda)$  is a FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs) in  $(\mathfrak{M}, Q, \tilde{\tau})$ , then  $((\tilde{\Omega}, \Lambda), \tilde{\tau}_{(\tilde{\Omega}, \Lambda)}, Q)$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_4$ -space.

**Proof.** Let  $(\mathfrak{M}, Q, \tilde{\tau})$  be a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_4$ -space and  $(\tilde{\Omega}, \Lambda)$  be a FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs) in  $(\mathfrak{M}, Q, \tilde{\tau})$ . Let  $(\tilde{\Omega}_1, \Lambda)$  and  $(\tilde{\Omega}_2, \Lambda)$  be two FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs)'s in  $((\tilde{\Omega}, \Lambda), \tilde{\tau}_{(\tilde{\Omega}, \Lambda)}, Q)$  such that  $(\tilde{\Omega}_1, \Lambda) \cap (\tilde{\Omega}_2, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . When  $(\tilde{\Omega}, \Lambda)$  is a FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs) in  $(\mathfrak{M}, Q, \tilde{\tau})$ ,  $(\tilde{\Omega}_1, \Lambda)$  and  $(\tilde{\Omega}_2, \Lambda)$  are FHS $\theta$ cs (resp. FHS $\theta$ Scs & FHS $\theta$ Pcs)'s in  $(\mathfrak{M}, Q, \tilde{\tau})$ . Since  $(\mathfrak{M}, Q, \tilde{\tau})$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_4$ -space, there exist FHS $\theta$ os (resp. FHS $\theta$ Sos & FHS $\theta$ Pos)'s  $(\tilde{Y}_1, \Lambda)$  and  $(\tilde{Y}_2, \Lambda)$  such that  $(\tilde{\Omega}_1, \Lambda) \subseteq (\tilde{Y}_1, \Lambda)$ ,  $(\tilde{\Omega}_2, \Lambda) \subseteq (\tilde{Y}_2, \Lambda)$  and  $(\tilde{Y}_1, \Lambda) \cap (\tilde{Y}_2, \Lambda) = 0_{(\mathfrak{M}, Q)}$ . Then  $(\tilde{\Omega}_1, \Lambda) = (\tilde{Y}_1, \Lambda) \cap (\tilde{\Omega}, \Lambda)$ ,  $(\tilde{\Omega}_2, \Lambda) = (\tilde{Y}_2, \Lambda) \cap (\tilde{\Omega}, \Lambda)$  and  $((\tilde{Y}_1, \Lambda) \cap (\tilde{\Omega}, \Lambda)) \cap ((\tilde{Y}_2, \Lambda) \cap (\tilde{\Omega}, \Lambda)) = 0_{(\mathfrak{M}, Q)}$ . Hence  $((\tilde{\Omega}, \Lambda), \tilde{\tau}_{(\tilde{\Omega}, \Lambda)}, Q)$  is a FHS $\theta$  (resp. FHS $\theta$ S & FHS $\theta$ P)-  $T_4$ -space.

#### 4 Conclusion

In this paper, FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-separation axioms in FHSts are introduced and studied using FHS $\theta$ 's. The relation and properties between FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)-  $T_i$ -spaces ( $i = 0, 1, 2, 3, 4$ ) are also discussed. The future work can involve the investigation of FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)- compactness, FHS $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)- connectedness and FHS contra  $\theta$  (resp.  $\theta$  semi &  $\theta$  pre)- continuous functions.

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