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# Topological Study on Revised Fuzzy Metric Spaces and Their Generalization

**[1]Thangathamizh R, [2]Abdul Razak, [3]Rajalakshmi S, [4]Gnanabala K, [5]Thangammal R, [6]Shanmugavel P**

Jeppiaar Institute of Technology, Sriperumpudhur, Kanchipuram, email: thamizh1418@gmail.com.,

K.Ramakrishnan College of Engineering, Samayapuram, Trichy, email: arrazak76@gmail.com.,

Mahalashmi Women's College of Arts and Science, Paruthipattu, Avadi, Chennai, email: gbalamaths@gmail.com.,

Chennai Institute of Technology, Chennai, email: rajalakshmi301991@gmail.com.,

Selvam college of Technology (Autonomous), Namakkal, email: rthangam1981@gmail.com.

Selvamm arts and science college (Autonomous), Namakkal, email: p.sham1988@gmail.com.

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**Abstract:**

**Introduction**

In this paper, we explore the concept of metric functions within a revised fuzzy metric space. The study focuses on understanding the relationships between these metric functions and the topological structures they generate. Specifically, we introduce the concept of a stratified function within this framework and investigate its implications.

**Objectives**

The main objectives of this paper are:

1. To define and analyze stratified functions in a revised fuzzy metric space.
2. To demonstrate that the topology generated by the family of stratified functions coincides with the topology generated by the revised fuzzy metric.
3. To derive the concrete form of the metric function under specific conditions.

**Methods**

We approach these objectives by first introducing the notion of a stratified function in a revised fuzzy metric space. Using this concept, we prove that the topology generated by the family of stratified functions is identical to the topology generated by the revised fuzzy metric. Additionally, we explore the conditions under which a specific form of the metric function can be determined.

**Results**

Our findings show that the topology generated by the stratified functions indeed coincides with the topology generated by the revised fuzzy metric. Moreover, under certain special conditions, we can obtain a concrete representation of the metric function.

**Conclusion**

This paper provides a deeper understanding of the structure of revised fuzzy metric spaces. The introduction of stratified functions serves as a key tool for analyzing the topology of these spaces, and our results offer a concrete form for the metric function under specific conditions, contributing to the broader study of fuzzy metric spaces.

**Keywords**: t-conorm, Revised fuzzy metric

**2020 Mathematical classification:** 37C25, 46S40, 46N20, 47H10

## 1. Introduction

Many scholars have created ideas of fuzzy metric spaces and examined their characteristics in various ways since Zadeh [30] initially suggested fuzzy set theory in 1965. In 1975, Kramosil and Michalek

[8] developed the concept of fuzzy metric, which is a fuzzy set in the Cartesian product that meets specific requirements. This concept was inspired by the idea of probabilistic metric spaces. Subsequently, George and Veeramani [2] modified this notion of fuzzy metric space by introducing the idea of continuous t-norms and shown that all fuzzy metric spaces produce a Hausdorff first-countable topology.

The theory of GV-fuzzy metric has been established thus far by several academics. A great deal of knowledge on classical metric spaces was extended to fuzzy metric spaces. It was discovered throughout this procedure that the fuzzy metric theory differed greatly from the traditional theory of metric. As an illustration, Gregori and Romaguera [3] demonstrated the existence of an incompletable GV fuzzy metric space. A classification of the class of completable strong fuzzy metric spaces was provided by [2, 4-7, 12-13, 23, 28-29].

Jainrong Wu and Hao Yang [11] found a stronger result in 2000, which was a little unexpected. They demonstrated that a metrizable topology is produced by each GV-fuzzy metric. This crucial finding establishes a link between the classical metric and the GV-fuzzy metric. Nonetheless, the metric function's shape hasn't been examined in any of the previous research works. That is only the current paper's primary objective.

Most interesting motivations is the introduction of Revised fuzzy metric spaces by Alexandar sostak [1]. Later on, Olga Grigorenko [17], Juan jose Minana, Alexander Sostak, Oscar Valero introduced "On t-conorm based Fuzzy (Pseudo) metrics", they develop the basics of the theory of CB-fuzzy (pseudo) metrics and compare them with "classic" fuzzy (pseudo) metrics [2020]. After that Muraliraj and Thangathamizh [21] proved the existence of fixed points in Revised fuzzy metric space. Good, related results about fixed point in fuzzy metric spaces were introduced recently [14, 16, 26-27]

In this study, we first present the idea of a stratified function in a revised fuzzy metric space, which differs somewhat from the RGV-fuzzy metric space. We next demonstrate that the metrizable topology may coexist with the topology produced by the family of stratified functions. The concrete metric function whose topology agrees with the metrizable topology is then provided, subject to certain restrictions.

The paper is organized as follows. We address the early ideas on revised fuzzy metrics in the next section. Section 3 presents our primary findings. Lastly, we conclude in Section 4 with some last thoughts.

## 2. Preliminaries

In this section, we first introduce some basic concepts and properties of revised fuzzy metric spaces.

**Definition 1[30].**

A binary operation $\oplus: [0,1]^2 \longrightarrow [0,1]$ is a continuous t-conorm if it satisfies the following conditions:

(1) $\oplus$ is associative and commutative

(2) $\oplus$ is continuous

(3) $0 \oplus \mathbb{p} = \mathbb{p}$ for each $\mathbb{p} \in [0,1]$

(4) $\mathbb{p} \oplus \mathbb{q} \leq \mathbb{r} \oplus \mathbb{s}$ whenever $\mathbb{p} \leq \mathbb{r}$ and $\mathbb{q} \leq \mathbb{s}$ with $\mathbb{p}, \mathbb{q}, \mathbb{r}, \mathbb{s} \in [0,1]$.

The following continuous t-conorms are used in this paper:

$\mathbb{p} \vee_1 \mathbb{q} = max\{\mathbb{p},\ \mathbb{q}\}$,
$\mathbb{p} \vee_2 \mathbb{q} = \mathbb{p} + \mathbb{q} - a\mathbb{q}$,
$\mathbb{p} \vee_3 \mathbb{q} = min\{\mathbb{p} + \mathbb{q}, 1\}$. (1)

In the sense of Alexander sostack, a GV-fuzzy metric is defined by the follows.

**Definition 2[1].**

Let $\mathfrak{U}$ be a nonempty set and $\oplus$ be a continuous $t$-conorm. A revised fuzzy metric $\mathbb{W}$ on the set $\mathfrak{U}$ is a mapping $\mathbb{W}: \mathfrak{U}^2 \times (0, \infty) \longrightarrow (0, 1]$ satisfying the following conditions:
for all $\mathbb{a}, \mathbb{b}, \mathbb{c} \in \mathfrak{U},\ and\ t, s > 0$:
($\boldsymbol{\mathcal{RGV}}$ **1**) $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) < 1$,
($\boldsymbol{\mathcal{RGV}}$ **2**) $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) = 0$ if and only if $\mathbb{a} = \mathbb{b}$,
($\boldsymbol{\mathcal{RGV}}$ **3**) $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) = \mathbb{W}(\mathbb{b}, \mathbb{a}, t)$, (2)
($\boldsymbol{\mathcal{RGV}}$ **4**) $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) \oplus \mathbb{W}(\mathbb{b}, \mathbb{c}, s) \geq \mathbb{W}(\mathbb{a}, \mathbb{b}, t)$,
($\boldsymbol{\mathcal{RGV}}$ **5**) $\mathbb{W}(\mathbb{a}, \mathbb{b}, -): (0, \infty) \longrightarrow (0,1]$ is continuous.

If $\mathbb{W}$ is a $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric on $\mathfrak{U}$, then the 3-tuple $(\mathfrak{U}, \mathbb{W}, \oplus)$ is said to be a $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric space. In that case, if confusion is not possible, we call $\mathfrak{U}$ a $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric space for short. The following is a well-known result.

**Lemma 1.**

Let $\mathbb{W}(\mathbb{a}, \mathbb{b}, -)$ is non-increasing for all $\mathbb{a}, \mathbb{b} \in \mathfrak{U}$. Alexander sostack in that every $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric $\mathbb{W}$ on $\mathfrak{U}$ generates a topology $\tau_{\mathbb{W}}$ which has as a base

$\{\mathfrak{B}_{\mathbb{W}}(\mathbb{a}, \mathbb{r}, t): \mathbb{a} \in \mathfrak{U}, \mathbb{r}(0,1), t > 0\}$ (3)

were

$\mathfrak{B}_{\mathbb{W}}(\mathbb{a}, \mathbb{r}, t) = \{\mathbb{b} \in \mathfrak{U}: \mathbb{W}(\mathbb{a}, \mathbb{b}, t) < \mathbb{r}\}$, for all $\mathbb{a} \in \mathfrak{U}, \mathbb{r} \in (0,1)$, and $t > 0$. (4)

They proved that for each $\mathbb{a} \in \mathfrak{U}$, the family $\left\{\mathfrak{B}_{\mathbb{W}}\left(\mathbb{a}, \left(\frac{1}{\mathbb{m}}\right), \left(\frac{1}{\mathbb{m}}\right)\right): \mathbb{m} \in \mathbb{N}\right\}$ is a local base at $\mathbb{a}$.

A sequence $\{\mathbb{a}_{\mathbb{m}}\}$ in $\{\mathfrak{U}, \tau_{\mathbb{W}})\}$ converges to $\mathbb{a} \in \mathfrak{U}$ if and only if $\underset{n \longrightarrow \infty}{\log} \mathbb{W}(\mathbb{a}_{\mathbb{m}}, \mathbb{a}, t) = 0$ for all $t > 0$. Also, by using Kelley metrization lemma [21], they also proved that $\tau_{\mathbb{W}}$ is a metrizable topology.

## 3. Main Results

First, we introduce the concept of a stratified function in a $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric space.

**Definition 3.**

Let $(\mathfrak{U}, \mathbb{W}, \oplus)$ is a $\boldsymbol{\mathcal{RGV}}$ – revised fuzzy metric space. Let $\mathbb{r} \in (0,1)$ and $\mathbb{a}, \mathbb{b} \in \mathfrak{U}$; set

$\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) = sup\{t > 0: \mathbb{W}(\mathbb{a}, \mathbb{b}, t) < \mathbb{r}\}$. (5)

then, $\mathbb{d}_{\mathbb{r}}$ is called a $\mathbb{r}$ -stratified function with respect to $(\mathfrak{U}, \mathbb{W}, \oplus)$, $\{\mathbb{d}_{\mathbb{r}}: 0 < \mathbb{r} < 1\}$, the family of stratified functions.

To avoid the occurrence of the empty set, by a revised fuzzy metric in the rest of this paper, we mean an RGV-fuzzy metric satisfying

($\boldsymbol{\mathcal{RGV}}$ **6**) $\underset{n \longrightarrow \infty}{\log} \mathbb{W}(\mathbb{a}, \mathbb{b}, t) = 0, \forall\ \mathbb{a}, \mathbb{b} \in \mathfrak{U}$. (6)

**Lemma 2.**

Let $(\mathfrak{U}, \mathbb{W}, \oplus)$ be a revised fuzzy metric space, $\mathbb{r} \in (0,1)$, $t > 0$, $\mathbb{a}, \mathbb{b} \in \mathfrak{U}$. then,

(1) For any $\mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b}) < \lambda,\ \mathbb{W}(\mathbb{a},\mathbb{b},\lambda) \le \mathbb{r}$
(2) The function $\mathbb{d}_{\mathbb{r}}$ is non-decreasing with respect to $\mathbb{r} \in (0,1)$
(3) $\mathfrak{B}_{\mathbb{W}}(\mathbb{a},\mathbb{r},t) = \mathbb{N}_{\mathbb{W}}(\mathbb{a},t)$, were
$\mathbb{N}_{\mathbb{r}}(\mathbb{a},t) = \mathbb{b} \in \mathfrak{U}\colon \mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b}) < t$. (7)
(4) The function $\mathbb{W}(\mathbb{a},\mathbb{b},-)$ is strictly non-increasing for the fixed points $\mathbb{a},\mathbb{b} \in \mathfrak{U}$, if and only if for any $\mathbb{r} \in (0,1)$,
$\mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b}) = sup\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) < \mathbb{r}\}$. (8)

**Proof**

(1) Let $\lambda > \mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b})$. From Definition 3, there exists $0 < t < \lambda$ such that $\mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}$. So, $\mathbb{W}(\mathbb{a},\mathbb{b},\lambda) \le \mathbb{r}$.
(2) It follows from Lemma 1 directly.
(3) Let $\mathbb{b} \in \mathfrak{B}_{\mathbb{W}}(\mathbb{a},\mathbb{r},t)$, that is, $\mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}$. Since $\mathbb{W}(\mathbb{a},\mathbb{b},-)$ is continuous and non-increasing, there exists
$0 < t_1 < t$ such that $\mathbb{W}(\mathbb{a},\mathbb{b},t_1) \le \mathbb{r}$. From (5), we know $\mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b}) \le t_1 < t$. So, $\mathbb{b} \in \mathbb{d}_{\mathbb{r}}(\mathbb{a},t)$. From the arbitrariness of $\mathbb{b}$, we know that
$\mathfrak{B}_{\mathbb{W}}(\mathbb{a},\mathbb{r},t) \subseteq \mathbb{N}_{\mathbb{r}}(\mathbb{a},t)$. (9)
Let $\mathbb{c} \in \mathbb{N}_{\mathbb{r}}(\mathbb{a},t)$; then, $\mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{c}) < t$. From (5), there exists $0 < t_2 < t$ such that $\mathbb{W}(\mathbb{a},\mathbb{b},t_2) \le \mathbb{r}$. therefore, $\mathbb{W}(\mathbb{a},\mathbb{c},t) \le \mathbb{W}(\mathbb{a},\mathbb{b},t_2) \le \mathbb{r}$. So, $\mathbb{c} \in \mathfrak{B}_{\mathbb{W}}(\mathbb{a},\mathbb{r},t)$. From the arbitrariness of $\mathbb{c}$, we know that $\mathbb{N}_{\mathbb{r}}(\mathbb{a},t) \subseteq \mathfrak{B}_{\mathbb{W}}(\mathbb{a},\mathbb{r},t)$.
(4) Suppose that (8) holds; however, $\mathbb{W}(\mathbb{a},\mathbb{b},-)$ is not strictly decreasing.
Then, there exist $t_1, t_2 \in \{t > 0\colon 0 < \mathbb{W}(\mathbb{a},\mathbb{b},t) < 1\}$ such that $t_1 < t_2$ and $\mathbb{W}(\mathbb{a},\mathbb{b},-) \equiv \mathbb{r}_0$ on $[t_1,t_2]$. thus,
$sup\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \ge \mathbb{r}_0\} \ge t_2 > t_1 \ge\ inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}_0\}$. (10)
It is easy to see that
$sup\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \ge \mathbb{r}_0\} = inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) < \mathbb{r}_0\}$. (11)
therefore, $\mathbb{d}_{\mathbb{r}_0}(\mathbb{a},\mathbb{b}) > inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) < \mathbb{r}_0\}$, which conflicts with (8).
Conversely, suppose $\mathbb{W}(\mathbb{a},\mathbb{b},-)$ is strictly decreasing.
Let, $t_0 = \mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b})\ = inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) < r\}$. (12)
Obviously, $inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}\} \le t_0$. If $inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) < r\} < t_0$, then there is $0 < t_2 < t_1$ such that $\mathbb{W}(\mathbb{a},\mathbb{b},t_2) \le \mathbb{r}$, so $\mathbb{W}(\mathbb{a},\mathbb{b},t_2) < r$. Since $\mathbb{W}(\mathbb{a},\mathbb{b},-)$ is
Right continuous at $t_0$, there is $\delta > 0$ such that
$\mathbb{W}(\mathbb{a},\mathbb{b},t_0 - \delta) < r$, which conflicts with the definition of $t_0$.
Thus, $inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}\} \ge t_0$. So,
$\mathbb{d}_{\mathbb{r}}(\mathbb{a},\mathbb{b}) = t_0 = inf\{t > 0\colon \mathbb{W}(\mathbb{a},\mathbb{b},t) \le \mathbb{r}\}$. (13)

Now, from Lemma 2 (3), it is easy to see that the topology $\tau_{\mathbb{W}}$ can be induced by the family of stratified functions. That is, we obtain the following theorem.

**Theorem 1.**

Let $\mathfrak{M} = \{\mathbb{d}_{\mathbb{r}} : 0 < \mathbb{r} < 1\}$ be the family of stratified functions with respect to a revised fuzzy metric space $(\mathfrak{U}, \mathbb{W}, \oplus)$, $\mathbb{N}_{\mathbb{r}}(\mathbb{a}, t)$ be defined by (8), and

$\mathfrak{B}_{\mathbb{a}} = \{\mathbb{N}_{\mathbb{r}}(\mathbb{a}, t) : \mathbb{r} \in (0,1),\ t > 0\}$.

(14)

Then,

(1) $\mathfrak{B}_{\mathbb{a}}$ is a base of neighborhoods at $\mathbb{a} \in \mathbb{A}$.

(2) The topology $\tau_{\mathbb{W}}$ generated by$\{\mathfrak{B}_{\mathbb{a}} : \mathbb{a} \in \mathbb{A}\}$ coincides with the topology $\tau_{\mathbb{W}}$.

Generally, a stratified function is not a pseudo metric. In fact, we have the following result.

**Theorem 2.**

Let $(\mathfrak{U}, \mathbb{W}, \oplus)$ be a revised fuzzy metric space. A stratified function $\left(\mathbb{d}_{\mathbb{r}}\left(\mathbb{r} \in (0,1)\right)\right)$ is a pseudometric on $\mathbb{A}$ if and only if $\mathbb{W}$ satisfies the following condition: for any $\mathbb{a}, \mathbb{b}, \mathbb{c} \in \mathbb{A}$, $t_1, t_2 > 0$, if $\mathbb{W}(\mathbb{a}, \mathbb{c}, t_1) < r$, $\mathbb{W}(\mathbb{c}, \mathbb{b}, t_2) < r$, then

$\mathbb{W}(\mathbb{a}, \mathbb{b}, t_1 + t_2) < r$.

(15)

**Proof.** For any $\mathbb{r} \in (0,1)$, it is obvious that $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \geq 0$,

$\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) = \mathbb{d}_{\mathbb{r}}(\mathbb{b}, \mathbb{a})$, and $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) = 0$ when $\mathbb{a} = \mathbb{b}$. Thus, to complete the proof, we only must prove that

$\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \leq \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b})$ if and only if $\mathbb{W}$ satisfies condition (15).

Sufficiency. For any $\varepsilon > 0$, from Lemma 2 (1), we obtain

$\mathbb{W}\left(\mathbb{a}, \mathbb{c}, \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \frac{\varepsilon}{2}\right) < r$, $\mathbb{W}\left(\mathbb{c}, \mathbb{b}, \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b}) + \frac{\varepsilon}{2}\right) < r$.

(16)

From (15), we have

$\mathbb{W}\left(\mathbb{a}, \mathbb{b}, \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b}) + \frac{\varepsilon}{2}\right) < r$.

(17)

therefore,

$\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \leq \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b}) + \varepsilon$.

(18)

From the arbitrariness of $\varepsilon > 0$, we know $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \leq \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b})$.

Necessity. Suppose that $\mathbb{W}(\mathbb{a}, \mathbb{c}, t_1) < r$, $\mathbb{W}(\mathbb{c}, \mathbb{b}, t_2) < r$. By (RGV5), there exists $\delta > 0$ such that $\mathbb{W}(\mathbb{a}, \mathbb{c}, t_1 - \delta) < r$, $\mathbb{W}(\mathbb{c}, \mathbb{b}, t_2 - \delta) < r$,

(19)

So, $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) \leq t_1 - \delta$ and $\mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b}) \leq t_2 - \delta$.

Since $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \leq \mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{c}) + \mathbb{d}_{\mathbb{r}}(\mathbb{c}, \mathbb{b})$, we have $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b}) \leq t_1 + t_2 - 2\delta < t_1 + t_2$. By the definition of $\mathbb{d}_{\mathbb{r}}(\mathbb{a}, \mathbb{b})$, there exists $t_0 < t_1 + t_2$ such that $\mathbb{W}(\mathbb{a}, \mathbb{b}, t_0) < r$. Thus, $\mathbb{W}(\mathbb{a}, \mathbb{b}, t_1 + t_2) < r$.

**Remark 1.**

Let $\mathbb{W}$ satisfies (15) if $\oplus = \vee_3$. Now, we explore the metric which induces the topology $\tau_{\mathbb{W}}$.

**Definition 4.**
Let $\mathbb{R}^+ = [0, \infty)$. We call function $\mathcal{G}: \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ satisfies
(i) the condition $\mathfrak{C}_1$, if $\mathcal{G}(0) = 0$, $\mathcal{G}(t) \equiv 0$, and $\mathcal{G}$ is non increasing and continuous at $0$.
(ii) the condition $\mathfrak{C}_2$, if $\mathcal{G}(0) = 0$, $\mathcal{G}(0) > 0$ as $t > 0$,
$\lim_{t \longrightarrow +\infty} \mathcal{G}(t) = +\infty, \mathcal{G}(t_1 + t_2) \leq \mathcal{G}(t_1) + \mathcal{G}(t_2)$ for any $t_1 + t_2 \in \mathbb{R}^+$, and $\mathcal{G}$ is right continuous and non-increasing.

**Theorem 3.**
Let $(\mathfrak{U}, \mathbb{W}, \oplus)$ is revised fuzzy metric space; the functions $\mathcal{K}$ and $\mathcal{G}$ satisfy the conditions $\mathfrak{C}_1$ *and* $\mathfrak{C}_2$, respectively. Define a function $\mathbb{d}$ on $\mathfrak{U}^2$ as
$\mathbb{d}(\mathbb{a}, \mathbb{b}) = inf\{s > 0\text{: if } \mathcal{G}(t) > s, then\ \mathbb{W}(\mathbb{a}, \mathbb{b}, t) + \mathcal{K}(t) \leq 0, \forall \mathbb{a}, \mathbb{b} \in \mathfrak{U}\}$.
(20)
If one of the following conditions is satisfied:
(I) $\mathbb{W}$ satisfies condition (15)
(II) $\oplus \geq \Delta_1, \forall r_1, r_2 \in [0, \infty)$
$\mathcal{K}(r_1) + \mathcal{K}(r_2) \leq \mathcal{K}(r_1) + r_2,$
(21)
then $\mathbb{d}$ is a metric on $\mathfrak{U}$.

**Proof.**
First, we prove the following fact: if
$\mathcal{G}(t) < r < \mathbb{d}(\mathbb{a}, \mathbb{b}),$
(22)
then
$\mathbb{W}(\mathbb{a}, \mathbb{b}, t) < r \leq \mathcal{K}(r).$
(23)
In fact, if $\mathcal{G}(t) < r < \mathbb{d}(\mathbb{a}, \mathbb{b})$, from the definition of $\mathbb{d}$, we obtain there exists $0 < s < r$ such that if $\mathcal{G}(t) > s$, then $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) + \mathcal{K}(r) \leq 0$.
Therefore, if $\mathcal{G}(t) > r$, then $\mathcal{G}(t) > s$, and hence,
$\mathbb{W}(\mathbb{a}, \mathbb{b}, t) + \mathcal{K}(r) \leq \mathbb{W}(\mathbb{a}, \mathbb{b}, t) + \mathcal{K}(s) \leq 0.$
That is, $\mathbb{W}(\mathbb{a}, \mathbb{b}, t) \leq \mathcal{K}(r)$.
Next, we prove $\mathbb{d}$ is a metric on $\mathfrak{U}$, that is, $\mathbb{d}$ satisfies the following properties: for any $\mathbb{a}, \mathbb{b}, \mathbb{c} \in \mathfrak{U}$,
(M1) $\mathbb{d}(\mathbb{a}, \mathbb{b}) \geq 0$, $\mathbb{d}(\mathbb{a}, \mathbb{b}) = \mathbb{d}(\mathbb{b}, \mathbb{a})$,
(M2) $\mathbb{d}(\mathbb{a}, \mathbb{b}) = 0$ if and only if $\mathbb{a} = \mathbb{b}$,
(M3) $\mathbb{d}(\mathbb{a}, \mathbb{b}) \leq \mathbb{d}(\mathbb{a}, \mathbb{c}) + \mathbb{d}(\mathbb{c}, \mathbb{b})$. the conclusion $(M1)$ is obvious.
For the conclusion $(M2)$, it is easy to see that $\mathbb{d}(\mathbb{a}, \mathbb{b}) = 0$ if $\mathbb{a} = \mathbb{b}$.
Now, we suppose $\mathbb{d}(\mathbb{a}, \mathbb{b}) \geq 0$; however, $\mathbb{a} \neq \mathbb{b}$. then, there exists $t_0 > 0$ such that $\mathbb{W}(\mathbb{a}, \mathbb{b}, t_0) \neq 0$, that is, $\mathbb{W}(\mathbb{a}, \mathbb{b}, t_0) \geq 0$.
Since $\mathcal{K}$ is continuous at 0, there exists $0 < r < \mathcal{G}(t)$ such that $\mathcal{K}(r_0) > \mathbb{W}(\mathbb{a}, \mathbb{b}, t_0)$.
This is indirect contradiction to (23). Thus, $\mathbb{d}(\mathbb{a}, \mathbb{b}) = 0$ implies that $\mathbb{a} = \mathbb{b}$.
To prove $(M3)$, we take $\mathbb{d}(\mathbb{a}, \mathbb{c}) < r_1$ and $\mathbb{d}(\mathbb{c}, \mathbb{b}) < r_2$ arbitrarily. From (23), we know
(i) If $\mathcal{G}(t) > r_1$, then $\mathbb{W}(\mathbb{a}, \mathbb{c}, t) \leq \mathcal{K}(r_1) \leq \mathcal{K}(r_1) + r_2$.
(24)

(ii) If $\mathcal{G}(\mathscr{t}) > \mathscr{r}_2$, then $\mathbb{W}(\mathbb{c}, \mathbb{b}, \mathscr{t}) \le \mathcal{K}(\mathscr{r}_2) \le \mathcal{K}(\mathscr{r}_1) + \mathscr{r}_2$.
(25)

Now, suppose that $\mathscr{r}_1 + \mathscr{r}_2 < \mathcal{G}(\mathscr{t})$. Let
$\mathscr{t}_0 = sup\{\mathscr{s}: 0 < \mathscr{s} \le \mathscr{t}, \mathcal{G}(\mathscr{s}) > \mathscr{r}_1\}$.
(26)

Obviously, $\mathscr{t}_0 \le \mathscr{t}$. It is easy to prove that $\mathcal{G}(\mathscr{t}_0) \le \mathscr{r}_1$.
In fact, if $\mathcal{G}(\mathscr{t}_0) \le \mathscr{r}_1$, from the right continuity of $\mathcal{G}$, there exists $\eta > 0$, such that $\mathcal{G}(\mathscr{t}_0 - \eta) \le \mathscr{r}_1$.
By the definition of $\mathscr{t}_0$, we conclude that $\mathscr{t}_0 \le \mathscr{t}_0 - \eta$, a contradiction.
The fact $\mathcal{G}(\mathscr{t}_0) \le \mathscr{r}_1$ implies that $\mathcal{G}(\mathscr{t} - \mathscr{t}_0) \le (\mathcal{G}\mathscr{t} - \mathcal{G}\mathscr{t}_0) \le \mathscr{r}_2$
By the right continuity of $\mathcal{G}$ again, we know there exists $\varepsilon > 0$ such that $\mathcal{G}(\mathscr{t} - (\mathscr{t}_0 + \varepsilon)) \le \mathcal{G}(\mathscr{t} - (\mathscr{t}_0 - \varepsilon)) < \mathscr{r}_2$.
By the definition of $\mathscr{t}_0$, there exists $0 < \mathscr{s}_1 \le t$ such that $\mathcal{G}(\mathscr{s}_1) \le \mathscr{r}_1$ and $\mathscr{s}_1 \le \mathscr{t}_0 + \varepsilon$.
Noting that $G$ is increasing, we obtain that $\mathcal{G}(\mathscr{t} - \varepsilon)) \le \mathcal{G}(\mathscr{s}_1) < \mathscr{r}_1$. From (i) and (ii), we have
$\mathbb{W}(\mathbb{c}, \mathbb{b}, (\mathscr{t}_0 + \varepsilon)) \le \mathcal{K}(\mathscr{r}_1) \le \mathcal{K}(\mathscr{r}_1 + \mathscr{r}_2)$
$\mathbb{W}(\mathbb{c}, \mathbb{b}, \mathscr{t} - (\mathscr{t}_0 + \varepsilon)) \le \mathcal{K}(\mathscr{r}_2) \le \mathcal{K}(\mathscr{r}_1) + \mathscr{r}_2$.
(27)

Combining conditions (I) and (II), we get $\mathbb{W}(\mathbb{a}, \mathbb{b}, \mathscr{t}) \le \mathcal{K}(\mathscr{r}_1 + \mathscr{r}_2)$.
By the definition of $\mathbb{d}$, we know $\mathbb{d}(\mathbb{a}, \mathbb{b}) \le \mathscr{r}_1 + \mathscr{r}_2$.
Since $\mathbb{d}(\mathbb{a}, \mathbb{c}) < \mathscr{r}_1$ and $\mathbb{d}(\mathbb{c}, \mathbb{b}) < \mathscr{r}_2$, we obtain
$\mathbb{d}(\mathbb{a}, \mathbb{b}) \le \mathbb{d}(\mathbb{a}, \mathbb{c}) + \mathbb{d}(\mathbb{c}, \mathbb{b})$.
(28)

directly.

**Theorem 4.**

$(\mathfrak{U}, \mathbb{W}, \oplus)$ *is a revised fuzzy metric space.* $\mathbb{W}$ *satisfies condition (15) or* $\oplus \ge \Delta_1$. *Let*
$\rho(\mathbb{a}, \mathbb{b}) = sup\{\mathbb{x} > 0: \mathbb{W}(\mathbb{a}, \mathbb{b}, \mathbb{x}) + \mathbb{x} \le 0\}, \forall \mathbb{a}, \mathbb{b} \in \mathfrak{U}$.
(29)

Then, $\rho$ is a metric on $\mathfrak{U}$ and the topology $\tau_\rho$ induced by $\rho$ coincides with the topology $\tau_{\mathbb{W}}$.

***Proof.***

Let $\mathcal{G}(\mathscr{t}) = \mathscr{t}, \mathcal{K}(\mathscr{t}) = \mathscr{t}$, $\forall t \ge 0$. Then, $\mathcal{K}$ and $\mathcal{G}$ satisfy the conditions $\mathfrak{C}_1$ *and* $\mathfrak{C}_2$, respectively.
Besides, $\mathcal{K}$ satisfies condition (21). From Theorem 3, we know that
$\mathbb{d}(\mathbb{a}, \mathbb{b}) = sup\{\mathscr{s} > 0: if\ \mathscr{t} > \mathscr{s}, \mathbb{W}(\mathbb{a}, \mathbb{b}, \mathscr{t}) + \mathscr{s} \le 0\}, \forall \mathbb{a}, \mathbb{b} \in \mathfrak{U}$.
(30)

is a metric on $\mathfrak{U}$.
Thus, to show that $\rho$ is a metric, we only need to show $\mathbb{d}(\mathbb{a}, \mathbb{b}) \le \rho(\mathbb{a}, \mathbb{c}), \forall \mathbb{a}, \mathbb{b} \in \mathfrak{U}$.
In fact, if $\mathbb{x} > 0$, $\mathbb{W}(\mathbb{a}, \mathbb{b}, \mathbb{x}) + \mathbb{x} \le 0$ and $\mathscr{t} > \mathbb{x}$, then $\mathbb{W}(\mathbb{a}, \mathbb{b}, \mathscr{t}) + \mathbb{x} \le \mathbb{W}(\mathbb{a}, \mathbb{b}, \mathbb{x}) + \mathbb{x} \le 0$.
Therefore,
$\{\mathbb{x} > 0: \mathbb{W}(\mathbb{a}, \mathbb{b}, \mathbb{x}) + \mathbb{x} \le 0\} \subseteq \{\mathscr{s} > 0: if\ \mathscr{t} > \mathscr{s}, then\ \mathbb{W}(\mathbb{a}, \mathbb{b}, \mathscr{t}) + \mathscr{s} \le 0\}$.
(31)

From (29) and (30), we get $\rho(\mathbb{a}, \mathbb{b}) \ge \mathbb{d}(\mathbb{a}, \mathbb{b})$.
On the other hand, from (30), for any $\varepsilon > 0$, there exists $\mathscr{s} > 0$ such that $\mathscr{s} > \mathbb{d}(\mathbb{a}, \mathbb{b}) + \varepsilon$ and $\mathbb{W}(\mathbb{a}, \mathbb{b}, \mathscr{t}) + \mathscr{s} \le 0$ when $\mathscr{t} > \mathscr{s}$. thus,

$\mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{d}(\mathbb{a},\mathbb{b})+\varepsilon)+\mathscr{s}\leq 0.$

(32)

and hence

$\mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{d}(\mathbb{a},\mathbb{b})+\varepsilon)+\mathbb{d}(\mathbb{a},\mathbb{b})+\varepsilon)<0.$

(33)

From (29), we get $\rho(\mathbb{a},\mathbb{b})\leq\mathbb{d}(\mathbb{a},\mathbb{b})+\ \varepsilon$.

From the arbitrariness of $\varepsilon > 0$, we have $\rho(\mathbb{a},\mathbb{b})\leq\mathbb{d}(\mathbb{a},\mathbb{b})$. thus, $\rho(\mathbb{a},\mathbb{b})=\mathbb{d}(\mathbb{a},\mathbb{b})$.

Let $\mathscr{r}\in(0,1),\mathbb{a}\in\mathfrak{U}$. We put $\mathcal{U}(\mathbb{a},\mathscr{r})=\{\mathbb{b}\in\mathfrak{U}:\rho(\mathbb{a},\mathbb{b})<\mathscr{r}\}$. It is easy to show that

$\mathcal{U}(\mathbb{a},\mathscr{r})\subseteq\mathcal{B}_{\mathbb{W}}(\mathbb{a},\mathscr{r},\mathscr{r})\subseteq\mathcal{U}(\mathbb{a},2\mathscr{r})$.

(34)

In fact, for any $\mathbb{b}\in\mathcal{U}(\mathbb{a},\mathscr{r})$, $sup\{\mathbb{x}>0:\mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{x})+\mathbb{x}\leq 0\}\ <\mathscr{r}$.

Thus, there is $\mathbb{x}>0$ such that $\mathbb{x}<\mathscr{r}$ and

$\mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{x})+\mathbb{x}\leq 0$. So, $\mathbb{W}(\mathbb{a},\mathbb{b},\mathscr{r})+\mathscr{r}\leq 0$, that is, $\mathbb{b}\in\mathcal{B}_{\mathbb{W}}(\mathbb{a},\mathscr{r},\mathscr{r})$. Hence,

$\mathcal{U}(\mathbb{a},\mathscr{r})\subseteq\mathcal{B}_{\mathbb{W}}(\mathbb{a},\mathscr{r},\mathscr{r})$.

(35)

On the other hand, for any $\mathbb{c}\in\mathcal{B}_{\mathbb{W}}(\mathbb{a},\mathscr{r},\mathscr{r})$, $\mathbb{W}(\mathbb{a},\mathbb{c},\mathscr{r})+\mathscr{r}<0$.

From (29), we get $\rho(\mathbb{a},\mathbb{c})\leq\mathscr{r}<\mathscr{r}$. So, $\mathbb{c}\in\mathcal{U}(\mathbb{a},2\mathscr{r})$.

Hence,

$\mathcal{B}_{\mathbb{W}}(\mathbb{a},\mathscr{r},\mathscr{r})\subseteq\mathcal{U}(\mathbb{a},\mathscr{r})$.

(36)

And (34) holds, which implies that $\tau_{\rho}=\tau_{\mathbb{W}}$ directly.

**Lemma 3.**

Let $(\mathfrak{U},\mathbb{W},\oplus)$ *is revised fuzzy metric space; the functions* $\mathcal{K}$ and $\mathcal{G}$ *satisfy the conditions* $\mathfrak{C}_1$ *and* $\mathfrak{C}_2$, *respectively. then, the function d defined by (20) can be represented as follows,* $\forall\ \mathbb{a},\mathbb{b}\in\mathfrak{U}$*:*

$\mathbb{d}(\mathbb{a},\mathbb{b})=inf\{\mathscr{s}>0: if\ \mathcal{G}(\mathscr{t})>\mathscr{s}, then\ \mathbb{W}(\mathbb{a},\mathbb{b},\mathscr{t})+\mathcal{K}(\mathscr{s})<0\}$.

(37)

***Proof.***

In fact, we only need to show that $inf\mathbb{A}=sup\mathbb{B}$,

Where,

$\mathbb{A}=\{\mathbb{x}>0: if\ \mathcal{G}(\mathscr{t})>\mathbb{x}, then\ \mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{x})+\mathcal{K}(\mathbb{x})\leq 0\}$,

$\mathbb{B}=\{\mathbb{y}>0: if\ \mathcal{G}(\mathscr{t})>\mathbb{y}, then\ \mathbb{W}(\mathbb{a},\mathbb{b},\mathbb{x})+\mathcal{K}(\mathbb{y})\leq 0\}$.

(38)

Take $\mathbb{x}\in\mathbb{A},\mathbb{y}\in\mathbb{B}$ arbitrarily. Suppose $\mathbb{y}>\mathbb{x}$. If $\mathcal{G}(\mathscr{t})>\mathbb{y}$, then $\mathcal{G}(\mathscr{t})>\mathbb{x}$. From the definitions of $\mathbb{A}$ and $\mathbb{B}$, we obtain

$\mathbb{W}(\mathbb{a},\mathbb{b},\mathscr{t})+\mathcal{K}(\mathbb{x})\leq 0>\mathbb{W}(\mathbb{a},\mathbb{b},\mathscr{t})+\mathcal{K}(\mathbb{y})$.

(39)

this is in direct contradiction to the condition that $\mathcal{K}$ is non increasing. So, $\mathbb{x}\geq\mathbb{y}$, and hence, $inf\mathbb{A}\geq sup\mathbb{B}$.

Now, let $inf\mathbb{A}=\alpha, sup\mathbb{B}=\beta$. For any $\delta>0$, we know $\beta+\delta\notin\mathbb{B}$, that is, $\beta+\delta\in\mathbb{A}$,

which implies that $\inf\mathbb{A}\leq\beta+\delta$.

By the arbitrariness of $\delta$, we know that $inf\mathbb{A}\leq\ \beta=sup\mathbb{B}$. This completes the proof.

**Theorem 5.**
Let $(\mathfrak{U}, \mathbb{W}, \oplus)$ *is a revised fuzzy metric space. If* $\mathbb{W}$ *satisfies condition (15) or* $\oplus \geq \Delta_1$, *then* $\forall \mathbb{a}, \mathbb{b} \in \mathfrak{U}$*:*

$$\rho(\mathbb{a}, \mathbb{b}) = inf\{t > 0 \colon \mathbb{W}(\mathbb{a}, \mathbb{b}, t) + t < 0\}, \tag{40}$$

*where ρ is defined by (29).*

***Proof.*** Let $\mathcal{G}(t) = t, \mathcal{K}(t) = t$, ∀t ≥ 0. Then, (40) follows from Lemma 3 directly.

## 4. Conclusions

We study several metric structures in a revised fuzzy metric space in current research. We provide the explicit form of the metric function about the metrizable topology for a revised fuzzy metric in two exceptional scenarios. Interesting future research investigations concerning linked themes may be prospective, based on the paper's conclusions. Furthermore, this paper's methodology suggests discussing the relevant issue in a broader context.

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