Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)

GRA Approach for the Solution of Mitigating Air Pollutant Emissions from Animal Barns with the Help of fuzzy Knowledge Measure

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Article History:

***Received:** 13-11-2024*

***Revised:**25-12-2024*

***Accepted:**09-01-2025*

Abstract: The air pollutants emissions from concentrated animal husbandry contribute to environmental pollution and global warming problems. A number of air pollutants, including NH_3, H_2S, CH_4 and CO_2, can be produced via land application, animal housing, and manure storage. It is necessary to reduce these contaminants while maintaining the supply of animal protein. In addition to safeguarding the environment, this mitigation will enhance indoor air quality, which is vital for the welfare and health of animals as well as the workers' safety. For this problem various methods are compared that can resolve this problem and a knowledge measure based on fuzzy sets for managing ambiguity and uncertainty in real life problems. This work uses fuzzy sets to present a knowledge measure to handle these type of ambiguity and uncertainty. The fuzzy sets that satisfies the Knowledge measure has accumulated significant focus yet remains pending due to its importance in evaluating fuzzy information. This paper presents a knowledge measure under fuzzy sets and rigorously evaluates its validity through a systematic, axiomatic analysis. The effectiveness of the suggested measure for ambiguity and linguistic comprise is explained via quantitative example, depend upon the discussed measure. Any fuzzy set that is co-relate to Knowledge is assess by a knowledge measure. Initiation of a new knowledge measure has been introduced for the current problems to get more effective results. The prime target of this study is to get foremost ideal solution which is much better than the orthodox fuzzy approach. The validity and usefulness of knowledge measure can be evaluated by numerical examples. Additionally, with the aid of Multi Criteria decision-making and GRA approach this knowledge measure becomes a useful methodology for comparison and can be applied to improve a solution to a problem. We present the suggested approach's real-world implementation especially by reducing CO_2 (Carbon Dioxide) and NH_3 (Ammonia) gas. A case study on the existence of pollution and how can a producer reduce CO_2 and NH_3 concentration in animal barns with this approach. Throughout the entire study a comparison is made between the enlisted approach and current approach in order to assess the proposed technique's efficacy and competence.

Keywords: Fuzzy Set, Fuzzy Knowledge Measure, GRA, Information Measure, MCDM.

1. Introduction

Lotfi Aliasker Zadeh [1] was a notable person known for his contribution in the field of computer science and mathematics, especially in the fuzzy set theory and many more related concepts like fuzzy logic, fuzzy algorithms, etc. Only the degree of membership which is provided by set of elements in the closed interval [0,1] is considered in fuzzy sets. Distance, similarity, and entropy measurements of fuzzy sets were defined axiomatically by Xuecheng

[2]. The fundamental relationship between these measures was also methodically examined. Four new postulates were also introduced by Luca and Termini [3] for the fuzzy information measure. It was the new version of the Shannon entropy [4] transformed into a fuzzy information measure. After that, Kaufmann [5] and Yager [6] proposed methods one by one to quantify the difference in the fuzzy set's membership function relative to its closest structured set and also determined their fuzzy set membership function and its complement, respectively. At the same time, the contradiction to the above belief, knowledge get measured with the help of knowledge measure that termed as reverse of fuzzy information measure or uncertainty. The contrast between an extreme fuzzy set and a fuzzy set can be represented through knowledge measure since an entropy measure is unable to capture all of the uncertainties of a fuzzy set. They, rather than focusing on the relationship between these two one of them is knowledge measure and other is entropy, we introduce knowledge measure in this study to address the problem that entropy had not frequently evaluated. Das et al. [7] found that while addressing multi-criterion decision-making (MCDM) problems, the knowledge measure was applied to determine the weight of each and every attribute. In an MCDM problem, we search the various alternatives to choose a particular choice that meets the requirement of predetermined criteria. Every MCDM problem conclusion includes a crucial term, such as weight of the criteria. We can use the weights for the justified criteria to determine, which choice is best. In (1998) a new technique which is known as VIKOR was introduced by Opricovic [8] for addressing MCDM problems. In order to resolve a disagreement, compromise is accepted, the best option is sought after by the decision maker, and all the predetermined criteria are taken into account when evaluating the options. By ranking the options, VIKOR finds the compromise that comes the closest to the ideal. The fuzzy accuracy measure is used in the proposed approach rather than distance measure the outcomes are highly positive. Most researchers used distance measure to compute maximum benefit to the group and least amount of personal sorrow in the earlier VIKOR approach, however, in certain places, A few handy standardized The reason why the standard results of distance measure are not followed is current measure are counter intuitive. As a result, the MCDM topics that depend on them lose credibility and become less valuable and the solution got by applying these measure has lost its properties of proximity to the optimal, ideal solution. A variety of techniques can be selected to get best opinion in MCDM problems. These are termed as VIKOR, ELECTRA, TOPSIS, ARAS, TODIM etc. In this we also use the GRA method. Some of the researchers used combined approach of two different methods. VIKOR technique was used by Shemshadi et al. [9] to solve the issue of supplier selection and some is used by Wan et al. [10] in resolving group decision making issues with several attributes. Todim-Electra method to hire a local partner for a multinational footwear company was used by earlier researcher. After the COVID-19 pandemic, Kadian and Kumar [11] proposed a method for pattern recognition and outbreaks of COVID-19. Everyone is impacted by environmental contamination and the variables that trigger pollution in contemporary times. Thus, reducing the amount of elements that create pollution is vital. In this study, we address optimization approaches for reducing carbon dioxide and ammonia and how to accomplish. Thus, a multi-criteria approach is needed to prioritize the alternatives for reducing pollutants through this. Multi-criteria decision-making (MCDM) is a useful technique

for contrary in this context. This paper uses the Grey Relational Analysis (GRA) with a multi-criteria assessment model and a new knowledge measure formula and other earlier approaches. Recent developments in the decision-making sciences have increased the acceptability and accessibility of MCDM techniques. The acceptance and accessibility of MCDM approaches have increased recently. The creation of MCDM techniques is driven by a variety of real-world issues that call for the evaluation of several factors. This paper presents an effort to determine the ideal concentration of ammonia and carbon dioxide. Specifically, algal cell concentration was used to test biological performance, whereas carbon dioxide fixation rate, ammonia fixation efficiency, carbon dioxide removal efficiency, and ammonia removal efficiency were used to measure environmental performance. Multiple-step processes are carried out consistently. This study's primary goal is to develop a fuzzy set of measure and provide instances to support its validity. It also introduces an accuracy measure, which is a generalized version of the stated knowledge measure. The stated measure is then implement to MCDM problems that undergone fuzzy conditions as well as its major properties are demonstrated through practical examples and comparative analysis. Finally, the proposed approach is tested in MCDM problem using the GRA method.

2. Some Preliminaries

A couple of key definitions are provided in this segment.

Definition 1

Let us assume a finite set P($\neq \phi$). Zadeh [1] defined the fuzzy set Q as

$$Q = \{\langle p_i, \mu_Q(p_i)\rangle : p_i \in P\},$$
(1)

where it provides μ_Q : P → $[0,1]$ with its affinity of element p_i in given set Q which is known as its membership function.

Remark: We will see that Fs(P) is a collection of all Fs defined on P throughout this work.

Definition 2

Assuming that Q_1, Q_2 ∈ Fs(P), the following are the definitions of the main operations on fuzzy sets:

$$Q_1 \cup Q_2 = \left\{\langle p_i, max\left(\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right)\rangle : p_i \in P\right\}.$$
(2)

$$Q_1 \cap Q_2 = \left\{\langle p_i, min\left(\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right)\rangle : p_i \in P\right\}.$$
(3)

$$Q^j = \{\langle p_i, 1 - \mu_Q(p_i)\rangle : p_i \in P\}.$$
(4)

$Q_1 \subseteq Q_2 \Leftrightarrow \mu_{Q_1}(p_i) \leq \mu_{Q_2}(p_i), where\ p_i \in P.$
(5)

Definition 3

A function R: Fs(P) → [0,1] can be defined as a measure of information if it upholds the subsequent four axioms which are given by Luca and Termini [3].

1. For maximum R(Q) ⇔ $\mu_Q(p_i)$ =0.5 ∀ $p_i \in P$, i.e., the most fuzzy set.**(Maximality)**
2. R(Q)=0 ⇔ $\mu_Q(p_i) \in \{0,1\}$ ∀ $p_i \in P$. **(Minimality)**
3. Since the fuzzy set Q has been sharpened, R($\tilde{Q}$) ≤ R(Q). **(Resolution)**
4. Given a fuzzy set Q and its complement Q^j, $R(Q) = R(Q^j)$. **(Symmetry)**

The degree of fuzziness in the fuzzy collection is measured by the entropy known as fuzzy entropy. Furthermore, the quantity of knowledge is determined by a knowledge measure Singh et al [12] claim that there is two beliefs complement one another effectively.

Definition 4

As stated by Singh et al. [12] Y is a function that we can define. → Fs(P) [0,1] If the following four axioms are redeemed, as FKM.

1. Y(Q) represents the maximum ↔ $\mu_Q(p_i) \in \{0,1\}$ ∀ $p_i \in P$; that is, Q can be any unambiguous set. **(Maximality)**
2. Y(Q)=0 ⇔ $\mu_Q(p_i)$ =0.5 ∀ $p_i \in P$, i.e.,the most fuzzy set is Q.**(Minimality)**
3. sharpened version of $\tilde{Q}$ there is fuzzy set Q, Y(Q) ≤ Y($\tilde{Q}$). **(Resolution)**
4. If Q^j show its complement for the fuzzy set Q, there is $Y(Q) = Y(Q^j)$. **(Symmetry)**

Definition 5

we have two fuzzy sets Q_1 and Q_2, there is hamming distance $v(Q_1, Q_2)$ is given as follows

$v(Q_1, Q_2) = \frac{1}{2}\sum_{i=1}^{k} |\mu_{Q_1}(p_i) - \mu_{Q_2}(p_i)|;$
(6)

where $p_i \in P$.

Note: The sharpened version $\tilde{Q}$ for an Fs Q should meet the next two requirements:

$$\begin{cases} \mu_{\tilde{Q}}(p_i) \leq \mu_Q(p_i) & if \quad \mu_Q(p_i) \leq \dfrac{1}{2}, \\ \mu_{\tilde{Q}}(p_i) \geq \mu_Q(p_i) & if \quad \mu_Q(p_i) \geq \dfrac{1}{2}. \end{cases}$$

3 Existing Entropy Information Measures

Assume

$$\vartheta_k = \left\{ F = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_k) | \sum_{i=1}^{k} \alpha_i = 1; 0 \le \alpha_i \le 1 \forall i = 1,2, \dots k \right\},$$

act as entire set that is $k \ge 2$ of probability distributions. According to Shannon [4], the entropy measure is

$$H(F) = -\sum_{i=1}^{k} (\alpha_i) \log_T(\alpha_i); \tag{7}$$

for some F ∈ ϑ_k. Renyi et al. generalize the Shannon entropy [4]. Boekee and Vander Lubbe [13]; Havrda and Charvat [14]; Tsallis [15]; etc. The Shannon entropy was generalized by Boekee and Vander Lubbe [13] in terms of R-norm entropy.

$$H_R(F) = \frac{R}{R-1}\left[1 - \left(\sum_{i=1}^{k} \alpha_i^R\right)^{\frac{1}{R}}\right]; R \in (0, \infty) - \{1\} \tag{8}$$

Also, $\lim_{R\to 1} H_R(F) = H(F)$. Additionally, the R-norm information measure was examined by Joshi and Kumar [16]; Kumar [17]; and Kumar et al. [18]. Zadeh [1] provided a metric for determining an Fs's fuzziness. Numerous researchers generalized certain new fuzzy information measures after Zadeh. As an illustration, consider Joshi and Kumar [16]; Hooda [19]; Luca and Termini [3]; etc. Hooda [19] investigated the R-norm entropy in the following fuzzy setting:

$$H_R(F) = \frac{R}{R-1}\sum_{i=1}^{k} \left[1 - ((\mu_T(s_i))^R + (1-\mu_T(s_i))^R)^{\frac{1}{R}}\right]; R \in (0, \infty) - \{1\}. \tag{9}$$

The ideas of FKM and FIM enhance one another. The FIM measures Fs's fuzziness, while the FKM assesses Fs's knowledge.

3.1 Novel fuzzy Non-Probabilistic Knowledge Measure

Using Hooda's fuzzy R-norm entropy idea [20], we presented a Novel KM in this part that is defined as

$$Y^B(Q) = \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y} \sum_{i=1}^{k} \left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right)} - 1\right]; \tag{10}$$

for some Q ∈ Fs(P). We now verify the suggested FKM's authenticity Y^B.

Theorem 1 Let P(≠ ϕ) be a finite set and $Q = \{\langle p_i, \mu_Q(p_i)\rangle : p_i \in P\}$ be the element of Fs(P). Consider a mapping Y^B: Fs(P) → [0,1] defined by Eq. (10). If Y^B meets the requirements listed below, then it is a legitimate fuzzy knowledge measure (Y1)-(Y4):

Y1. Y^B(Q) is maximum ⇔ $\mu_Q(p_i) \in \{0,1\}$ ∀ $p_i \in P$, i.e., Q can be any unambiguous set. **(Maximality)**

Y2. Y^B(Q)=0 ⇔ $\mu_Q(p_i)$ =0.5 ∀ $p_i \in P$, i.e., the most fuzzy is set Q . **(Minimality)**

Y3. Sharpened version of $\tilde{Q}$ there is fuzzy set Q, Y^B(Q) ≤ $Y^B(\tilde{Q})$. **(Resolution)**

Y4. If Q^j is the complement for the fuzzy set Q, there is $Y^B(Q) = Y^B(Y^j)$. **(Symmetry)**

Proof. *(Y1)*. Let Q is any unambiguous fuzzy sets, i.e., Either 0 or 1 represents the membership function μ_Q. Afterward, Eq. (10) turns into

$$Y^B(Q) = \frac{y(\sqrt[3]{8}-1)^{-1}(\sqrt[3]{8}-1)}{y} = 1.$$

However, contrary to this, suppose that Y^B(Q)=1. Eq. (10) thus suggests

$$\frac{(\sqrt[3]{8}-1)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4+\left(1-\mu_Q(p_i)\right)^4\right)}-1\right]=1.$$
(11)

After we compute Eq. (11), we get

$$\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4+\left(1-\mu_Q(p_i)\right)^4\right)}=\sqrt[3]{8}, \quad \forall p_i \in P;$$
(12)

which gives

$$\left(\left(\mu_Q(p_i)\right)^4+(1-\mu_T(s_i))^4\right)=1, \quad \forall p_i \in P;$$
(13)

whenever you use this function membership μ_Q is either 0 or 1 is this true. Thus, this supposition $Y1$ is proved.

(Y2). Suppose Q be the most fuzzy set, i.e., $\mu_Q(p_i)$=$\frac{1}{2}$ ∀ $p_i \in P$.

Put $\mu_Q(p_i)$=$\frac{1}{2}$ in Eq. (10), we get Y^B(Q)=0.

In the opposite case, let's suppose that Y^B(Q)=0. Then Eq. (10) suggests

$$\frac{(\sqrt[3]{8}-1)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4+\left(1-\mu_Q(p_i)\right)^4\right)}-1\right]=0.$$
(14)

It gives

$$\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4+\left(1-\mu_Q(p_i)\right)^4\right)}=1, \quad \forall p_i \in P;$$

which is possible only if $\mu_Q(p_i)$=0.5 ∀ $p_i \in P$. Thus, axiom *Y2* is proved.

(Y3). Initially using Eq. (10), we show that Y^B(Q), declines if the interval $[0,\frac{1}{2})$ contains the membership function and inclines when it is exist in the interval $(\frac{1}{2},1]$ in order to prove axiom $Y3$.

So, define a function by

$$I(\mu_Q(p_i)) = \sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right)}.$$

(15)

Differentiate Eq. (15) w.r.t. $\mu_Q(p_i)$, we get

$$\frac{dG(\mu_Q(p_i))}{d\mu_Q(p_i)} = \frac{\sqrt[3]{8}\left(\mu_Q(p_i)-\frac{1}{2}\right)}{3\sqrt[3]{\left(\left(\mu_Q(p_i)\right)^4+\left(1-\mu_Q(p_i)\right)^4\right)^3}}.$$

(16)

The denominator in Eq. (16) remains positive, and the numerator is dependent on the term's sign $\left(3\mu_Q(p_i)-1\right)$.

Now,

$$\left(3\mu_Q(p_i)-1\right) = .\begin{cases} Positive & if \quad \mu_Q(p_i) \in \left(\frac{1}{2},1\right], \\ Negative & if \quad \mu_Q(p_i) \in \left[0,\frac{1}{2}\right). \end{cases}$$

this indicates that G increase at (0.5,1] and at this [0,0.5) it is decreased.

Let's now assume a Fs Q and $\tilde{Q}$ are there sharpened version. $G(\mu_Q(p_i)) \geq$ G($\mu_{\tilde{Q}}(p_i)$) for the interval (0.5,1) since G is an increasing function of $\mu_Q(p_i)$. The result is $Y^B(\tilde{Q})$≥ Y^B(Q). $G(\mu_Q(p_i)) \leq$ G($\mu_{\tilde{Q}}(p_i)$) likewise holds if G states that it is a function that is decreasing $\mu_Q(p_i)$ in this interval [0,0.5]. The result is $Y^B(\tilde{Q})$≥ Y^B(Q). The axiom *(Y3)* is thus proved.

(Y4).By substituting Q^j for Q in Eq. (10) to demonstrate axiom Y4, we have

$$\begin{aligned} Y^B(Q^j) &= \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_{Q^j}(p_i)\right)^4+\left(1-\mu_{Q^j}(p_i)\right)^4\right)}-1\right], \\ &= \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(1-\mu_Q(p_i)\right)^4+\left(\mu_Q(p_i)\right)^4\right)}-1\right], \\ &= Y^B(Q). \end{aligned}$$

(17)

This proves axiom *(Y4)*.

Therefore, Y^B(Q) is a legitimate FKM.

3.2 Properties

The properties of the proposed KM Y^B(Q) are examined in this section.

Theorem 2 The idea for the knowledge measure Y^{B}(Q) satisfies the properties listed below:

- Y^B(Q)=$Y^B(Q^j)$.
- For a fuzzy set Q, Y^B(Q) ∈[0,1].

- $Y^B(Q_1 \cup Q_2)$+$Y^B(Q_1 \cap Q_2)$=$Y^B(Q_1)$+$Y^B(Q_2)$ regarding any two independent fuzzy sets Q_1, Q_2.

Proof. *(1).* From the axiom *(Y4)*, the proof is clear.

(2). Given that $\left(1-\mu_Q(p_i)\right)$ resides in [0,1], and that $\mu_Q(p_i)$ is known for each member p_i in P so, $0 \le \left(\mu_Q(p_i)\right)^4 \le 1$ and $0 \le \left(1-\mu_Q(p_i)\right)^4 \le 1$ $\forall p_i \in P$.

$$\Rightarrow 0 \le \left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right) \le 1,$$

$$\Rightarrow 0 \le 8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right) \le 8,$$

$$\Rightarrow 0 \le \sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right)} \le \sqrt[3]{8},$$

$$\Rightarrow 0 \le \sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right)} - 1 \le \sqrt[3]{8} - 1,$$

$$\Rightarrow 0 \le \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^3 + \left(1-\mu_Q(p_i)\right)^4\right)} - 1\right] \le 1,$$

$$\Rightarrow 0 \le Y^B(Q) \le 1.$$

(3). Let $Q_1, Q_2 \in$ Fs(P). Divide P into two parts as follows:

$$\begin{aligned} P_1 &= \left\{p_i \in P \mid \mu_{Q_1}(p_i) \ge \mu_{Q_2}(p_i)\right\}, \\ P_2 &= \left\{p_i \in P \mid \mu_{Q_1}(p_i) < \mu_{Q_2}(p_i)\right\}; \end{aligned}$$
(18)

For the Fs Q_1 and Q_2, the membership functions are $\mu_{Q_1}(p_i)$ and $\mu_{Q_2}(p_i)$, respectively."

"Let's now assume that $p_i \in P_1$.

$$\begin{aligned} \mu_{Q_1 \cup Q_2}(p_i) &= max\left\{\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right\} = \mu_{Q_1}(p_i), \\ \mu_{Q_1 \cap Q_2}(p_i) &= min\left\{\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right\} = \mu_{Q_2}(p_i); \end{aligned}$$
(19)

and if $p_i \in P_2$, then

$$\begin{aligned} \mu_{Q_1 \cup Q_2}(p_i) &= max\left\{\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right\} = \mu_{Q_2}(p_i), \\ \mu_{Q_1 \cap Q_2}(p_i) &= min\left\{\mu_{Q_1}(p_i), \mu_{Q_2}(p_i)\right\} = \mu_{Q_1}(p_i). \end{aligned}$$
(20)

Now, $\forall p_i \in P$,

$$Y^B(Q_1 \cup Q_2) + Y^B(Q_1 \cap Q_2)$$
$$= \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_{Q_1 \cup Q_2}(p_i)\right)^4 + \left(1-\mu_{Q_1 \cup Q_2}(p_i)\right)^4\right)} - 1\right]$$

$$+\frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y}\left[\sqrt[3]{8\left(\left(\mu_{Q_1\cap Q_2}(p_i)\right)^4+\left(1-\mu_{Q_1\cap Q_2}(p_i)\right)^4\right)}-1\right].$$

It gives

$$\begin{aligned}Y^B(Q_1\cup Q_2)+Y^B(Q_1\cap Q_2) &= \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum\nolimits_{p_1}\left[\sqrt[3]{8\left(\left(\mu_{Q_1}(p_i)\right)^4+\left(1-\mu_{Q_1}(p_i)\right)^4\right)}-1\right]\\ &+\frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum\nolimits_{p_1}\left[\sqrt[3]{8\left(\left(\mu_{Q_2}(p_i)\right)^4+\left(1-\mu_{Q_2}(p_i)\right)^4\right)}-1\right]\\ &+\frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum\nolimits_{p_2}\left[\sqrt[3]{8\left(\left(\mu_{Q_1}(p_i)\right)^4+\left(1-\mu_{Q_1}(p_i)\right)^4\right)}-1\right]\\ &+\frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum\nolimits_{p_2}\left[\sqrt[3]{8\left(\left(\mu_{Q_2}(p_i)\right)^4+\left(1-\mu_{Q_2}(p_i)\right)^4\right)}-1\right].\end{aligned}$$

(21)

On solving, we have

$$Y^B(Q_1\cup Q_2)+Y^B(Q_1\cap Q_2)=Y^B(Q_1)+Y^B(Q_2).$$

(22)

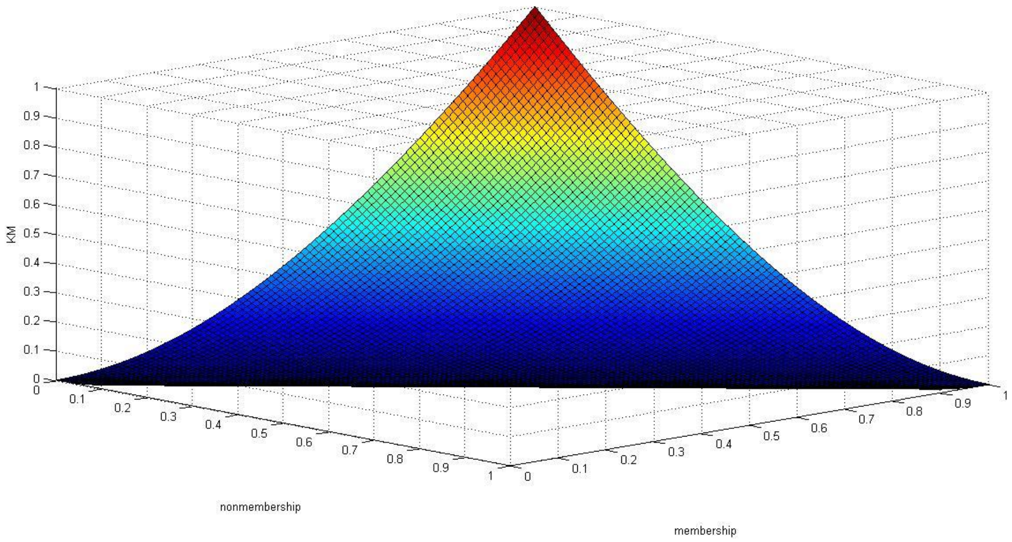

Figure 1: Proposed Knowledge Measure.

3.3 Evaluative Comparison

The recommended FKM is now compared against the existing FKMs and FIMs. The proposed FKM's benefits are examined. We study these benefits in using structured language variables, determining the content of uncertainty in an Fs, and allocating the weights of attribute in MCDM issues. The following are some examples of modern policies that are discussed in the literature:

$$R_Y^r(Q) = 1 - \frac{d_r(Q,Q^j)}{y^{\frac{1}{r}}}; \quad \text{(Yager [6])}, \tag{23}$$

where

$$d_r(Q_1, Q_2) = \left[\sum_{i=1}^{y} |\mu_{Q_1}(p_i) - \mu_{Q_2}(p_i)|^r\right]^{\frac{1}{r}},$$

$$R_Y^p(Q) = \frac{d_r(Q,Q_{near})}{d_r(Q,Q_{far})}; \quad \text{(Kosko [21])},$$

$$\mu_{Q_{near}}(p_i) = \begin{cases} 1 & \text{if } \mu_Q(p_i) \ge \frac{1}{2} \\ 0 & \text{if } \mu_Q(p_i) < \frac{1}{2} \end{cases} \text{and } \mu_{Q_{far}}(p_i) = \begin{cases} 1 & \text{if } \mu_Q(p_i) < \frac{1}{2} \\ 0 & \text{if } \mu_Q(p_i) \ge \frac{1}{2} \end{cases}.$$

(24)

$$R_U(Q) = \frac{1}{y}\sum_{i=1}^{y} \left[\mu_Q(p_i)e^{1-\mu_Q(p_i)} + \left(1 - \mu_Q(p_i)\right)e^{\mu_Q(p_i)}\right]; \quad \text{(Pal and Pal [22])}.$$

(25)

$$Y_P(Q) = \frac{1}{y}\sum_{i=1}^{y} 2\left[\left(\mu_Q(p_i)\right)^2 + \left(1 - \mu_Q(p_i)\right)^2\right] - 1; \quad \text{(Singh et al. [12])}.$$

(26)

$$Y_P^{\beta}(Q) = \frac{1}{y}\sum_{i=1}^{y} 2\left[\left(\mu_Q(p_i)\right)^{\beta} + \left(1 - \mu_Q(p_i)\right)^{\beta}\right] - 1; \quad \beta > 1 \text{(Singh et al[23])}.$$

(27)

$$Y_{OY}(Q) = \log_2\left[\frac{2}{y}\sum_{i=1}^{y} \left(\left(\mu_Q(p_i)\right)^2 + \left(1 - \mu_Q(p_i)\right)^2\right)\right]; \text{(Arya and Kumar[24])}.$$

(28)

$$Y_P^{\beta,\gamma}(Q) = \frac{1}{y}\sum_{i=1}^{y} 2\left[\left(\mu_Q(p_i)\right)^{\beta} + \left(1 - \mu_Q(p_i)\right)^{\beta}\right]^{\frac{\beta-1}{\gamma-1}} - 1; \quad \beta \in (1,2], \gamma \ge \beta,$$

(Singh and Ganie[25]). (29)

$$Y^B(Q) = \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{y}\sum_{i=1}^{y} \left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1 - \mu_Q(p_i)\right)^4\right)} - 1\right]; \quad \text{(proposed one)}.$$

(30)

3.3.1 Computation of Uncertainty in fuzzy sets

There is a distinction in the ambiguity around the two Fs. Conversely, several FIM provide the same vagueness values for several Fs. As a result, a new measure that builds on the ones that have previously been developed must be adopted. The suggested measure’s operation is demonstrated in the following example.

Example 1 Q_i *(for i=1,2,3,4)* $P_i=\{p_1, p_2, p_3, p_4, p_5\}$ *given as*

$$Q_1 = \{(p_1, 0.045), (p_2, 0.132), (p_3, 0.6), (p_4, 0.625), (p_5, 0.643)\};$$
$$Q_2 = \{(p_1, 0.560), (p_2, 0.568), (p_3, 0.047), (p_4, 0.541), (p_5, 0.343)\};$$
$$Q_3 = \{(p_1, 0.641), (p_2, 0.418), (p_3, 0.608), (p_4, 0.05), (p_5, 0.634)\};$$
$$Q_4 = \{(p_1, 0.047), (p_2, 0.679), (p_3, 0.315), (p_4, 0.547), (p_5, 0.793)\}.$$

At the moment, we establish the uncertain matter of these Fss utilizing the already determined metrics as well as the suggested KM. The estimated findings are presented in Table 1. It shows that the ambiguity content for different Fss is average according to some metrics. The suggested KM, however, can distinguish between these Fs with ease. Thus, a fresh approach is always needed.

3.3.2 Evaluating weights of Attribute

The weights of attribute are crucial in an MCDM scenario. Now, we calculate weights of attribute by utilizing these two suggested measure and the previously existing measurements. Think about an example for this.

Table 1: Calculated values for a range of metrics corresponding to different fuzzy sets that were provided in Example 1.

Measures ↓	← fuzzy sets →			
	Q_1	Q_2	Q_3	Q_4
$R_H^1 Y(Q)$	**0.523**	**0.688**	0.634	0.537
$R_K^1 Y^{(}Q)$	**0.354**	**0.524**	0.464	0.367
$R_P V(Q)$	1.450	1.525	**1.516**	**1.465**
$Y_P^{\beta}(Q)$	0.266	0.138	**0.154**	**0.241**
$Y_{OY}(Q)$	0.390	0.253	**0.271**	**0.364**
$Y_P^{\beta,\gamma}(Q)$	0.542	**0.453**	**0.466**	**0.527**
$Y^B(Q)$	0.373	0.224	0.254	0.360

We take β=2.1 for $Y_S P^{\beta}$(Q), and β=2.1, γ=3 for $Y_P^{\beta,\gamma}$(Q).

Example 2 *In a fuzzy environment, take a decision matrix M that has a collection of attributes* $\{A_1, A_2, A_3, A_4\}$ *and a set of alternatives* $\{C_1, C_2, C_3, C_4, C_5\}$.

$$M = \begin{bmatrix} 0.541 & 0.582 & 0.037 & 0.066 \\ 0.482 & 0.658 & 0.579 & 0.142 \\ 0.708 & 0.037 & 0.415 & 0.500 \\ 0.040 & 0.591 & 0.647 & 0.525 \\ 0.534 & 0.434 & 0.593 & 0.543 \end{bmatrix}$$

The attribute weights shown below can be computed in two ways:

- Entropy-based approach: We can ascertain the weights given to different traits by using the formula.

$$w_e = \frac{F(A_e) - 1}{\sum_{e=1}^{n} F(A_e) - n}, e = 1,2,3, \dots, n;$$

Here F shows FIM.

- Knowledge-based approach - We identify the weights that correspond to different attributes by applying formulas.

$$w_e = \frac{Y(A_e)}{\sum_{e=1}^{n} Y(A_e)}, e = 1,2,3, \dots, n;$$

Here Y shows FKM.

These two methods are used to construct the attribute weights in Table 1.

Table 2: The weights of the qualities that correspond to Example 2

Measures ↓	← Weights of Criteria →			
	w_1	w_2	w_3	w_4
$R_H^1 Y(Q)$	0.2614	**0.2465**	0.2455	**0.2465**
$R_K^1 Y(Q)$	0.2674	**0.2447**	0.2432	**0.2447**
$R_P V(Q)$	**0.2513**	0.2512	**0.2513**	0.2462
$Y_P^{\beta}(Q)$	**0.2302**	0.2310	**0.2302**	0.3086
$Y_{OY}(Q)$	**0.2373**	**0.2373**	0.2374	0.2874
$Y_P^{\beta,\gamma}(Q)$	**0.2465**	0.2467	**0.2465**	0.2602
$Y^B(Q)$	0.2334	0.2389	0.2374	0.2888

We take β=1.9 for Y_P^{β}(Q), and β=1.9, γ=2.1 for $Y_P^{\beta,\gamma}$(Q).

It can be shown from Table 1 that there is inconsistency in the weights of the attributes determined with a few of the existing FKMs and FIMs. There are instances where the weights assigned to multiple properties coincide. By the way, the weights that the suggested KM assigns to these traits vary. Thus, it is necessary to implement a measure.

3.3.3 Structured linguistic comparison

Tahani [26] initially created an arrangement for processing fuzzy queries using fuzzy sets. Fuzzy linguistic quantitative terms for database queries are next that were proposed by Kacprzyk and Ziolkowski [27]. Petry's book [28] finally had a fuzzy database containing its concepts and uses. Linguistic hedges, such as MORE, FEW, VERY, SLIGHTLY, and LESS, are used to represent linguistic variables. It shows Linguistic variables by the notion of Fs, and these linguistic hedges represent the operations on an Fs. We looked at these operations in the current scenario and contrasted the effectiveness of the suggested KM with alternative metrics.

Define a Fs $Q = \{\langle p_i, \mu_Q(p_i)\rangle : p_i \in P\}$ on P and according to this Fs "Q" , as Big on P, the definition of its modifier is then

$$Q^n = \{\langle p_i, (\mu_Q(p_i))^n\rangle : p_i \in P\}.$$
(31)

Hung and Yang [29] and Hwang and Yang [30] calculated the concentration and dilatation for a Fs Q.

$$CON(Q) = Q^2, \text{and } DIL(Q) = Q^{0.5}. \tag{32}$$

Regarding variables, dilation and focus are worked. After that terminology are shortened in the sake of clarity : BIG is represented by B, VERY BIG by VB, MORE/LESS BIG by MLB, QUITE VERY BIG by QVB, and VERY VERY BIG by VVB. We may now characterize Fs Q's hedges as follows

$$\begin{cases} MLB & \text{for} Q^{0.5} \\ B & \text{for} Q \\ VB & \text{for} Q^2 \\ QVB & \text{for} Q^3 \\ VVB & \text{for} Q^4 \end{cases} \tag{33}$$

They were used to evaluate various FIM by Liu and Ren [31], Hung and Yang [29], Hwang and Yang [30], and Xia and Xu [32]. For optimal performance, an Fs Q's FIM R(Q) must satisfy the following order

$$R(VVB) < R(QVB) < R(VB) < R(B) < R(MLB);$$
(34)

where R(Q) is defined as FIM of Fs Q.

On the other hand, according to Singh et al. [12], a KM should adhere to the following order:

$$Y(VVB) > Y(QVB) > Y(VB) > Y(B) > Y(MLB);$$
(35)

Here Y^B(Q) is known as FKM of Fs Q.

We now use the example below to assess the efficacy of the proposed KM Y^B(Q):

Example 3 *Suppose P=* $\{p_i; 1 \le i \le 5\}$ *be a countable set and similarly Q ∈ Fs(P) is defined as*

$$Q = \{(\wp_1, 0.3), (\wp_2, 0.5), (\wp_3, 0.7), (\wp_4, 0.8), (\wp_5, 1)\}.$$
(36)

Considering Fs "Q" as Big on P and assuming the linguistic variables in accordance with Eq. (33). It allows us to produce the following Fs Eq. (31).

$$\begin{aligned}
Q^{0.5} &= \{(\wp_1, 0.5477), (\wp_2, 0.7071), (\wp_3, 0.8366), (\wp_4, 0.8944), (\wp_5, 1)\}; \\
Q &= \{(\wp_1, 0.3), (\wp_2, 0.5), (\wp_3, 0.7), (\wp_4, 0.8), (\wp_5, 1)\}; \\
Q^2 &= \{(\wp_1, 0.09), (\wp_2, 0.25), (\wp_3, 0.49), (\wp_4, 0.64), (\wp_5, 1)\}; \\
Q^3 &= \{(\wp_1, 0.027), (\wp_2, 0.125), (\wp_3, 0.343), (\wp_4, 0.512), (\wp_5, 1)\}; \\
Q^4 &= \{(\wp_1, 0.0081), (\wp_2, 0.0625), (\wp_3, 0.2401), (\wp_4, 0.4096), (\wp_5, 1)\}.
\end{aligned}$$

(37)

We now use the suggested KM given in Eq. (10) to analyze the efficacy of the existing measures. Together with the suggested KM, we now compute the values of the various metrics. The computed values are in table 3.

Table 3: Evaluated values that are specified in Eqs. (23) - (30).

of different measures are shown here.

fuzzy set ↓	← Different measures →						
	$R_H^1 Y(Q)$	$R_K^1 Y(Q)$	$R_P V(Q)$	$Y_P^{\beta}(Q)$	$Y_{OY}(Q)$	$Y_P^{\beta,\gamma}(Q)$	$Y^B(Q)$
MLB	0.4056	0.2544	1.3598	0.5372	0.6400	0.5096	0.4182
B	0.5200	0.3513	1.4338	0.4179	0.5585	0.3892	0.4402
VB	0.4760	0.3123	1.3980	0.4856	0.6014	0.4463	0.4543
QVB	0.3932	0.2447	1.3946	0.4997	0.6757	0.5492	0.5546
VVB	0.2881	0.1683	1.2574	0.5820	0.6846	0.6471	0.6533

We take β=1.5 for Y_P^{β}(Q), and β=1.9, γ=5.5 for $Y_P^{\beta,\gamma}$(Q).

From Table 3, we get the following observations:

$$\begin{aligned}
& R_H^1 Y(VVB) < R_H^1 Y(QVB) < R_H^1 Y(VB) < R_H^1 Y(B) > R_H^1 Y(MLB); \\
& R_K^1 Y(VVB) < R_K^1 Y(QVB) < R_K^1 Y(VB) < R_K^1 Y(B) > R_K^1 Y(MLB); \\
& R_P V(VVB) < R_P V(QVB) < R_P V(VB) < R_P V(B) > R_P V(MLB); \\
& Y_P^{\beta}(VVB) > Y_P^{\beta}(QVB) > Y_P^{\beta}(VB) > Y_P^{\beta}(B) < Y_P^{\beta}(MLB); \\
& Y_{OY}(VVB) > Y_{OY}(QVB) > Y_{OY}(VB) > Y_{OY}(B) < Y_{OY}(MLB); \\
& Y_P^{\beta,\gamma}(VVB) > Y_P^{\beta,\gamma}(QVB) > Y_P^{\beta,\gamma}(VB) > Y_P^{\beta,\gamma}(B) < Y_P^{\beta,\gamma}(MLB); \\
& Y^B(VVB) > Y^B(QVB) > Y^B(VB) > Y^B(B) > Y^B(MLB).
\end{aligned}$$

(38)

We then look at another example. This has been noted that neither any FIM adhere to the order provided in Eq. (34), and nor any FKMs satisfy the order provided in Eq. (35). This implies that they are not particularly accurate. Nonetheless, the sequence is satisfied by the suggested measure $Y^{B}(Q)$ provided in Eq. (35).

To do this, we use an additional Fs provided by

$T=\{(s_1,0.1),(s_2,0.5),(s_3,0.6),(s_4,0.7),(s_5,0.9)\}$.
(39)

The measures' calculated values are displayed by Table 3.

Table 4: The values determined for the different metrics listed in Eqs. It is (23) - (30).

fuzzy set ↓	← Different measures →						
	$R_H^1Y(Q)$	$R_K^1Y(Q)$	$R_PV(Q)$	$Y_P^{\beta}(Q)$	$Y_{OY}(Q)$	$Y_P^{\beta,\gamma}(Q)$	$Y^B(Q)$
MLB	0.4197	0.2656	1.4119	0.3333	0.4575	0.5931	0.3525
B	0.5600	0.3889	1.4605	0.2504	0.3740	0.5334	0.6152
VB	0.5200	0.3513	1.4348	0.2914	0.4165	0.5582	0.3985
QVB	0.3824	0.2364	1.3690	0.4019	0.5239	0.6337	0.5220
VVB	0.3104	0.1837	1.3051	0.5070	0.6194	0.7002	0.6113

We take β=1.5 for Y_P^{β}(Q), and β=1.9, γ=3.5 for $Y_P^{\beta,\gamma}$(Q).

The listed observations are obtained from Table 3:

$$\begin{aligned}
&R_H^1Y(VVB) < R_H^1Y(QVB) < R_H^1Y(VB) < R_H^1Y(B)H_Y^1(MLB);\\
&R_K^1Y(VVB) < R_K^1Y(QVB) < R_K^1Y(VB) < R_K^1Y(B) < R_K^1Y(MLB);\\
&R_PV(VVB) < R_PV(QVB) < R_PV(VB) < R_PV(B) > R_PV(MLB);\\
&Y_P^{\beta}(VVB) > Y_P^{\beta}(QVB) > Y_P^{\beta}(VB) > Y_P^{\beta}(B) < Y_P^{\beta}(MLB);\\
&Y_{OY}(VVB) > Y_{OY}(QVB) > Y_{OY}(VB) > Y_{OY}(B) < Y_{OY}(MLB);\\
&Y_P^{\beta,\gamma}(VVB) > Y_P^{\beta,\gamma}(QVB) > Y_P^{\beta,\gamma}(VB) > Y_P^{\beta,\gamma}(B) < Y_P^{\beta,\gamma}(MLB);\\
&Y^B(VVB) > Y^B(QVB) > Y^B(VB) > Y^B(B) > Y^B(MLB).
\end{aligned} \tag{40}$$

With the exception of the suggested one, There are no FKMs that meet the specified order in Eq. (35), and this is now seen that FIMs $H_Y^1(T)$ and $H_K^1(T)$ follow the sequence stated in Eq. (34). Consequently, KM's efficacy is astounding.

3.4 Proposed fuzzy knowledge measure for the enhancement of New Information measure

We introduced a FIM in this part based on the suggested FKM Y^B(Q). Examine the following fuzzy information measure: $(\overline{FIM})$.

$$\begin{aligned}\overline{FIM}(Q) &= 1 - Y^{B}(Q),\\ &= 1 - \frac{\left(\sqrt[3]{8}-1\right)^{-1}}{k}\sum_{i=1}^{k}\left[\sqrt[3]{8\left(\left(\mu_Q(p_i)\right)^4 + \left(1-\mu_Q(p_i)\right)^4\right)} - 1\right].\end{aligned}$$

(41)

Now, it is clear that the FIM provided in Eq. (41) is legitimate FIM.

Theorem 3 *The following characteristics are satisfied by the suggested fuzzy information measure in Eq. (41):*

- $0 \leq \overline{FIMs}(Q) \leq 1$.
- $\overline{FIMs}(Q) = 1$ iff Q defined as the most fuzzy set.
- $\overline{FIMs}(Q) = 0$ iff Q is unambiguous fuzzy set.
- $\overline{FIMs}(Q) = \overline{FIMs}(Q^{j})$.

Here Q^{j} is defined as the complement of any fuzzy set Q in FS(P).

Proof. The FIM described in Eq. (41) clearly satisfies each of these requirements.

4 Application of FKM for Solving MCDM Problems.

The potential uses of the recommended KM within the MCDM scenarios are presented in this section. MCDM is a technique for selecting the best option out of all those available. Numerous real-world problems are described using a variety of criteria. The criteria for this particular design are as follows:

- Collection of alternatives.
- Collection of decision criteria (attributes).
- Weights for criteria and attributes.
- Factors that could influence each alternative's rank of preference.

4.1 The Commenced approach

Take an MCDM problem where the grouping of all the attributes is represented by $BT = \{N_j | 1 \leq j \leq q\}$ and the group of all alternatives is represented by $BL = \{C_i | 1 \leq i \leq p\}$. Let the collection of invited experts be represented by F=$\{F_v | 1 \leq v \leq n\}$. Let's think about $FY = \{FY_1, FY_2, \ldots, FY_q\}$ denotes the attribute weight vectors. $\sum_{j=1}^{q} FY_j = 1$. s.t. A_j Following the receipt of expert questionnaire replies, we may create the fuzzy environment decision matrix as follows:

Allow a total of 'n' experts to be present in order to make a decision. The formula determines the membership degrees v_{ij} if p_{ij} is the number of experts who support a certain option M_i according to criteria N_j.

$$v_{ij} = \frac{p_{ij}}{n}, \forall i = 1,2,3, \ldots, p, j = 1,2,3, \ldots, q.$$

Table 5: Decision matrix that exist in fuzzy environment $D_{p\times q}$.

$D_{p\times q}$	A_1	A_2	$\cdots$	A_q
M_1	v_{11}	v_{12}	$\cdots$	v_{1q}
M_2	v_{21}	v_{22}	$\cdots$	v_{2q}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
M_p	v_{p1}	v_{p2}	$\cdots$	v_{pq}

4.2 GRA approach that lies on suggested fuzzy Knowledge Measure

Since distance measures neglect the correlation between criteria, we use knowledge measures instead of distance measures in the suggested approach. The suggested method is constructed using the subsequent steps:

• Crafting a decisions matrix .The $m \times n$ decision matrix D, with m criteria and n choices, is displayed in table 4. In this case, v_{12} represents the first alternative's second criteria value, while v_{21} represents the second alternative's first criteria score.

• We employ the given approach to normalize the fuzzy decision matrix once it has been created.

$$dn_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{p}(d_{ij})^2}}, \forall i = 1,2,3,\dots,p, \quad j = 1,2,3,\dots,q. \tag{42}$$

After a normalized fuzzy decision matrix is constructed, the amount of knowledge that satisfies each criteria is found using Eq. (10).

• Any MCDM problem must take into account the weights given to the criteria. Any particular problem's results can be changed by adjusting the criteria's weights. There are two methods available in the literature to determine criteria weights:

(a) $$FE_j = \frac{1-E_j}{q-\sum_{j=1}^{q}E_j}, \forall j = 1,2,\dots,q; \tag{43}$$

fuzzy information of the i^{th} alternative is represented by $E_j = \sum_{i=1}^{p} H(M_i, N_j)$ and $H(M_i, N_j)$, which are equivalent to the j^{th} criteria. Given our knowledge of the complementing nature of the concepts of FIM and FKM, we utilize the following approach to determine the criteria weights FK_j.

$$FK_j = \frac{k_{ij}}{\sum_{j=1}^{q}k_{ij}}, \forall j = 1,2,\dots,q; \tag{44}$$

where $K(M_i, N_j)$ and $k_{ij} = \sum_{i=1}^{p} K(M_i, N_j)$ denote the knowledge derived from the i^{th} alternative, which is comparable to the j^{th} criteria.

Deng [33] first proposed grey system theory, a control theory that has had a significant impact on many engineering and management fields. Grey relational analysis (GRA) was created as a result of the theory [34]. GRA is a powerful method that effectively resolves intricate correlations between a variety of performance metrics.As with almost all MCDM systems, The decision matrix, which includes each of the decision criteria, serves as the foundation for resolving the GRA problem. An overview of GRA's approach to problem-solving is given below [34] : Creating a matrix that make comparison . Equation 43 is employed in the comparison matrix to calculate a reference series (for criteria comparison). $X_0(l)$ indicates the optimum value of the lth criterion among the normalized values.

$$x_0 = (x_0(1), x_0(2), \cdots, x_0(l)), l = 1,2, \cdots, n$$
(45)

. This series is produced by taking the best value for every criterion in the decision matrix.

• Establishing a normalized decision matrix and normalizing by definition, decision issues involve criteria with various units and goals. Therefore, in order to solve the choice difficulties, a normalizing method is used. The GRA approach allows for normalization in three different scenarios.

- The Greater and Compared Case: Equation 46 is used to achieve normalization if the criterion utilized is the most acceptable for the intended usage .

$$x_i^* = \frac{x(l) - minx_i(l)}{maxx_i(l) - minx_i(l)}$$
(46)

- .The Problems Who Are Smaller and Better: Equation is used to accomplish normalization if the minimal appropriateness requirement for the purpose is applied .

$$x_i^* = \frac{maxx_i(l) - x_i(l)}{maxx_i(l) - minx_i(l)}$$
(47)

where $x_i(l)$ is x_i^*, the normalised criterion value, and is the numerical representation of the lth parameter in the initial choice of matrix.

- .The better the situation, the closer it is to the desired value: Normalization is carried out utilizing Equation if the criterion is the most acceptable (most suited) for the goal .

$$x_i^* = \frac{|x_i(l) - x_{0b}(l)|}{maxx_i(l) - x_{0b}(l)}$$
(48)

where lth denotes the criterion's target value and $x_{0b}(l)$ denotes the determined optimal value. The range $\min_l x_i(l) \leq x_{0b}(l) \leq \max_l x_i(l)$ is where this ideal value can fall.

- In order to calculate a matrix of an absolute value The values that are normalized of the decision matrix, are reduced from the reference series which have normalized values (Equation 49) to generate a matrix of absolute values (Equation (50)).

$$\delta_{0i} = x_0^*(l) - x_i^*(l)$$
(49)

$$\delta_{0i} = \begin{bmatrix} \delta_{01}(1) & \delta_{01}(2) & \cdots & \delta_{01}(n) \\ \delta_{02}(1) & \delta_{02}(2) & \cdots & \delta_{02}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{0m}(1) & \delta_{0m}(2) & \cdots & \delta_{0m}(n) \end{bmatrix}$$
(50)

where δ_{0i} symbolises the absolute value matrix's values.

- Making a matrix of grey relational coefficients

The grey relational coefficient matrix is created using equation (51), and the values of $\delta_{\max}$ and $\delta_{\min}$ are determined using equations (52) and (53), respectively.

$$\gamma_{0i}(j) = \frac{\delta_{\min} + \zeta \cdot \delta_{\max}}{\delta_{0i}(l) + \zeta \cdot \delta_{\max}}$$
(51)

$$\delta_{\max} = \max_i \max_l \delta_{0i}(l)$$
(52)

$$\delta_{\min} = \min_i \min_l \delta_{0i}(l)$$
(53)

And the values of the grey relational coefficient matrix are represented by the symbol $\gamma_{0i}(l)$. In Equation (49), ζ, also known as the "discriminant coefficient" or "contrast control coefficient," has a value between 0 and 1. In this investigation, $\zeta = 0.5$ was used for pertinent studies in order to be consistent with the literature.

- Making a computation to find grey relational degrees

The degree of geometric resemblance between the x_i^* series in a grey system and the reference series x_0^* is known as the grey relational degree, which makes it possible to compare the series. A significant correlation between the comparison and reference series is indicated by a big gray relational degree. The gray correlation degree is one if the two series under comparison are identical.The weight status of the criteria affects how different gray relationship degrees are calculated. Equation (54) is used to determine the grey relational degrees when all criteria weights are similar, while Equation (55) is used when criteria weights differ.

$$\Gamma_{0i} = \frac{1}{n}\sum_{l=1}^{n} \gamma_{0i}(l)$$
(54)

$$\Gamma_{0i} = \sum_{l=1}^{n} \left(w_i(l) \cdot \gamma_{0i}(l)\right)$$
(55)

where the weight of the lth criterion is $w_i(l)$, and the grey relational degrees are denoted by Γ_{0i}. ($\sum_{l=1}^{n} w_l = 1$) is the total of the criterion weights.

5 Case Study

Sixteen trials with various combinations of $\boldsymbol{CO_2}$ and $\boldsymbol{NH_3}$ gas concentrations were carried out for the study. To track algal growth and $\boldsymbol{CO_2}$ and $\boldsymbol{NH_3}$ mitigation efficiency, the following parameters were calculated at the conclusion of Every trial: cell number, dry weight, cell weight, growth rate, and $\boldsymbol{CO_2}$ and $\boldsymbol{NH_3}$ fixation and removal rates. These variables were used as standards for the experiments' MCDM analysis. The GRA technique was then used to determine the weights of each criterion. The weight of each chosen criterion in the GRA approach is displayed in Figure 1. The weight figures that were produced were then fed into GRA. As previously noted, three scenarios/output goals were compared in the analyses, and each scenario included a number of performance indicators. The indicator data from sixteen batches of algal cultivation trials are compiled in Table **6** have been taken from [35].

Table 6: Decision matrix in fuzzy environment $D_{p\times q}$.

Criteria	CO_2 and NH_3 combinations	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$	$\Re_8$
EXP1	0 ppm NH3-350 ppm CO2	0.54	0.60	0.31	3.39	0	0	0	0
EXP2	12 ppm NH3-350 ppm CO2	1.15	1.12	0.44	0.96	99.5	22.3	5.48	76.6
EXP3	25 ppm NH3-350 ppm CO2	1.27	1.20	0.6	2.08	81.8	18.3	4.5	36.1

EXP4	50 ppm NH3-350 ppm CO2	1.23	1.24	0.45	2.26	71.0	15.9	3.9	15.6
EXP5	0 ppm NH3-1200 ppm CO2	0.69	0.72	0.05	1.28	0	0	0	0
EXP6	12 ppm NH3-1200 ppm CO2	1.78	1.80	0.35	1.01	193.0	12.6	10.6	94.4
EXP7	25 ppm NH3-1200 ppm CO2	1.95	2.00	0.62	0.53	364.8	23.8	20.1	85.3
EXP8	50 ppm NH3-1200 ppm CO2	1.41	1.45	0.26	1.2	128.7	8.4	7.09	28.4
EXP9	0 ppm NH3-2350 ppm CO2	0.85	0.88	0.28	1.73	6.30	0.41	0.34	0
EXP10	12 ppm NH3-2350 ppm CO2	1.76	1.77	0.34	1.06	163.8	5.46	9.02	80.0
EXP11	25 ppm NH3-2350 ppm CO2	2.04	2.16	0.43	0.58	432.2	14.4	23.8	99.8
EXP12	50 ppm NH3-2350 ppm CO2	1.32	1.33	0.35	0.73	252.9	8.43	13.9	55.7
EXP13	0 ppm NH3-3500 ppm CO2	1.17	1.19	0.31	1.26	54.4	1.21	2.99	0
EXP14	12 ppm NH3-3500 ppm CO2	2.21	2.22	0.49	0.75	267.3	5.98	14.77	97.2
EXP15	25 ppm NH3-3500 ppm CO2	1.56	1.53	0.83	1.58	105.8	2.36	5.83	46.7

EXP16	50 ppm NH_3-3500 ppm CO_2	1.21	1.23	0.51	1.23	105.6	2.36	5.81	23.3

step2 We Normalized the decision matrix given in table 7 by using equation (42).

Table 7: Normalized Decision matrix

Experiments/Criteria	$\Re_1$	$\Re_2$	$\Re_3$	$\Re_4$	$\Re_5$	$\Re_6$	$\Re_7$	$\Re_8$
$\Im_1$	0	0	0.3333	1.0000	0	0	0	0
$\Im_2$	0.3653	0.3210	0.5000	0.1503	0.2302	0.9370	0.2303	0.7675
$\Im_3$	0.4371	0.3704	0.7051	0.5420	0.1893	0.7689	0.1891	0.3617
$\Im_4$	0.4132	0.3951	0.5128	0.6049	0.1643	0.6681	0.1639	0.1563
$\Im_5$	0.0898	0.0741	0	0.2622	0	0	0	0
$\Im_6$	0.7425	0.7407	0.3846	0.1678	0.4466	0.5294	0.4454	0.9459
$\Im_7$	0.8443	0.8642	0.7308	0	0.8441	1.0000	0.8445	0.8547
$\Im_8$	0.5210	0.5247	0.2692	0.2343	0.2978	0.3529	0.2979	0.2846
$\Im_9$	0.1856	0.1728	0.2949	0.4196	0.0146	0.0172	0.0143	0
$\Im_{10}$	0.7305	0.7222	0.3718	0.1853	0.3790	0.2294	0.3790	0.8016
$\Im_{11}$	0.8982	0.9630	0.4872	0.0175	1.0000	0.6050	1.0000	1.0000
$\Im_{12}$	0.4671	0.4506	0.3846	0.0699	0.5851	0.3542	0.5840	0.5581
$\Im_{13}$	0.3772	0.3642	0.3333	0.2552	0.1259	0.0508	0.1256	0
$\Im_{14}$	1.0000	1.0000	0.5641	0.0769	0.6185	0.2513	0.6176	0.9739
$\Im_{15}$	0.6108	0.5741	1.0000	0.3671	0.2448	0.0992	0.2450	0.4679
$\Im_{16}$	0.4012	0.3889	0.5897	0.2448	0.2443	0.0992	0.2441	0.2335

Step 3 We compute the criteria weights. Let us consider criteria weights are incompletely known or unknown, then by using Eq. (44), we have

$$Wt = \{0.0243, 0.0505, 0.0671, 0.1007, 0.1343, 0.1716, 0.2053, 0.2462\}. \quad (56)$$

Step 4 We find absolute coefficient matrix given in table (8) using equation (49)

Table 8: Absolute coefficient matrix .

Experiments/Criteria	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$	$\mathfrak{R}_5$	$\mathfrak{R}_6$	$\mathfrak{R}_7$	$\mathfrak{R}_8$
$\mathfrak{I}_1$	1.0000	1.0000	0.6667	0	1.0000	1.0000	1.0000	1.0000
$\mathfrak{I}_2$	0.6347	0.6790	0.5000	0.8497	0.7698	0.0630	0.7697	0.2325
$\mathfrak{I}_3$	0.5629	0.6296	0.2949	0.4580	0.8107	0.2311	0.8109	0.6383
$\mathfrak{I}_4$	0.5868	0.6049	0.4872	0.3951	0.8357	0.3319	0.8361	0.8437
$\mathfrak{I}_5$	0.9102	0.9259	1.0000	0.7378	1.0000	1.0000	1.0000	1.0000
$\mathfrak{I}_6$	0.2575	0.2593	0.6154	0.8322	0.5534	0.4706	0.5546	0.0541
$\mathfrak{I}_7$	0.1557	0.1358	0.2692	1.0000	0.1559	0	0.1555	0.1453
$\mathfrak{I}_8$	0.4790	0.4753	0.7308	0.7657	0.7022	0.6471	0.7021	0.7154
$\mathfrak{I}_9$	0.8144	0.8272	0.7051	0.5804	0.9854	0.9828	0.9857	1.0000
$\mathfrak{I}_{10}$	0.2695	0.2778	0.6282	0.8147	0.6210	0.7706	0.6210	0.1984
$\mathfrak{I}_{11}$	0.1018	0.0370	0.5128	0.9825	0	0.3950	0	0
$\mathfrak{I}_{12}$	0.5329	0.5494	0.6154	0.9301	0.4149	0.6458	0.4160	0.4419
$\mathfrak{I}_{13}$	0.6228	0.6358	0.6667	0.7448	0.8741	0.9492	0.8744	1.0000
$\mathfrak{I}_{14}$	0	0	0.4359	0.9231	0.3815	0.7487	0.3824	0.0261
$\mathfrak{I}_{15}$	0.3892	0.4259	0	0.6329	0.7552	0.9008	0.7550	0.5321
$\mathfrak{I}_{16}$	0.5988	0.6111	0.4103	0.7552	0.7557	0.9008	0.7559	0.7665

step 5 Calculating Grey relation coefficient matrix using equation (51) given in table 9,

step 6 calculating Grey relation degrees using equation (55) given in table 9.

Table 9: Grey Relation coefficient matrix.

Experiments/Criteria	$\mathfrak{R}_1$	$\mathfrak{R}_2$	$\mathfrak{R}_3$	$\mathfrak{R}_4$	$\mathfrak{R}_5$	$\mathfrak{R}_6$	$\mathfrak{R}_7$	$\mathfrak{R}_8$	Γ_{0i}	Rank
$\mathfrak{I}_1$	0.3333	0.3333	0.4286	1.0000	0.3333	0.3333	0.3333	0.3333	0.4068	12
$\mathfrak{I}_2$	0.4406	0.4241	0.5000	0.3705	0.3938	0.8881	0.3938	0.6826	0.5572	5
$\mathfrak{I}_3$	0.4704	0.4426	0.6290	0.5219	0.3815	0.6839	0.3814	0.4393	0.4836	8
$\mathfrak{I}_4$	0.4601	0.4525	0.5065	0.5586	0.3743	0.6010	0.3742	0.3721	0.4461	10
$\mathfrak{I}_5$	0.3546	0.3506	0.3333	0.4040	0.3333	0.3333	0.3333	0.3333	0.3418	16

$\mathfrak{I}_6$	0.6601	0.6585	0.4483	0.3753	0.4746	0.5152	0.4741	0.9024	0.5888	4
$\mathfrak{I}_7$	0.7626	0.7864	0.6500	0.3333	0.7623	1.0000	0.7628	0.7748	0.7568	2
$\mathfrak{I}_8$	0.5107	0.5127	0.4063	0.3950	0.4159	0.4359	0.4159	0.4114	0.4227	11
$\mathfrak{I}_9$	0.3804	0.3767	0.4149	0.4628	0.3366	0.3372	0.3365	0.3333	0.3569	15
$\mathfrak{I}_{10}$	0.6498	0.6429	0.4432	0.3803	0.4460	0.3935	0.4460	0.7159	0.5115	6
$\mathfrak{I}_{11}$	0.8308	0.9310	0.4937	0.3373	1.0000	0.5587	1.0000	1.0000	0.8160	1
$\mathfrak{I}_{12}$	0.4841	0.4765	0.4483	0.3496	0.5465	0.4364	0.5459	0.5309	0.4922	7
$\mathfrak{I}_{13}$	0.4453	0.4402	0.4286	0.4017	0.3639	0.3450	0.3638	0.3333	0.3671	14
$\mathfrak{I}_{14}$	1.0000	1.0000	0.5342	0.3514	0.5672	0.4004	0.5667	0.9505	0.6412	3
$\mathfrak{I}_{15}$	0.5623	0.5400	1.0000	0.4414	0.3983	0.3569	0.3984	0.4845	0.4683	9
$\mathfrak{I}_{16}$	0.4550	0.4500	0.5493	0.3983	0.3982	0.3569	0.3981	0.3948	0.4044	13

6 Results and Discussion

Reducing the atmospheric pollutants that livestock farms send into the atmosphere is the primary goal and the most efficient way to lower NH_3 and CO_2 gasses in the barn environment is through micro-algae. On the other hand, the reduction amounts of ammonia and carbon dioxide gases do not alter correspondingly in all studies, based on the findings of 16 distinct $NH_3 - CO_2$ concentrations. For instance, for changing gas concentrations, the CO_2 and NH_3 removal efficiencies change independently chlorella sp. was found to have higher cell concentrations with elevated CO_2 concentrations by Ryu et al. [36], but Reduced CO_2 fixation efficiency was the outcome of higher CO_2 concentrations. The experiments with the highest values of environmental parameters were shown to be independent of each other through statistical analysis of the study's data. But a more efficient method of lowering the gases produced by barns would be to find the optimal condition while concurrently considering every environmental factor. The air pollutants (NH_3 and CO_2) emitted from animal feeding operations have an impact on the environment, the health of the animals and workers, and the quality of the air in the area. These air pollution emissions are currently governed by international agreements and national laws that aim to reduce air pollution emissions in intense farming of livestock [37]. The primary goal of any environmental regulation is to lower the concentrations of air pollutants to levels that are safe for both human health and the environment protection organization or oversight organization in industrialized or underdeveloped nations[38]. In bio-mitigation, micro-algae can be employed to extract these air pollutants and create useful products. Based on the findings, EXP11 was determined to be the best experiment. The second and third-ranked entries were EXP7 and EXP14, respectively. It was discovered that experiments EXP13, EXP9, and EXP5 performed the worst. The 16 experiments are ranked in Table 11.

6.1 Sensitive Analysis

Sensitive analysis is a process used to gauge the power and consistency of decision issue outcomes. Sensitive analysis was used in this investigation to assess the power and consistency of the GRA results. There was a comparison between the outcomes of GRA and the other MCDM techniques. The following techniques were applied for the sensitivity analysis: Additive Ratio Assessment (ARAS), Multi-Objective Optimization on the basis of ratio analysis (MOORA), Technique For Order Of Preference By Similarity To Ideal Solution (TOPSIS) , Weighted Aggregated Sum Product Assessment (WASPAS), Deep Mixing Method (DMM), and Grey Relational Analysis (GRA). The weights determined using the suggested approach were applied to all methods. The association between the above-mentioned methodologies and the GRA method results was examined using Spearman's rho rank correlation. The outcomes are displayed with rho rank correlation of Spearman. The table shows the following results 12.

Table 10: Comparision Table.

Experiments / Method	WASPAS	ARAS	MOORA	TOPSIS(AM)	TOPIS(DM)	DMM	GRA(CLIOS)	GRA
$\mathfrak{I}_1$	0.0656	0.1875	0.1230	0.5158	0.2074	0.1231	0.567	0.4068
$\mathfrak{I}_2$	0.4972	0.6246	0.5017	0.5169	0.5382	0.5017	0.434	0.5572
$\mathfrak{I}_3$	0.4028	0.5342	0.4164	0.5166	0.4085	0.4165	0.529	0.4836
$\mathfrak{I}_4$	0.3108	0.4427	0.3341	0.5164	0.3263	0.3342	0.503	0.4461
$\mathfrak{I}_5$	0.0300	0.0787	0.0323	0.5156	0.0622	0.0323	0.363	0.3418
$\mathfrak{I}_6$	0.5817	0.6958	0.5733	0.5170	0.6005	0.5733	0.511	0.5888
$\mathfrak{I}_7$	0.7797	0.9893	0.7820	0.5174	0.7515	0.7820	0.600	0.7568
$\mathfrak{I}_8$	0.3306	0.4126	0.3126	0.5163	0.3040	0.3126	0.445	0.4227
$\mathfrak{I}_9$	0.0514	0.1360	0.0831	0.5157	0.1084	0.0831	0.415	0.3569

$\mathfrak{I}_{10}$	0.4651	0.5613	0.4632	0.5167	0.4998	0.4633	0.506	0.5115
$\mathfrak{I}_{11}$	0.7868	1.0000	0.7946	0.5173	0.7468	0.7945	0.592	0.8160
$\mathfrak{I}_{12}$	0.4856	0.6069	0.4636	0.5168	0.4960	0.4636	0.431	0.4922
$\mathfrak{I}_{13}$	0.0734	0.1917	0.1270	0.5160	0.1194	0.1270	0.426	0.3671
$\mathfrak{I}_{14}$	0.5999	0.7416	0.6131	0.5172	0.6124	0.6132	0.665	0.6412
$\mathfrak{I}_{15}$	0.3445	0.4497	0.3633	0.5165	0.3544	0.3633	0.648	0.4683
$\mathfrak{I}_{16}$	0.2568	0.3372	0.2510	0.5162	0.2373	0.2510	0.464	0.4044

Table 11: Ranking by different methods.

Experiments/ Method	WAPAS	ARAS	MOORA	TOPSIS(AM)	TOPIS(DM)	DMM	GRA(CLIOS)	GRA
$\mathfrak{I}_{1}$	14	14	14	14	13	14	13	12
$\mathfrak{I}_{2}$	5	5	5	5	5	5	8	5
$\mathfrak{I}_{3}$	8	8	8	8	8	8	7	8
$\mathfrak{I}_{4}$	11	10	10	10	10	10	10	10
$\mathfrak{I}_{5}$	16	16	16	16	16	16	16	16
$\mathfrak{I}_{6}$	4	4	4	4	4	4	4	4
$\mathfrak{I}_{7}$	2	2	2	1	1	2	3	2
$\mathfrak{I}_{8}$	10	11	11	11	11	11	11	11
$\mathfrak{I}_{9}$	15	15	15	15	15	15	15	15
$\mathfrak{I}_{10}$	7	7	7	7	6	7	5	6
$\mathfrak{I}_{11}$	1	1	1	2	2	1	1	1
$\mathfrak{I}_{12}$	6	6	6	6	7	6	9	7
$\mathfrak{I}_{13}$	13	13	13	13	14	13	14	14
$\mathfrak{I}_{14}$	3	3	3	3	3	3	2	3

$\Im_{15}$	9	9	9	9	9	9	6	9
$\Im_{16}$	12	12	12	12	12	12	12	13

Table 12: Spearman’s Rho correlation values between methods.

Method	WAPAS	ARAS	MOORA	TOPSIS(AM)	TOPIS(DM)	DMM	GRA(CLIOS)
GRA	0.9853	0.9882	0.9882	0.9853	0.9941	0.9882	0.9588

There are strong relationships between the rankings produced by the GRA method and the other MCDM approaches when Spearman’s Rho values are analyzed. These findings demonstrate the excellent measuring power and consistency of GRA.

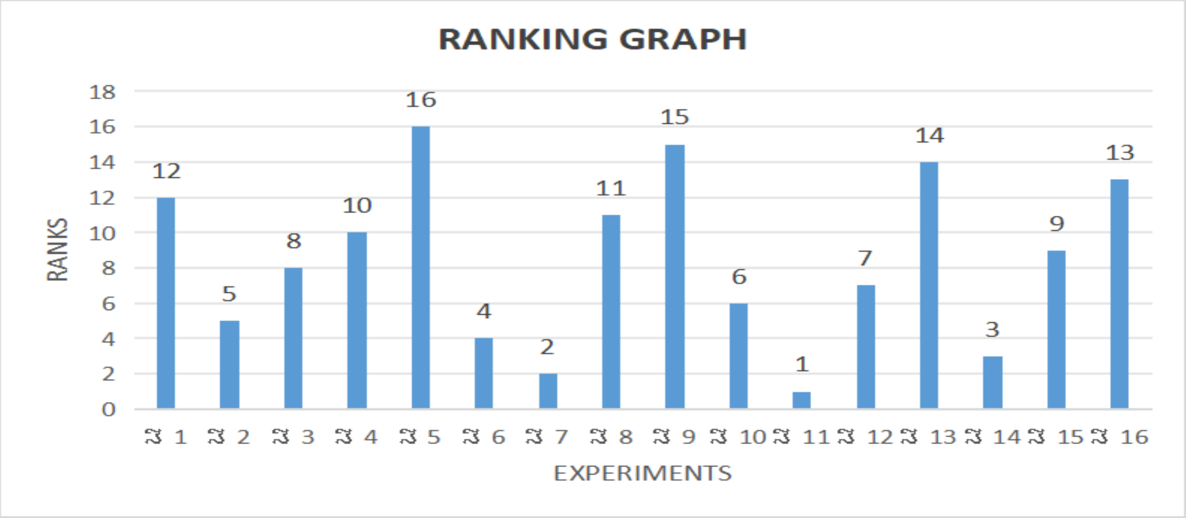

Figure 2: Ranks by GRA method.

7 Conclusions

This study examined and validated a fuzzy Knowledge Measure (FKM), demonstrating its effectiveness through numerical examples. The proposed FKM provides a robust alternative for addressing biological and environmental performance issues by assimilate structured linguistic variables, assessing ambiguity between two distinct fuzzy sets (Fss), and calculating objective weights. Comparative analysis with various well-known fuzzy Information Measures (FIMs) confirmed the effectiveness of the proposed FKM. To overcome the limitations of the traditional approaches, this study introduces a novel solution to Multi-Criteria Decision-Making (MCDM) problems by combining the proposed FKM with a fuzzy Aggregation Model (FAM) and utilizing the GRA approach for more effective results. The findings are promising during this study. The approach calculates criteria weights using two methods: one for unknown criteria weights and another for partially known weights. Finally the case study demonstrates its application in reducing CO_2 and NH_3 levels within an animal barn. The proposed approach has significant potential for identifying the best alternatives that satisfy nearly all benefit criteria and guiding experts on which criteria may hinder the effectiveness of specific alternatives. Additionally, it provides insights into why certain alternatives are preferred in decision-making contexts. The method's flexibility allows for application across various fuzzy scenarios without requiring complex computations. Future research could extend such tool which simplify the air pollutant mitigation process by using fuzzy set concepts to include Intuitionistic fuzzy Sets (IFS), Hesitant fuzzy Sets (HFS), Interval-Valued Intuitionistic fuzzy Sets (IvIFS), Picture fuzzy Sets (PFS), and Neutrosophic fuzzy Sets (NFS). This would broaden the potential applications of the proposed measures to include areas like feature detection, voice recognition, and image threshold which make a benchmark in the future study.

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