Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)

A Study on Performance Measure of FM/FD/1 Queueing Problem using LR-type Trapezoidal Fuzzy Numbers by LR Method

S. Vijaya[1], Dr. N. Srinivasan[2], Dr. M. V. Suresh[3], Dr. G. Balaji[4], D. Jayasri[5]

[1]Research Scholar, Mathematics Department, St. Peter's Institute of Higher Education and Research, Avadi, Chennai – 54.
[2]Professor, Mathematics Department, St. Peter's Institute of Higher Education and Research, Avadi, Chennai – 54.Chennai - 600 123.
[3]Assistant Professor, Department of Mathematics, Panimalar Engineering College, Poonamalle,
[4]Professor, Mathematics Department, Thangavelu Engineering College, Chennai - 97
[5]Research Scholar, Mathematics Department, St. Peter's Institute of Higher Education and Research, Avadi, Chennai – 54.
18svijaya@gmail.com[1], nsri24455@gmail.com[2], dr.m.v.suresh1708[3]

Article History:

***Received:** 15-11-2024*

***Revised:**28-12-2024*

***Accepted:**11-01-2025*

Abstract: Many Queueing models have been solved under fuzzy environment using triangular and trapezoidal numbers of fuzzy type where they are extensively used in real-life applications which are implemented frequently in day-to-day life. LR method has been presented in this paper to examine FM/FD/1 queueing model that directs to system performance measure implementing capacity in limited form and the calling source with unlimited form. The expected inter-arrival interval, service interval follow fuzzy manner having single server with no bounds and are expressed as LR-type trapezoidal fuzzy numbers. Fuzzy environment has been preserved in this model with unlimited bounds with the circumstances related in direction to randomness and connected under fuzziness of LR-type. A numerical illustration has been analysed and solved in this model using trapezoidal fuzzy numbers of LR type by LR method and taken for comparison with DSW(Dong, Shah & Wong) Algorithm. The comparative example illustrates the applicability, efficiency and sustainability of the LR type model.

Keywords: Fuzzy Queueing, system performance measure, LR method, LR-type Trapezoidal fuzzy numbers.

1. Introduction

Queueing patterns are incorporated with multiple application under fuzzy pattern having several dimensions in actual normal existence. Performance measures play a significant role in Queueing system that helps and allows us to the analyse the nature of mean inter arrival time and mean service time under various situations determined by different queueing patterns. Structural Performance measure for Queueing theory patterns are computed and evaluated by various methods namely alpha-cut method with the help of non-linear parametric programming approach, Robust ranking technique, DSW algorithm, LR method. Queueing models under Fuzzy was proposed by many researchers Li and Lee[1], Negi and Lee[2], Usha Prameela Karupothu and Pavan Kumar [3] perceptionizing FM/FD/1 queueing model under various fuzzy numbers , J. P. Mukeba Kanyinda et al [4] , W. Ritha and S. Josephine Vinnarasi [5] analyzing Queueing model by LR method, B. Palpandi, G.Geetharamani [6] examining Queueing model by Robust Ranking technique, S. Shanmugasundaram et al [7] , R. Srinivasan [8] studying Queueing problem by DSW Algorithm under fuzzy pattern.

This article focuses on $(FM/FD/1) : (\infty/FCFS)$ queueing pattern with average expected arrival and service intervals undergoing distributions exponentially with mean rate 'λ' , 'μ' under FCFS queueing technique following source population infinitely with FD being constant fuzzified dispersion having the proposed rates. To prioritize the efficiency of LR method, a numerical illustration has been computed using LR method and analyzed using DSW algorithm.

2. Preliminaries

A. Fuzzy set:

A fuzzy set $\widetilde{P}$ specified on R is categorized by a membership function having its elements of a specified space 'X' in the specified range [0, 1]. It is represented as

$$\widetilde{P} = \{ (Z, \mu_{\widetilde{P}}(z)) \, ; \, z \in Z \}$$

B. Trapezoidal fuzzy number by LR – type :

A fuzzy number $\widetilde{P} = (p, q, r, s)_{LR}$ is defined to be trapezoidal fuzzy numbers of LR type if its membership function is specified by

$$\mu_{\widetilde{P}}(x) = \begin{cases} L\left(\frac{p-x}{r}\right); & \text{if} x \le p \, ; \, r > 0 \\ R\left(\frac{x-q}{s}\right); & \text{if} x \ge q \, ; \, s > 0 \\ 1 & ; \text{otherwise} \end{cases}$$

C. Basic operations in trapezoidal fuzzy numbers of LR type:

Let $M' = (m_1, m_2, m_3, m_4)$, $V' = (v_1, v_2, v_3, v_4)$ & 'λ' is a parameter, then

(i) $M' + V' = (m_1, m_2, m_3, m_4) + (v_1, v_2, v_3, v_4) = (m_1+v_1, m_2+v_2, m_3+v_3, m_4+v_4)$

(ii) $M' - V' = (m_1, m_2, m_3, m_4) - (v_1, v_2, v_3, v_4) = (m_1-v_2, m_2-v_1, m_3+v_4, m_4+v_3)$

(iii) $M'V' = (m_1, m_2, m_3, m_4). (v_1, v_2, v_3, v_4)=(m_1v_1, m_2v_2, m_1v_3 + m_3v_1, m_2v_4+v_2m_4)$

(iv) $\frac{M'}{V'} = \frac{(m1,m2,m3,m4)}{(v_1,v_2,v_3,v_4)} = \left(\frac{m_1}{v_2}, \frac{m_2}{v_1}, \frac{m_3}{v_4}, \frac{m_4}{v_3} \right)$

(v) $\lambda (m_1, m_2, m_3, m_4) = (\lambda m_1, \lambda m_2, \lambda m_3, \lambda m_4)$

(vi) $1/\lambda (m_1, m_2, m_3, m_4) = (m_1/\lambda, m_2/\lambda, m_3/\lambda, m_4/\lambda)$

D. Representation of trapezoidal fuzzy numbers of LR type:

Trapezoidal Fuzzy numbers are represented as $\widetilde{F}$(s, t, m, v). These trapezoidal fuzzy numbers can be written as $\widetilde{F}$(s, t, m, v) = < t, m, t-s, v-m > Where t, m are spreads under left and right patterns respectively using LR representation technique.

3. Performance Measure Under Constant Fuzzy Dispersion With Infinite Source Population:

The Queueing pattern(FM/FD/1) : (∞/FCFS) is considered with population source under unlimited pattern with the constant fuzzified dispersion having average expected arrival and service intervals undergoing distributions exponentially with arrival and service rates having 'λ' and 'μ'respectively .

The structural performance measures specifically expected customer rate and delayed customer rate in the system, queue are given by the respective formulas.

- $L_s = \frac{\rho^2}{2(1-\rho)} + \rho$
- $L_q = \frac{\rho^2}{2(1-\rho)}$
- $W_q = \frac{\rho}{2(1-\rho)\mu}$
- $W_s = \frac{\rho}{2(1-\rho)\mu} + \frac{1}{\mu}$

Where $\rho = \frac{\lambda}{\mu}$, the utilization factor or traffic intensity.

4. Numerical Example

A Queueing model (FM/FD/1):(∞/FCFS) is examined with the average expected arrival and service intervals using trapezoidal fuzzy numbers followed by λ = (1,2,3,4) and μ = (11,12,13,14).

In LR type, the average expected arrival and service intervals using trapezoidal fuzzy numbers of LR type followed by λ = (2,3,1,1) and μ= (12,13,1,1).

To find the value of :

$$\rho = \frac{\lambda}{\mu} = \frac{(2,3,1,1)}{(12,13,1,1)} = \left(\frac{2}{13}, \frac{3}{12}, 1, 1\right)$$

$$= (0.1538, 0.25, 1, 1)$$

Average customer rate in the system (L_s):

$$L_s = \frac{\rho^2}{2(1-\rho)} + \rho$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(1-\frac{\lambda}{\mu})} + (0.1538, 0.25, 1, 1)$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{\mu-\lambda}{\mu})} + (0.1538, 0.25, 1, 1)$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{(12,13,1,1)-(2,3,1,1)}{(12,13,1,1)})} + (0.1538, 0.25, 1, 1)$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{(9,11,2,2)}{(12,13,1,1)})} + (0.1538, 0.25, 1, 1)$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(0.6923,0.9166,2,2)} + (0.1538, 0.25, 1, 1)$$

$$= \frac{(0.0236,0.0625,0.3076,0.5)}{(1.3846,1.8332,4,4)} + (0.1538, 0.25, 1, 1)$$

$$= (0.0128, 0.0451, 0.0769, 0.125) + (0.1538, 0.25, 1, 1)$$

$$= (0.1666, 0.2951, 1.0769, 1.125)$$

Therefore, L_s ranges in (0.1666, 0.2951).

Average customer rate in the Queue (L_q):

➢ $$L_q = \frac{\rho^2}{2(1-\rho)}$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(1-\frac{\lambda}{\mu})}$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{\mu-\lambda}{\mu})}$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{(12,13,1,1)-(2,3,1,1)}{(12,13,1,1)})}$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(\frac{(9,11,2,2)}{(12,13,1,1)})}$$

$$= \frac{(0.1538,0.25,1,1)^2}{2(0.6923,0.9166,2,2)}$$

$$= \frac{(0.0236,0.0625,0.3076,0.5)}{(1.3846,1.8332,4,4)}$$

$$= (0.0128, 0.0451, 0.0769, 0.125)$$

Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)
Therefore, L_q ranges in (0.0128 and 0.0451).

Average delayed customer rate in the Queue (W_q):

➢ $W_q = \frac{\rho}{2(1-\rho)\mu}$

$= \frac{(0.1538,0.25,1,1)}{2(1-\frac{\lambda}{\mu})\mu}$

$= \frac{(0.1538,0.25,1,1)}{(1.3846,1.8332,4,4)(12,13,1,1)}$

$= \frac{(0.1538,0.25,1,1)}{(16.6152,23.8316,49.3846,53.8332)}$

= (0.0064, 0.0150, 0.0185, 0.0202)

Therefore, W_q ranges in (0.0064, 0.0150).

Average delayed customer rate in the system(W_s):

➢ $W_s = \frac{\rho}{2(1-\rho)\mu} + \frac{1}{\mu}$

$= \frac{2(1-\rho)+\rho}{2(1-\rho)\mu}$

$= \frac{2(1-\frac{\lambda}{\mu})+\frac{\lambda}{\mu}}{2(1-\frac{\lambda}{\mu})\mu}$

$= \frac{2(\frac{\mu-\lambda}{\mu})+\frac{\lambda}{\mu}}{2(\frac{\mu-\lambda}{\mu})\mu}$

$= \frac{2(\mu-\lambda)+\lambda}{2(\mu-\lambda)\mu}$

$= \frac{2\mu-\lambda}{2\mu^2-2\lambda\mu}$

$= \frac{2(12,13,1,1)-(2,3,1,1)}{2(12,13,1,1)^2-2(2,3,1,1)(12,13,1,1)}$

$= \frac{(24,26,2,2)-(2,3,1,1)}{2(144,169,24,26)-2(24,39,14,16)}$

$= \frac{(21,24,3,3)}{(288,338,48,52)-(48,78,28,32)}$

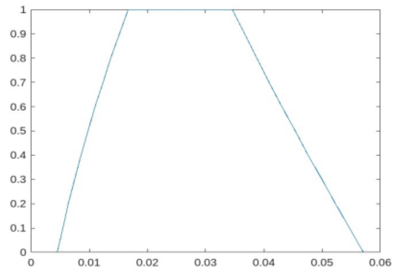
$= \frac{(21,24,3,3)}{(210,290,80,80)}$

= (0.0724, 0.1142, 0.0375, 0.0375)

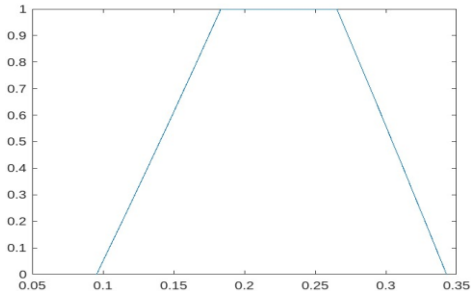
Therefore, W_s ranges in (0.0724, 0.1142).

The structural performance measure of L_q , L_s, W_s and W_q has been given in graphical representation using MATLAB R2023a.

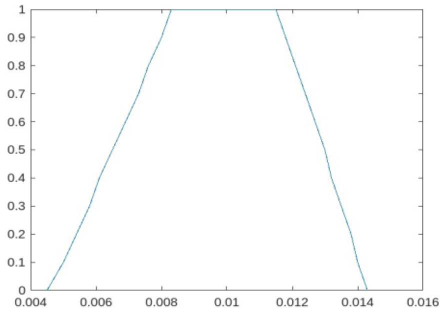
Membership Function of Lq:



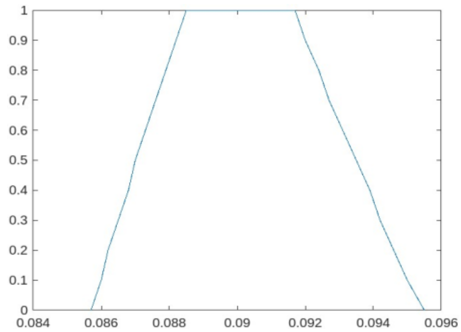
Membership Function of Ls:



Membership Function of Wq:



Membership Function of Ws:



Comparison for FM/FD/1 Queueing Model:

	DSW Algorithm	**LR Method**
Without Fuzzy number	An algorithm using Interval Analysis Arithmetic is implemented for execution that comprises of four steps in calculating the system performance measure.	Easy and simple manual computation has been carried out for system performance measure using trapezoidal fuzzy number formulas under LR type.
L_q = 0.0166	Ranges in (0.0027 , 0.1039) and doesn't drop under 0.0023 and rise beyond 0.1893.	Ranges in (0.0128 , 0.0451).
L_s= 0.1832	Ranges in (0.0741 , 0.4675) and doesn't drop under 0.0690 and rise beyond 0.6439.	Ranges in (0.1666 , 0.2951).
W_s=0.0916	Ranges in (0.0741, 0.1168) and doesn't drop under 0.0690 and rise beyond 0.1287	Ranges in (0.0724 , 0.1142).
W_q= 0.0083	Ranges in (0.0027, 0.0259) and doesn't drop under 0.0023 and rise beyond 0.0378.	Ranges in (0.0064 , 0.0150).

5. Conclusion

In the present document, the estimate of system performance under FM/FD/1 queueing pattern with population under infinite source following FCFS pattern that has been computed with the help of trapezoidal fuzzy numbers of LR type. The spreads under left and right patterns are computed with the help of trapezoidal fuzzy numbers under LR type using LR method.

The average customer rate in the queue ranges in (0.0128, 0.0451), average customer rate in the system ranges in (0.1666, 0.2951), the average delayed period in the system ranges in (0.0724, 0.1142) and the average delayed period in the queue ranges in (0.0064,0.0150).

Hence, LR method helps in computing the structural performance measure of a queueing system model in a simple and flexible manner.

References

[1] R.J. Li, E.S. Lee, Analsis of Fuzzy queues, Computers and Mathematics with Applications, 17 (1989) 1143-1147.

[2] D. S. Negi, E.S. Lee, Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems, 46 (1992) 321-330.

[3] Usha Prameela Karupothu and Pavan kumar, (2020), "Perceptionization of FM/FD/1 queueing model under various fuzzy numbers", Croatian Operation Research Review, CRORR 11(2020), 135-144.

[4] J. P. Mukeba Kanyinda, R. Mabela Makengo Matendo, B. Ulungu Ekunda Lukata., (2015), "Computing Fuzzy Queueing Performance Measures by LR method", Journal of Fuzzy set valued Analysis, volume 2015, Issue 1, (57-67).

[5] W. Ritha and S. Josephine Vinnarasi, " Analysis of Priority Queueing Models: LR Method", Annals of Pure and Applied Mathematics, vol.15, No.2, 2017, 271-276.

[6] B. Palpandi, G.Geetharamani (2013), "Evaluation of performance measure of Bulk arrival queue with fuzzy parameters using Robust Ranking Technique", International Journal of Computational Engineering Research, Volume 3, Issue 10 (53-57).

[7] S. Shanmugasundaram, S. Thamodharan, M. Ragapriya , A study on single server fuzzy queueing model using DSW Algorithm, International Journal of Latest Trends in Engineering and Technology, vol.6 issue 1, september 2015, 162-169.

[8] R. Srinivasan, Fuzzy Queueing model using DSW Algorithm, International journal of Advanced Research in Mathematics and Application., vol. 1, issue 1, January 2014, 57-62.

[9] Chen SP, Parametric, nonlinear programming approach to fuzzy queues with bulk service, European Journal of Operation Research, 2005, 163:434-444.

[10] J. Vahidi, S. Rezvani(2013), " Arithmetic operations on Trapezoidal Fuzzy numbers", Journal Non-linear Analysis and Application 2013, volume 2013, (1-8).