Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)

Two Warehouse Inventory Model for Deteriorating Items with Trade Credit Under Inflation

Monika Rani[a], Sachin Kumar[b] , Vikas Tiwari[c], Vipin Kumar[d*]

a. Research Scholar, Department of Mathematics, AKTU, Lucknow, U.P., India.

b. Department of Applied Sciences, KIET Group of Institutions Delhi - NCR, Ghaziabad

c. Department of Applied Sciences, Rajkiya Engineering College, Sonbhadra, UP, India

d. Department of Mathematics, B.K. Birla Institute of Engineering and Technology, Pilani Raj. India (Corresponding author)

(drvkmaths@gmail.com)

Article History:

Received: *14-11-2024*

Revised:*26-12-2024*

Accepted:*10-01-2025*

Abstract:

Establishing warehouses is essential in societies where commercial activities have expanded significantly, making efficient storage a prerequisite for smooth exchange. Variable holding costs play a crucial role in determining warehouse expenses, as these costs typically increase over time. This study examines an inventory model for perishable goods stored in two types of warehouses (rented and owned) under conditions of partial backlogging and inflation. Along with exploring sustainable marketing strategies, the article investigates the connections between product pricing, advertising, and demand, highlight the considerable impact of price reductions and promotions by advertisement. Additionally, this study addresses a trade credit scheme where the supplier grants the retailer a fixed time frame to finalize the account. The research aims to enhance operational efficiency and reduce overall costs by optimizing key decision factors such as storage capacity, credit period, and replenishment time period. To explore the effects on the system's optimal total cost, along with managerial insights, numerical examples and sensitivity analyses are provided. The model's outcomes are validated through sensitivity analysis using Mathematica 13.0 software, ensuring their robustness and reliability.

Keywords: Inventory Model, Two-Warehouse, Deteriorating Items, Trade Credit, Inflation

1. Introduction

The establishment of efficient and strategically located warehouses is a fundamental necessity in modern commercial societies, where the rapid expansion of trade and commerce demands robust storage solution policies. Warehouses serve as pivotal nodes in the supply chain, facilitating the smooth exchange of goods between suppliers, retailers, and consumers. A significant factor in the management of warehouses is the variable holding cost, which represents the expenses incurred for storing goods over time. These costs are not static; they tend to escalate due to factors such as spoilage, obsolescence, and inflation. Particularly in the context of perishable goods, managing holding costs becomes even more critical, as these goods have a limited shelf life and are favorable to rapid depreciation. Thus, understanding and controlling these costs is crucial for maintaining the warehouse operations.

This study focuses on developing and analyzing an inventory model specifically designed for perishable goods. The model considers two types of warehouses: rented and owned. Each type of warehouse comes with its unique set of cost structures. Rented warehouses often involve variable rental costs and less control over the infrastructure, while owned warehouses require significant capital investment and incur maintenance expenses. The model addresses these differences and aims to provide a balanced approach to managing inventory across both types of storage facilities.

In addition to the core focus on warehouse management, the study explores sustainable marketing strategies that can influence the demand for perishable goods. The relationships between advertising, product pricing, and consumer demand are complex and impactful. Effective promotional activities and strategic price reductions can significantly boost demand, thereby reducing the time goods spend in storage and mitigating holding costs. The research emphasizes the importance of aligning marketing efforts with inventory management practices to achieve optimal results.

Another critical aspect introduced in this study is the trade credit policy, where suppliers grant retailers a specific period to settle their accounts. This policy is a vital point of financial management within the supply chain, which provides the flexibility to the retailers to manage the cash flows and optimize their inventory levels. By extending the payment period, retailers can invest in larger inventories without immediate financial strain, with the better bulk purchasing rates and improved inventory turnover.

The aim of this research is to enhance operational efficiency and reduce overall costs by optimizing key factors such as the storage capacity, credit period, and replenishment time. By these parameters, businesses can achieve a more cost-effective and responsive inventory management system, particularly suited to the challenges associated with perishable goods.

In summary, this study provides a comprehensive framework for managing perishable goods in a dual-warehouse environment. It highlights the significance of variable holding costs, the strategic role of advertising and pricing in demand management, and the benefits of flexible trade credit policies. The findings offer valuable insights and practical solutions for businesses seeking to enhance their operational frameworks, reduce costs, and improve their market responsiveness and sustainability. Through this research, we aim to contribute to the ongoing efforts in optimizing supply chain management and promoting more efficient commercial practices.

2. Literature Review:

We have presented an inventory model for deteriorating items under the effect of inflation along with two warehouse system and trade credit policy. Here in this section literature review is provided with Two warehouse system, deterioration, Trade credit period and inflation keywords.

Two Warehouse: The inclusion of two types of warehouses—rented and owned—in an inventory model for perishable goods is a strategic decision driven by various operational and financial considerations. The necessity of incorporating both rented and owned warehouses arises from the need to balance cost efficiency, flexibility, and risk management in inventory management. Jaggi et al. (2015) examined the effect of deterioration along with imperfect quality on an inventory model of two-warehouse. Tiwari et al. (2016) analyzed a retailer ordering policies with the effects of inflation and trade credit for deteriorating items in a two-warehouse setup. Palanivel et al. (2016) developed and incorporating non-instantaneous deterioration in a two-warehouse inventory model with stock-dependent demand, inflation and shortages. Jaggi et al. (2017) presented a two-warehouse inventory model for deteriorating items and incorporated permissible delay in payments with imperfect quality. Kumar and Chanda (2018) introduced a model for two-warehouse where demand is influenced by innovation, relevant to growing technology markets. Panda et al. (2019) proposed a two-warehouse inventory model incorporating partial backlogging, credit policy along with price- and stock-dependent demand. Sethy et al. (2020) included a two-warehouse production prototype for managing deteriorating inventory items within payment structures. Xu et al. (2021) formulated a model for items with infinite lifespan and warehouse mode selection with partial backlogging. Qiuet al. (2022) presented an optimization approach management in a dual-channel warehouse for multi-product inventory. Das et al. (2023) developed a dual-channel supply chain model with all-units discount and partial backordering under a two-warehouse setting. Sharma et al. (2024) formulated a green inventory model and included carbon emission with energy consumption in two ware house system. Sharma and Mandal (2024) explored as a sustainable two-warehouse inventory models with preservation technology investment.

Deterioration: Considering deterioration in an inventory model is crucial for maintaining realistic stock levels, optimizing costs, and ensuring customer satisfaction. Items like food, pharmaceuticals, and electronics degrade over time, and failing to account for this can lead to significant waste and financial loss. By integrating deterioration rates, businesses can make informed decisions about order quantities and timing, reducing holding costs and minimizing spoilage. This approach also supports regulatory compliance and enhances supply chain efficiency, ensuring that high-quality products are available when needed, ultimately fostering customer loyalty and trust. Bhunia et al. (2015) introduced an inventory model of two storage for perishable items incorporating variable demand. Pervin et al. (2016) formulated an inventory model tailored for perishable items in markets with declining demand, emphasizing the role of trade credit policy. Chan et al. (2017) integrated production and inventory management for deteriorating items, focusing on optimizing production rates while accounting for deterioration during delivery. Singha et al. (2018) presented a fuzzy model for decaying items along with stock-dependent demand rates. This approach allows for better handling of uncertainties in inventory levels and demand fluctuations, crucial for managing perishable products effectively. Braglia et al. (2019) introduced a continuous review inventory model for decaying items under uncertain demand and lead time. Khakzad and Gholamin (2020) explored an inventory model for deteriorating items with the effect of inspection on the deterioration rate. Sharma et al. (2021) described a production-based model for deteriorating items with price discount and inventory dependent demand. Further, Ghandehari and Karimi-Lenji (2022) formulated an optimal inventory policy for perishable items with a multivariate demand, addressing complex demand patterns and their effect on inventory management. Kumar et al. (2022) developed an inventory model with advertisement and price-based demand function along with deterioration. Mahato et al. (2023) presented inventory models for decaying items with fixed lifetimes and carbon emissions policies, which highlight the integration of environmental considerations into inventory management for deteriorating goods. Recently, San-Jose et al. (2024) proposed a sustainable inventory model under a carbon emission tax for perishable items with full backlogging.

Trade Credit Period: Incorporating trade credit into an inventory model is essential for optimizing financial management and improving supplier relationships. Trade credit allows businesses to delay payments for inventory purchases, providing crucial cash flow flexibility. This can enable companies to invest in other areas, such as marketing or expansion, while still maintaining adequate stock levels. By including trade credit terms in inventory models, businesses can more accurately assess the true cost of holding inventory, balance their working capital, and negotiate better terms with suppliers. This strategic financial tool helps in managing liquidity and can lead to more favorable purchasing conditions, enhancing overall operational efficiency. Sarkar et al. (2015) introduced an inventory model incorporating trade-credit period and variable deterioration rate. Mahata and De (2016) presented an EOQ inventory system for repairable items with price-dependent demand rates under a partial trade credit policy for retailer. This model offers insights into optimizing order quantities and credit utilization in inventory management. Tsao et al. (2017) explored explores sustainable newsvendor models under trade credit, emphasizing the environmental and financial benefits of integrating sustainability considerations into inventory decision-making. Tiwari et al. (2018) developed an inventory model with expiration dates for deteriorating items under two-level partial trade credits and with partial backlogging. It addresses the complexities of managing perishable goods efficiently. Pervin et al. (2019) proposed a two-echelon inventory model incorporating a trade-credit policy with price- and stock-dependent demand. Kumar et al. (2020) explored a model and included trade credit policy for deteriorating items and multivariable demand function. Barron et al. (2020) presented an inventory model with stock-dependent carrying costs and non-linear inventory-dependent demand with a trade credit period. This model accounts for the dynamic nature of inventory costs and demand fluctuations. Esmaeili and Nasrabadi (2021) proposed a model for multi-retailers consisting of trade credit and an inflationary environment. Sharma et al. (2022) explored an economic quantity model for decaying items under the effect of inflation along with trade credit policy. Shan and Shroff (2022) introduced a model for fixed-life products with a two-level trade credit policy and trapezoidal demand. Moradi et al. (2023) considered learning effects and partial trade credit policy in their study and presented an inventory model for imperfect quality items. Shah et al. (2024) proposed a model for non-instantaneous deteriorating items with advertisement-dependent probabilistic demand under trade credit financing.

Inflation: Incorporating inflation into an inventory policy is vital for maintaining accurate cost assessments and ensuring profitability. Inflation affects the purchasing power of money, causing prices of goods to rise over time. By accounting for inflation, businesses can better forecast future costs, adjust pricing strategies, and optimize order quantities to mitigate the impact of rising prices. This proactive approach helps in preserving profit margins, maintaining competitiveness, and ensuring long-term financial stability. Pal et al. (2015) formulated a production model with inflation along with shortage and ramp type demand. A bi-objective inventory model under inflation and discount was introduced by Mousavi et al. (2016). Yadav et al. (2017) investigated the effect of inflation on a two-warehouse model with time dependent demand. Shah and Vaghela (2018) proposed a production model for effort and time-based demand under inflationary environment. Yadav et al. (2019) presented a supply chain model under inflationary conditions for deteriorating items. It offers insights into optimizing inventory across the supply chain network. Further, Kumar et al. (2020) and Sundararajan et al. (2021) analyzed the effect of inflation in their various inventory models.

Recently Sarkar et al. (2022) applied neural network within an inventory model under uncertainty and inflation. It leverages advanced computational techniques to improve inventory decision-making accuracy. The effect of inflation with multivariate demand on the EOQ model along with partial backlogging and carbon tax policy was introduced by Singh et al. (2023). Kumar et al. (2023) investigated the combined effect of promotional efforts and selling price in an inventory model under inflation. Further Pal et al. (2024) proposed an inventory model of two-warehouse with credit policy and inflation effect. It addresses the financial aspects of inventory management in a multi-warehouse setting, considering inflationary pressures.

In this study we have formulated an inventory model for decaying items with two warehouse one is rented and another is owned. Demand is dependent on price and advertisement. Shortage is allowed and partially backlogged. Holding cost is variable in nature and time dependent. Additionally, various costs are considered in inflationary environment. To increase the flow of money a trade credit period is also offered to the retailer. Using examples and careful analysis, the study showed how these choices affect the overall cost and offered helpful tips for managers. Overall, it helps improve how companies manage their supply chains for perishable goods, making operations more efficient and cost-effective.

Table-1 Comparison table between Previous research work and current work.

References	**Variable Demand**	**Variable Holding cost**	**Shortage**	**Inflation**	**Deterio ration**	**Trade credit**
Jaggi et al. (2015)	Yes	No	No	Yes	Yes	No
Bhunia et al. (2015)	Yes	No	Yes	No	Yes	No
Tiwari et al. (2016)	No	No	No	Yes	Yes	Yes
Palanivel et al. (2016)	Yes	No	Yes	Yes	Yes	No
Jaggi et al. (2017)	No	No	No	No	Yes	Yes
Kumar and Chanda (2018)	Yes	No	No	No	Yes	No
Panda et al. (2019)	Yes	No	Yes	No	Yes	No
Sethy et al. (2020)	Yes	No	No	No	Yes	Yes
Xu et al. (2021)	Yes	No	Yes	No	No	No
Ghandehariet al. (2022)	Yes	No	No	No	Yes	No
Das et al. (2023)	Yes	No	Yes	No	No	Yes
San-Jose et al. (2024)	Yes	No	No	No	Yes	No
Pal et al. (2024)	No	No	Yes	Yes	Yes	Yes

This Paper	Yes	Yes	Yes	Yes	Yes	Yes

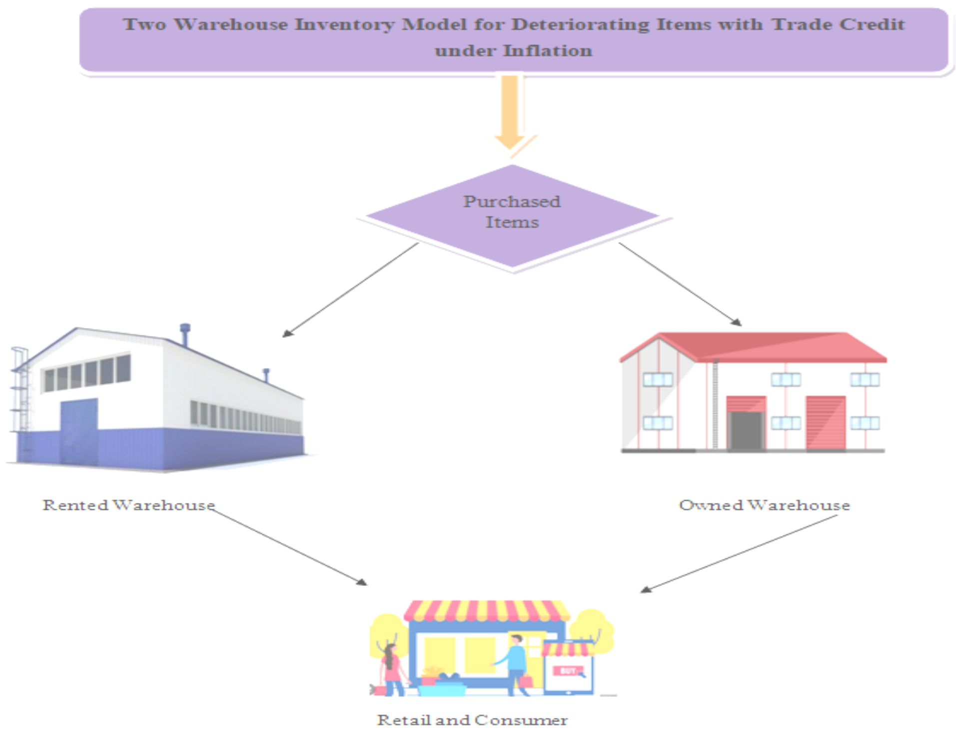


Fig.1 Graphical Representation of presented model

3. Assumptions and list of symbols: The manuscript uses the following conventions and

symbols to mathematically define the proposed inventory procedure.

3.2 Assumptions:

- To maintain the inventory level model is based on two warehouses with fixed capacity, one is rented (RW) and another is owned warehouse (OW).
- Demand is based on market advertisement and price of product that is $D = A^{\alpha}(a - bp)$, where a, b and α are demand parameter, A is the frequency of market advertisement and p is the product selling price.
- Shortages are allowed in the time period $[t_2, T]$, which is partially backlogged in the proportion $e^{-\delta(T-t)}$ of demand where $0 < \delta < 1$.
- θ_1 is the deterioration rate of inventory in the rented warehouse while θ_2 is the deterioration rate of owned warehouse.
- Holding cost of inventory is variable and time dependent.
- Replenishment rate of inventory is infinite and lead time is zero.

- In the model, deteriorating items are neither repaired nor replaced throughout the cycle length [0, T].
- Various costs are taken in the inflationary environment.
- Trade credit period M is offered to the supplier to improve the business.

3.2 List of symbols

- a, b Demand parameters $a > 0, b > 0$ (constant)
- A Advertisement frequency
- $h_0 + h_1 t$ variable holding cost for rented warehouse
- $h_2 + h_3 t$variable holding cost for owned warehouse
- I_e Rate of Interest earned
- I_c Rate of Interest charged
- p- Market price ($/unit)
- RMaximum backlogged amount (Per order)
- SInventory in the owned warehouse initially (units)
- Q Total level of inventory in the both warehouses at time $t = 0$(units)
- $Q - S$ Initial stock level of rented warehouse(units)
- TCTotal cost per cycle for the inventory procedure ($/time unit)
- δ Backorder parameter. $\delta > 0$
- θ_1Rate of deterioration of rented warehouse, $0 < \theta_1 < 1$(in %)
- θ_2Rate of deterioration of owned warehouse, $0 < \theta_2 < 1$(in %)
- I_r(t) Inventory in rented warehouse at any time t(units)
- I_{o_1}(t) Inventory in owned warehouse in time interval [0,t_1] (units)
- I_{o_2}(t) Inventory in owned warehouse in time interval [$t_{1,}t_{2,}$] (units)
- I_B(t)backorder inventory during shortage in time interval [$t_{2,}T$] (units)
- MOffered trade credit period
- C_l Lost sale cost($/order)
- C_d Deterioration cost($/order)
- C_s Shortage cost($/order)

3.3 Decision variables

- t_1 Time at which stock of rented warehouse become zero
- t_2 Time at which total stock of owned warehouse become zero
- T Cycle duration (time unit)

4. Mathematical Modeling:

Let us consider in this model Q units of inventory received at time $t = 0$. Out of which S units of inventory are stored in the owned warehouse OW and remaining $Q - S$ units are kept in rented warehouse RW. Now to meet the demand inventory of rented warehouse will be consumed first. Let in the time period $[0, t_1]$ inventory of rented warehouse become zero due to deterioration and demand. And inventory of OW depletes due to deterioration only during this period $[0, t_1]$.

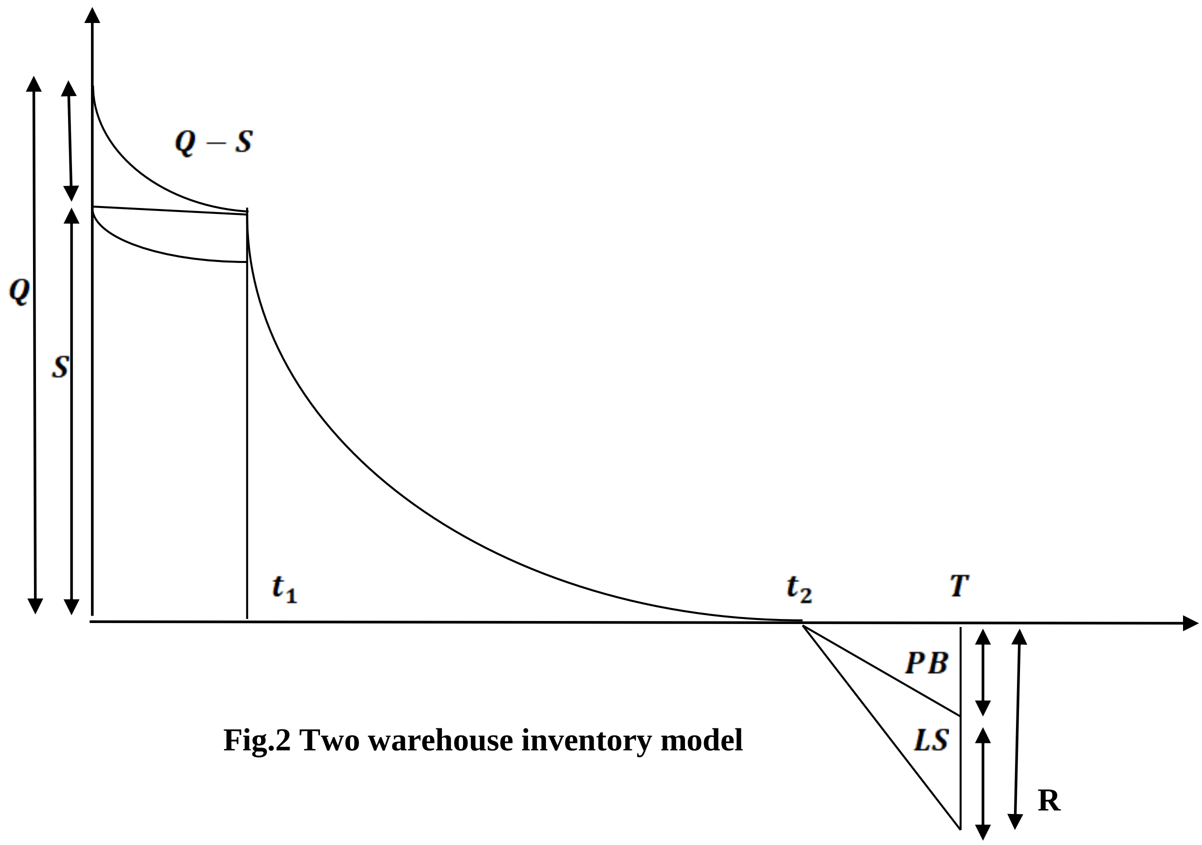


Fig.2 Two warehouse inventory model

Further during the period $[t_1, t_2]$ inventory of OW gets down due to demand and deterioration and become zero at $t = t_2$. After this shortage started during time period $[t_2, T]$ which is partially backlogged and R is the maximum shortage inventory.

Following differential equations are the representation of inventory level at any time t in $RW\ and\ OW$ in the duration $[t_2, T]$.

$$\frac{dI_r(t)}{dt} + \theta_1 I_r(t) = -A^{\alpha}(a-bp),\ 0 \le t \le t_1 \quad (1)$$

$$\frac{dI_{o_1}(t)}{dt} + \theta_2 I_{o_1}(t) = 0, \quad 0 \le t \le t_1 \quad (2)$$

$$\frac{dI_{o_2}(t)}{dt} + \theta_2 I_{o_2}(t) = -A^{\alpha}(a-bp),\ t_1 \le t \le t_2 \quad (3)$$

$$\frac{dI_B(t)}{dt} = -A^{\alpha}(a-bp)e^{-\delta(T-t)}, t_2 \le t \le T \quad (4)$$

With boundary conditions $I_r(0) = Q - S, I_r(t_1) = 0\ , I_{o_1}(0) = S\ , I_{o_2}(t_2) = 0, I_B(t_2) = 0\ and\ I_B(T) = -R$

On solving above equation (1), (2), (3) and (4) we get

$$I_r(t) = \frac{Q-S}{e^{\theta_1 t}} - \frac{A^{\alpha}(a-bp)}{\theta_1} + \frac{A^{\alpha}(a-bp)}{\theta_1 e^{\theta_1 t}}, \quad 0 \le t \le t_1 \quad (5)$$

$$I_{o_1}(t) = Se^{-\theta_2 t}, \quad 0 \le t \le t_1 \quad (6)$$

$$I_{o_2}(t) = \frac{A^{\alpha}(a-bp)}{\theta_2}\left(e^{\theta_2(t_2-t)} - 1\right), \quad t_1 \le t \le t_2 \quad (7)$$

$$I_B(t) = \frac{A^{\alpha}(a-bp)}{\delta}\left(e^{-\delta(T-t_2))} - e^{-\delta(T-t)}\right), \quad t_2 \le t \le T \quad (8)$$

Also $I_r(t_1) = 0$ therefore by (5) we get

$$S = Q - \frac{A^{\alpha}(a-bp)}{\theta_1}(e^{\theta_1 t_1} - 1) \quad (9)$$

Applying the condition of continuity $I_{o_1}(t_1) = I_{o_2}(t_1)$ we get

$$S = \frac{A^{\alpha}(a-bp)}{\theta_2}(e^{\theta_2 t_2} - e^{\theta_2 t_1}) \quad (10)$$

putting this value in equation (9) we get

$$Q = A^{\alpha}(a-bp)\left\{\frac{1}{\theta_1}\left(e^{\frac{e^{-it_1}}{i}t_1} - 1\right) + \frac{1}{\theta_2}\left(e^{\theta_2 t_2} - e^{\theta_2 t_1}\right)\right\} \quad (11)$$

In the next cycle the total quantity to be replenished is expressed as

$$TOQ = I_r(0) + I_o(t) - I_B(t)$$

$$TOQ = A^{\alpha}(a-bp)\left[\frac{1}{\theta_2}\left(e^{\theta_2 t_2} - e^{\theta_2 t_1} + e^{\theta_2(t_2-t)} - 1\right) - \frac{1}{\delta}\left(e^{-\delta(T-t_2))} - e^{-\delta(T-t)}\right)\right] (12)$$

Let us consider the ordering cost is OC

Cost of holding inventory in Rented warehouse

$$HC_R = \int_0^{t_1}(h_o + h_1 t)I_r(t)e^{-it}\,dt$$

$$HC_R = \frac{A^\alpha(a-bp)}{\theta_1}\left[(h_o + h_1 t_1)\left(\frac{e^{-it_1}}{i} - \frac{e^{-it_1}}{\theta_1+i}\right) - h_1\left(\frac{e^{-it_1}}{(\theta_1+i)^2} - \frac{e^{-it_1}}{i^2}\right) - h_o\left(\frac{1}{i} - \frac{e^{\theta_1 t_1}}{(\theta_1+i)}\right) + h_1\left(\frac{e^{\theta_1 t_1}}{(\theta_1+i)^2} - \frac{1}{i^2}\right)\right] \quad (13)$$

Holding cost in owned warehouse is given by

$$HC_o = \int_0^{t_1}(h_2 + h_3 t) I_{o_1}(t) e^{-it}\, dt + \int_{t_1}^{t_2}(h_2 + h_3 t) I_{o_2}(t) e^{-it}\, dt$$

$$= \frac{A^\alpha(a-bp)}{\theta_2}\left[\left(\frac{h_2}{\theta_2+i} + \frac{h_3}{(\theta_2+i)^2}\right)\left(e^{\theta_2 t_2} - e^{\theta_2 t_1} + e^{-it_1} - e^{-it_2}\right) + \frac{h3}{(\theta_2+i)}\left(t_1 e^{-it_1} - t_2 e^{-it_2}\right) + \frac{h_2}{i}\left(e^{-it_2} - e^{-it_1}\right) + \frac{h_3 t_2}{i} e^{-it_2} + \frac{h_3}{i^2}\left(e^{-it_2} - e^{-it_1}\right)\right] \quad (14)$$

Deterioration cost per cycle is

$$DC = A^\alpha(a-bp) C_d\left[\frac{1}{(\theta_1+i)}\left(e^{\theta_1 t_1} - e^{-it_1}\right) + \frac{1}{(\theta_2+i)}\left(e^{-it_1} - e^{\theta_2 t_2} + e^{\theta_2 t_1} - e^{-it_2}\right) + \frac{1}{i}\left(e^{-it_2} - 1\right)\right] \quad (15)$$

Shortage Cost per cycle is

$$SC = C_s \int_{t_2}^{T} I_B(t) e^{-it}\, dt$$

$$= C_s \frac{A^\alpha(a-bp)}{\delta}\left(\frac{e^{-\delta(T-t_2)-it_2}}{i} + \frac{e^{-\delta(T-t_2)-it_2}}{\delta-i} - \frac{e^{-\delta(T-t_2)-iT}}{i} - \frac{e^{-iT}}{\delta-i}\right) \quad (16)$$

Lost sale cost is given by

$$LSC = C_l \int_{t_2}^{T} I_B(t) e^{-it}\, dt$$

$$= C_l \frac{A^\alpha(a-bp)}{\delta}\left(\frac{(e^{-\delta(T-t_2)-it_2} - e^{-\delta(T-t_2)-iT})}{i} + \frac{(e^{-\delta(T-t_2)-it_2} - e^{-iT})}{\delta-i}\right) \quad (17)$$

Advertising cost = A_o

Case – I When$M \leq t_2$(permissible delay period is less than inventory period)

In this condition, since positive stock period is larger than the credit period, the retailer can earn interest on the sales revenue at an annual rate I_e in the time interval $[0, t_2]$. ThereforeInterest earned is

$$IE_1 = pI_e\left[\int_0^{t_1}(t_1 - t) A^\alpha(a-bp) e^{-it} dt + \int_{t_1}^{t_2}(t_2 - t) A^\alpha(a-bp) e^{-it} dt\right]$$

$$= pI_e A^\alpha(a-bp)\left[\frac{t_1}{i} - \frac{1}{i^2} + \frac{e^{-it_2}}{i^2} + (t_2 - t_1)\frac{e^{-it_1}}{i}\right] \quad (18)$$

And after this credit period M, retailer have to pay interest with annual interest rate $I_r$on the unsold stock and the payable interest IP_1 is Interest Payable

$$IP_1 = pI_p \int_M^{t_2} I_o(t)\, e^{-it} dt$$

$$= pI_p \frac{A^\alpha(a-bp)}{\theta_2}\left[\frac{e^{\theta_2(t_2-M)-iM}}{\theta_2+i} - \frac{e^{-iM}}{i} - \frac{e^{-it_2}}{\theta_2+i} + \frac{e^{-it_2}}{i}\right] \quad (19)$$

Therefore, total average cost is given by

$TC_1(t_1,t_2,T)=\frac{1}{T}[OC+HC_R+HC_o+DC+SC+LSC+AC+IP_1-IE_1]$

$$= \frac{1}{T}\Bigg[OC+\frac{A^{\alpha}(a-bp)}{\theta_1}\Big[(h_o+h_1t_1)\left(\frac{e^{-it_1}}{i}-\frac{e^{-it_1}}{\theta_1+i}\right)-h_1\left(\frac{e^{-it_1}}{(\theta_1+i)^2}-\frac{e^{-it_1}}{i^2}\right)-h_o\left(\frac{1}{i}-\frac{e^{\theta_1t_1}}{(\theta_1+i)}\right)+h_1\left(\frac{e^{\theta_1t_1}}{(\theta_1+i)^2}-\frac{1}{i^2}\right)\Big]+\frac{A^{\alpha}(a-bp)}{\theta_2}\Big[\left(\frac{h_2}{\theta_2+i}+\frac{h_3}{(\theta_2+i)^2}\right)\left(e^{\theta_2t_2}-e^{\theta_2t_1}+e^{-it_1}-e^{-it_2}\right)+\frac{h3}{(\theta_2+i)}\left(t_1e^{-it_1}-t_2e^{-it_2}\right)+\frac{h_2}{i}\left(e^{-it_2}-e^{-it_1}\right)+\frac{h_3t_2}{i}e^{-it_2}+\frac{h_3}{i^2}\left(e^{-it_2}-e^{-it_1}\right)\Big]+A^{\alpha}(a-bp)C_d\Big[\frac{1}{(\theta_1+i)}\left(e^{\theta_1t_1}-e^{-it_1}\right)+\frac{1}{(\theta_2+i)}\left(e^{-it_1}-e^{\theta_2t_2}+e^{\theta_2t_1}-e^{-it_2}\right)+\frac{1}{i}\left(e^{-it_2}-1\right)\Big]+C_s\frac{A^{\alpha}(a-bp)}{\delta}\left(\frac{e^{-\delta(T-t_2)-it_2}}{i}+\frac{e^{-\delta(T-t_2)-it_2}}{\delta-i}-\frac{e^{-\delta(T-t_2)-iT}}{i}-\frac{e^{-iT}}{\delta-i}\right)+C_l\frac{A^{\alpha}(a-bp)}{\delta}\left(\frac{(e^{-\delta(T-t_2)-it_2}-e^{-\delta(T-t_2)-iT})}{i}+\frac{(e^{-\delta(T-t_2)-it_2}-e^{-iT})}{\delta-i}\right)+A_o+pI_p\frac{A^{\alpha}(a-bp)}{\theta_2}\Big[\frac{e^{\theta_2(t_2-M)-iM}}{\theta_2+i}-\frac{e^{-iM}}{i}-\frac{e^{-it_2}}{\theta_2+i}+\frac{e^{-it_2}}{i}\Big]-pI_eA^{\alpha}(a-bp)\Big[\frac{t_1}{i}-\frac{1}{i^2}+\frac{e^{-it_2}}{i^2}+(t_2-t_1)\frac{e^{-it_1}}{i})\Big]\Bigg] \tag{20}$$

Case – II When $M>t_2$ i.e permissible delay period is greater than inventory period

As $M > t2$, the retailer has to pay no interest and can accumulate interest at an annual rate I_e during the time period $(0,M)$therefore earned interest IE_2 in this case is given by

$$IE_2= pI_e\left[\int_0^{t_1}(t_1-t)A^{\alpha}(a-bp)e^{-it}dt+\int_{t_1}^{t_2}(t_2-t)A^{\alpha}(a-bp)e^{-it}dt+(M-t_2)\int_0^{t_2}e^{-it}dt\right]$$

$$= pI_eA^{\alpha}(a-bp)\left[\frac{(e^{-it_2}-1)}{i^2}+\frac{t_1}{i}+\frac{(t_2-t_1)}{i}e^{-it_1}-\frac{1}{i^2}-\frac{(M-t_2)(e^{-it_2}-1)}{i}\right] \tag{21}$$

And $IP_2=0$

Hence total average cost is given by

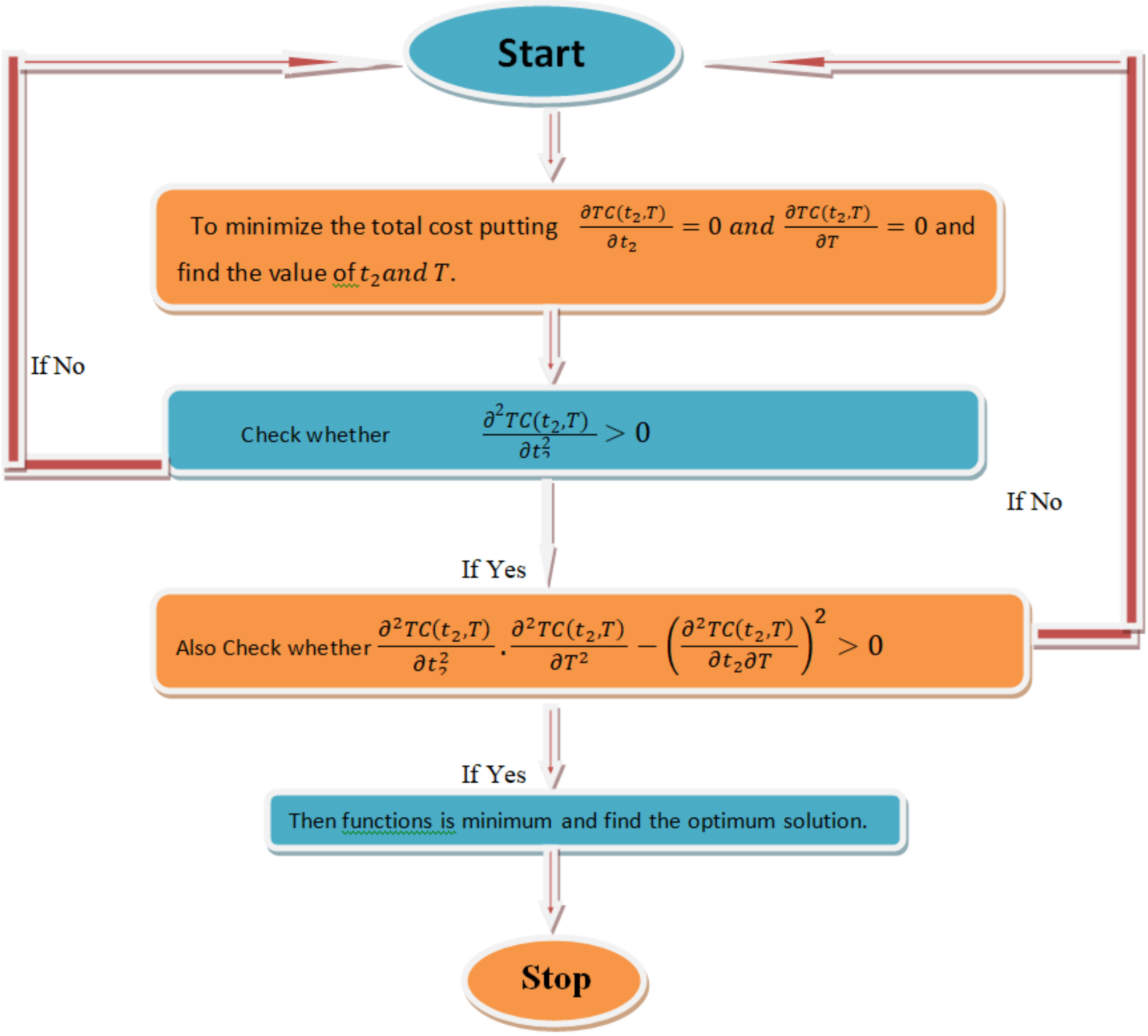
$TC_2(t_1,t_2,T)=\frac{1}{T}[OC+HC_R+HC_o+DC+SC+LSC+AC+IP_2-IE_2]$

$$=\frac{1}{T}\Bigg[OC+\frac{A^{\alpha}(a-bp)}{\theta_1}\Big[(h_o+h_1t_1)\left(\frac{e^{-it_1}}{i}-\frac{e^{-it_1}}{\theta_1+i}\right)-h_1\left(\frac{e^{-it_1}}{(\theta_1+i)^2}-\frac{e^{-it_1}}{i^2}\right)-h_o\left(\frac{1}{i}-\frac{e^{\theta_1t_1}}{(\theta_1+i)}\right)+h_1\left(\frac{e^{\theta_1t_1}}{(\theta_1+i)^2}-\frac{1}{i^2}\right)\Big]+\frac{A^{\alpha}(a-bp)}{\theta_2}\Big[\left(\frac{h_2}{\theta_2+i}+\frac{h_3}{(\theta_2+i)^2}\right)\left(e^{\theta_2t_2}-e^{\theta_2t_1}+e^{-it_1}-e^{-it_2}\right)+\frac{h3}{(\theta_2+i)}\left(t_1e^{-it_1}-t_2e^{-it_2}\right)+\frac{h_2}{i}\left(e^{-it_2}-e^{-it_1}\right)+\frac{h_3t_2}{i}e^{-it_2}+\frac{h_3}{i^2}\left(e^{-it_2}-e^{-it_1}\right)\Big]+A^{\alpha}(a-bp)C_d\Big[\frac{1}{(\theta_1+i)}\left(e^{\theta_1t_1}-e^{-it_1}\right)+\frac{1}{(\theta_2+i)}\left(e^{-it_1}-e^{\theta_2t_2}+e^{\theta_2t_1}-e^{-it_2}\right)+\frac{1}{i}\left(e^{-it_2}-1\right)\Big]+C_s\frac{A^{\alpha}(a-bp)}{\delta}\left(\frac{e^{-\delta(T-t_2)-it_2}}{i}+\frac{e^{-\delta(T-t_2)-it_2}}{\delta-i}-\frac{e^{-\delta(T-t_2)-iT}}{i}-\frac{e^{-iT}}{\delta-i}\right)+C_l\frac{A^{\alpha}(a-bp)}{\delta}\left(\frac{(e^{-\delta(T-t_2)-it_2}-e^{-\delta(T-t_2)-iT})}{i}+\frac{(e^{-\delta(T-t_2)-it_2}-e^{-iT})}{\delta-i}\right)+A_o+0+pI_eA^{\alpha}(a-bp)\left[\frac{(e^{-it_2}-1)}{i^2}+\frac{t_1}{i}+\frac{(t_2-t_1)}{i}e^{-it_1}-\frac{1}{i^2}-\frac{(M-t_2)(e^{-it_2}-1)}{i}\right]\Bigg] \tag{22}$$

Hence total average cost is $TC(t_1,t_2,T) = \begin{cases} TC_1(t_1,t_2,T) & M \le \boldsymbol{t_2} \\ TC_2(t_1,t_2,T), & M > \boldsymbol{t_2} \end{cases}$

5. Solution Procedure: By using equation (10) we can derive $TC(t_1,t_2,T) = TC(t_2,T)$

Now the solution algorithm is expressed by the following flowchart.



6. Numerical Example:

To authenticate the presented model, we employ the proposed algorithm to address the subsequent numerical instances. Mathematica 13.0 is utilized to ascertain the outcomes.

Example-1 When $M \le t_2$

The example is solved using the following inputs:

$A = 3, \alpha = 0.4., a = 110, p = \frac{50\$}{unit}, b = 2, A_o = \frac{50\$}{cycle}, \theta_1 = 0.09$ %, $\theta_2 = 0.13$ %, $h_o = \frac{500\$}{cycle}, \ h_1 = 500 \frac{\$}{cycle}, h_2 = \frac{40\$}{cycle}, h_3 = \frac{40\$}{cycle}, i = 0.8, C_s = \frac{5\$}{unit}, C_l = \frac{25\$}{unit}, C_d = 2 \frac{\$}{unit}, S = 50\ units, \delta = 0.9, M = 80\ days,\ I_p = 0.016,\ I_e = 0.012\ , OC = 100\ \$/order$

The optimum value of the decision variables are

$t_1 = 53.2418\ Days, t_2 = 113.207\ Days,\ T = 149.8113\ Days$ and the minimum total inventory cost is 942.9614 \$ $i.e.$approximately 942 \$ per order respectively

Example-2 When $M > t_2$

The example is solved using the following inputs:

$A = 3, \alpha = 0.4., a = 110, p = \frac{50\$}{unit}, b = 2, A_o = \frac{50\$}{cycle}, \theta_1 = 0.09$ %, $\theta_2 = 0.13$ %, $h_o = \frac{500\$}{cycle}, \ h_1 = 500 \frac{\$}{cycle}, h_2 = \frac{40\$}{cycle}, h_3 = \frac{40\$}{cycle}, i = 0.8, C_s = \frac{5\$}{unit}, C_l = \frac{25\$}{unit}, C_d = 2 \frac{\$}{unit}, S = 50\ units, \delta = 0.9, M = 120\ days,\ I_p = 0.016,\ I_e = 0.012\ , OC = 100\ \$/order$

The optimum value of the decision variables are

$t_1 = 62.3412\ Days,\ t_2 = 109.237\ Days,\ T = 154.483\ Days$ and the minimum total inventory cost is 847.2341 \$ $i.e.$ approximately 847 \$ per order respectively.

7. Sensitivity Analysis:

Table-2

Parameter	% Change in parameter	Change in Value	t_1	t_2	T	TC
	-40%	30	51.4096	61.335	109.422	988.7416
	-20 %	40	52.8652	87.921	128.536	968.5614
	0 %	50	53.2418	113.207	149.813	942.9614
A_o	20%	60	55.7458	143.352	151.797	931.2587
	40%	70	56.8246	149.021	154.814	922.5687
p	-40%	30	54.9087	114.625	155.985	938.4175
	-20 %	40	54.0123	114.112	152.354	940.6574
	0 %	50	53.2418	113.207	149.813	942.9614
	20%	60	52.9584	112.869	147.365	944.2457
	40%	70	51.5647	112.013	145.214	946.5460
	-40%	3	52.9876	110.568	149.1002	1548.5418

C_s	-20 %	4	53.0012	111.854	149.4657	1247.8714
	0 %	5	53.2418	113.207	149.8113	942.9614
	20%	6	53.4549	114.563	150.1236	658.4126
	40%	7	53.6847	115.102	150.5487	342.1458
M	-40%	48	53.5420	113.425	151.8385	754.6924
	-20 %	64	53.3401	113.315	150.8364	845.9836
	0 %	80	53.2418	113.207	149.8113	942.9614
	20%	96	53.0913	113.109	148.7883	1037.9523
	40%	112	52.9408	113.008	147.7635	1135.9253
δ	-40%	0.54	53.2497	113.389	150.5142	943.5544
	-20 %	0.72	53.2455	113.298	150.1615	943.2551
	0 %	0.90	53.2418	113.207	149.8113	942.9614
	20%	1.08	53.2368	113.116	149.4740	942.6567
	40%	1.26	53.2311	113.024	149.1235	942.3700
OC	-40%	60	53.1645	113.039	151.1459	1055.554
	-20 %	80	53.2016	113.124	150.4786	998.2317
	0 %	100	53.2418	113.207	149.8113	942.9614
	20%	120	53.2928	113.295	149.1440	878.6900
	40%	140	53.3514	113.375	148.4756	815.4201
I_p	-40%	0.0096	53.1867	113.123	149.6488	1102.2471
	-20 %	0.0128	53.2145	113.169	149.7748	1018.3574
	0 %	0.0160	53.2418	113.207	149.8113	942.9614
	20%	0.0192	53.2698	113.258	149.9369	870.5656
	40%	0.0224	53.2988	113.305	1549.9832	795.1698
I_e	-40%	0.0072	52.4743	112.540	149.3458	926.5847
	-20 %	0.0096	52.8744	112.898	149.5748	935.7469
	0 %	0.0120	53.2418	113.207	149.8113	942.9614
	20%	0.0144	53.5982	113.624	149.9369	950.2315
	40%	0.0168	54.024	114.036	150.1932	957.5024

Table-3

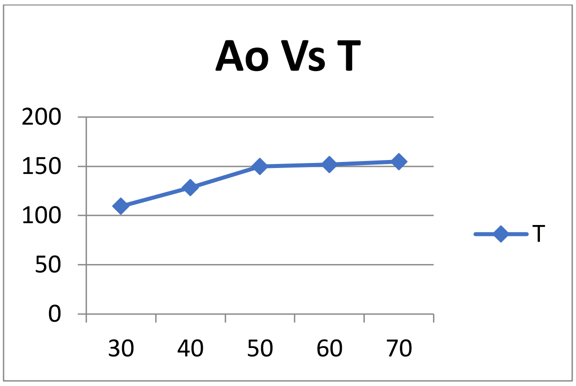
The Change of Directions in t_1, t_2, T and TC Due to change in Parameters.

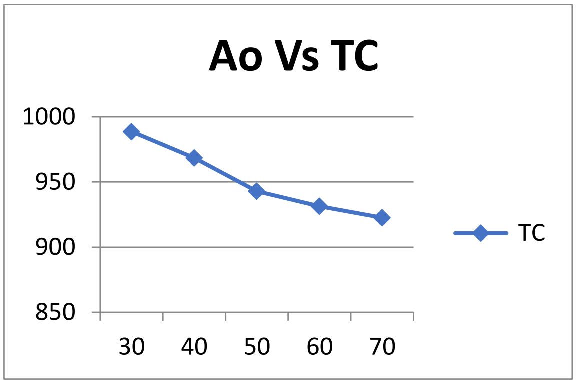
Change of Directions in Parameters	**Effect on t_1, t_2, T and TC**			
	t_1	t_2	T	TC
A_0 ↑	↑	↑	↑	↓
p ↑	↓	↓	↓	↑
C_s ↑	↑	↑	↑	↓
M ↑	↓	↓	↓	↑
δ ↑	↓	↓	↓	↓
OC ↑	↑	↑	↓	↓
I_p ↑	↑	↑	↑	↓
I_e ↑	↑	↑	↑	↑

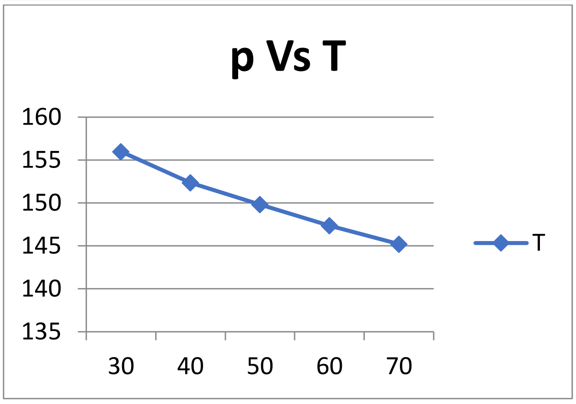
To assess the impact of altering parameters within a range of -40% to +40%, a sensitivity table was constructed. This involved adjusting one variable at a time while maintaining all other parameters constant at their base values, as illustrated in the example. The resulting outcomes shed light on the ramifications of these adjustments on the total cost and time required to fulfil demand.

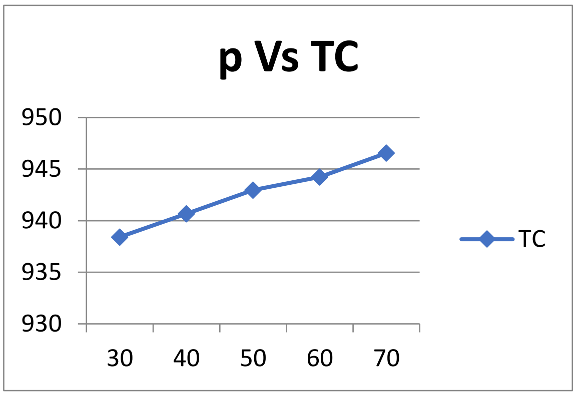
- Increasing advertisement frequency results in higher total cost and time, suggesting a need for cost-benefit analysis in marketing strategies.
- Lowering the selling price decreases total cost and time, and higher selling price increases the profit of business.
- Increasing shortage costs result in higher total cost and time, indicating potential losses due to stockouts.
- Shortening the trade credit period leads to a slight increase in total cost and time, i.e to decrease the total cost and make more profit trade credit period should be higher.
- Changes in the backorder parameter have minimal effects on total cost and time, indicating the stability of the system under varying backorder conditions.
- Decreasing ordering costs lead to lower total cost and time, emphasizing the importance of efficient procurement processes.
- Reducing interest payable decreases total cost and time, indicating the significance of financial management in cost optimization.
- Increasing interest earned leads to lower total cost and time, emphasizing the importance of maximizing returns on idle funds.

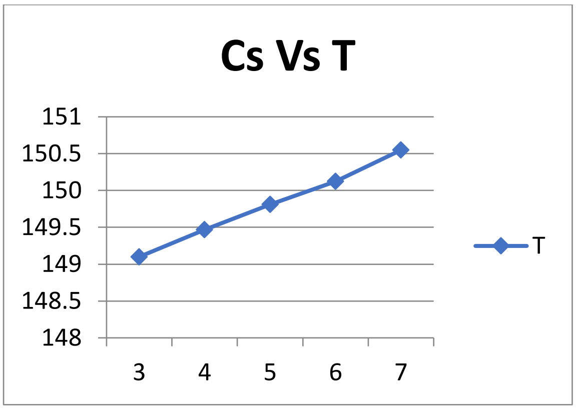
Graphical representation is as follows:

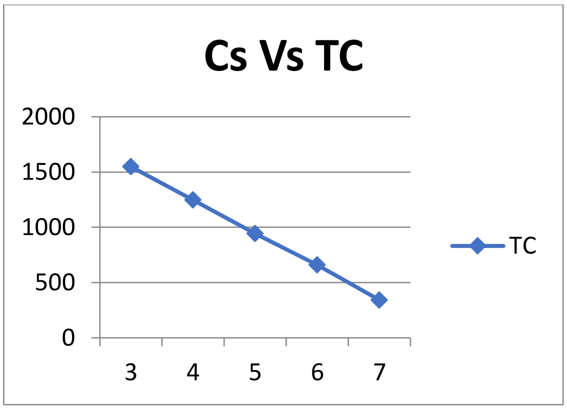

Ao Vs T
200
150
100
50
0
30
40
50
60
70
T

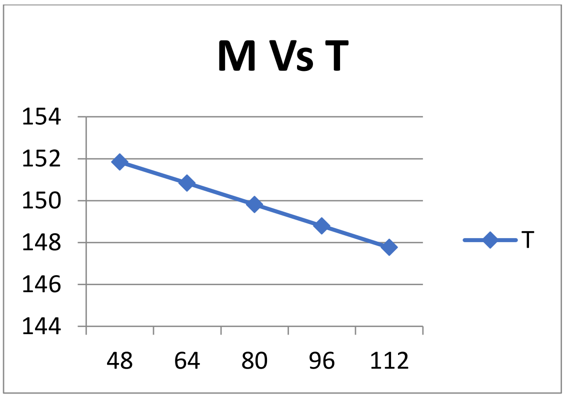

Ao Vs TC
1000
950
900
850
30
40
50
60
70
TC

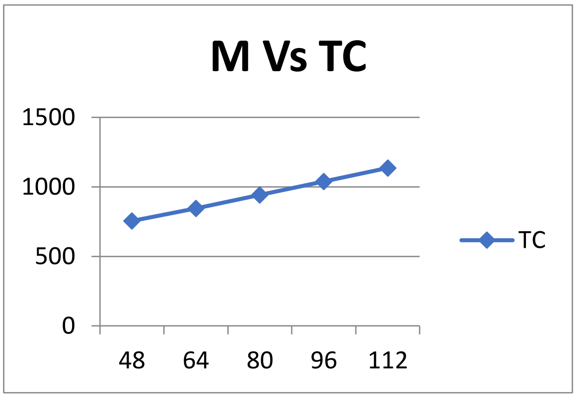

p Vs T
160
155
150
145
140
135
30
40
50
60
70
T

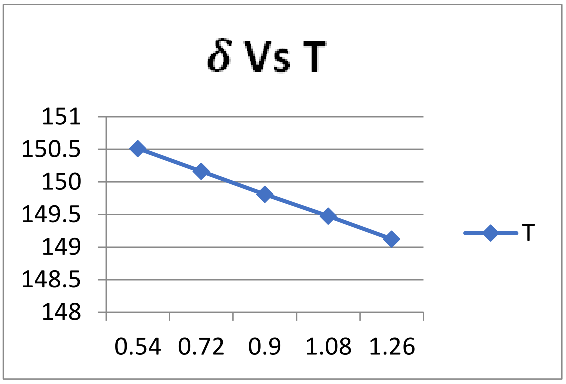

p Vs TC
950
945
940
935
930
30
40
50
60
70
TC

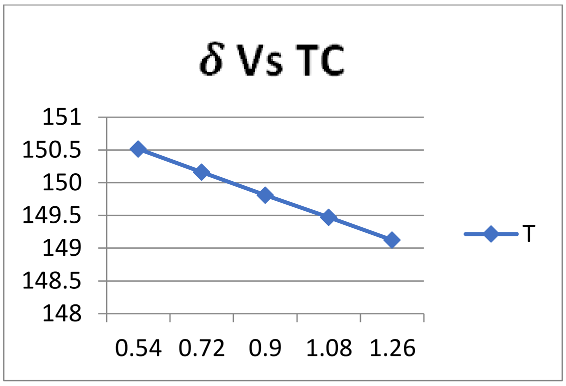

Cs Vs T
151
150.5
150
149.5
149
148.5
148
3
4
5
6
7
T

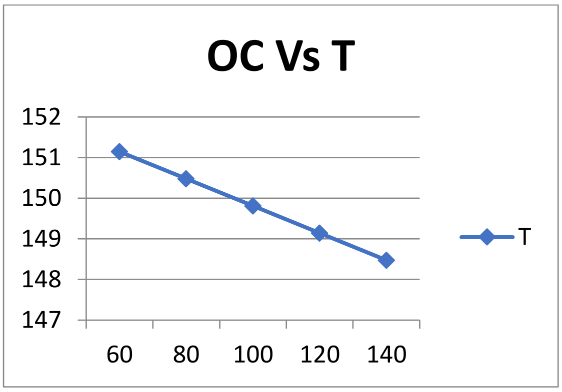

Cs Vs TC
2000
1500
1000
500
0
3
4
5
6
7
TC

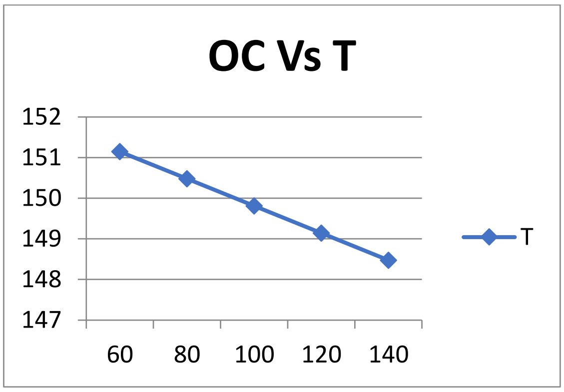

M Vs T
154
152
150
148
146
144
48
64
80
96
112
T

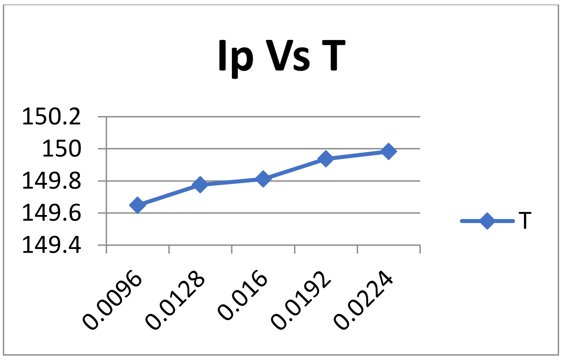

M Vs TC
1500
1000
500
0
48
64
80
96
112
TC

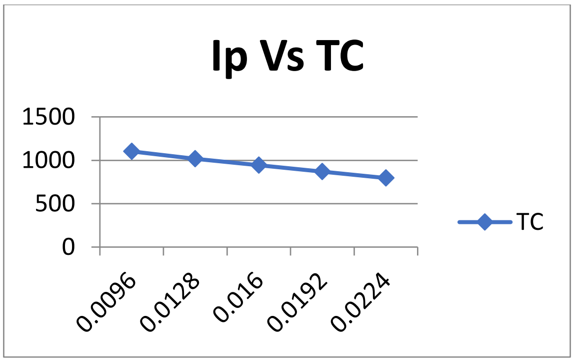


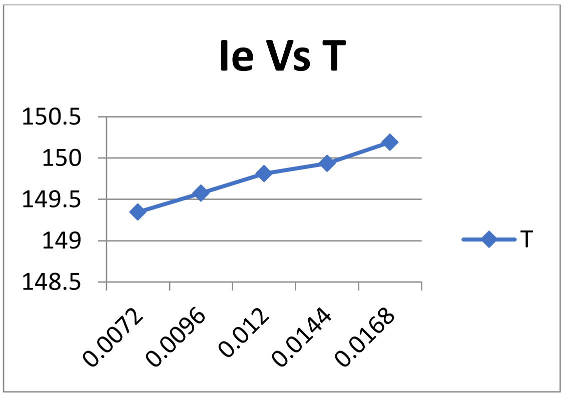


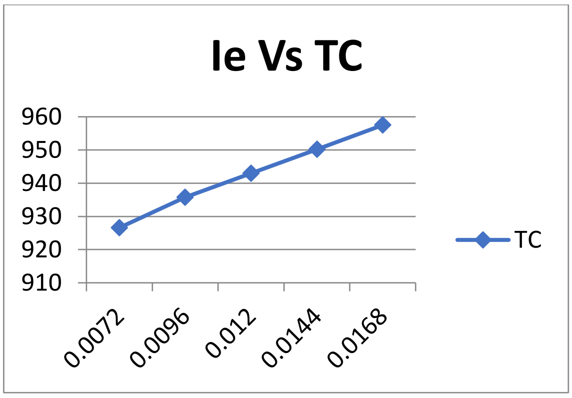












8. Managerial Insights: Managers should carefully evaluate the impact of advertisement frequency and pricing strategies on total cost and time to optimize marketing expenditures. Efficient inventory management practices, including minimizing shortage costs and optimizing trade credit periods, can lead to cost savings and improved operational efficiency. Procurement processes should be streamlined to reduce ordering costs and enhance cost-effectiveness. Financial management strategies, such as optimizing interest payable and maximizing interest earned, are crucial for minimizing total cost and improving profitability.

9. Conclusion: In this presented paper we have formulated an inventory model for instantaneous deteriorating items with two warehouse one is owned and another is rented. Holding cost is a variable and function of time t, which talks about how costs in different warehouses change over time and affect overall expenses. Holding cost of rented warehouse is higher than owned warehouse. Various costs are taken in the inflationary environment. Shortage is allowable and partially backlogged. By studying two different types of warehouses and considering partial backlogging (when orders can't be fulfilled completely) and inflation, the paper gives useful tips on how to make warehouses work better. Demand is taken multivariate function of advertisement and product price which talked about about how advertising, pricing, and promotions affect what people buy and how much, which is important for businesses. Trade credit period $M$is given to the retailer to enhance business. The goal of the research is to help businesses make smart decisions about things like how long to give credit, how much space to use in a warehouse, and how often to restock. It provides examples and analysis to help managers understand what choices might save them money and work best for their operations. By using Mathematica software to check its findings, the research makes sure its advice is trustworthy and can be used in real-world situations. Overall, it offers practical ideas to help businesses manage their warehouses better and save money.

References

[1] Bhunia, A. K., Shaikh, A. A., Sharma, G., &Pareek, S. (2015). A two storage inventory model for deteriorating items with variable demand and partial backlogging. *Journal of Industrial and Production Engineering*, *32*(4), 263-272. https://doi.org/10.1080/21681015.2015.1046508

[2] Braglia, M., Castellano, D., Marrazzini, L., & Song, D. (2019). A continuous review,(Q, r) inventory model for a deteriorating item with random demand and positive lead time. *Computers & Operations Research*, *109*, 102-121. https://doi.org/10.1016/j.cor.2019.04.019

[3] Cárdenas-Barrón, L. E., Shaikh, A. A., Tiwari, S., &Treviño-Garza, G. (2020). An EOQ inventory model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit. *Computers & Industrial Engineering*, *139*, 105557. https://doi.org/10.1016/j.cie.2018.12.004

[4] Chan, C. K., Wong, W. H., Langevin, A., & Lee, Y. C. E. (2017). An integrated production-inventory model for deteriorating items with consideration of optimal production rate and deterioration during delivery. *International Journal of Production Economics*, *189*, 1-13. https://doi.org/10.1016/j.ijpe.2017.04.001

[5] Das, S., Choudhury, M., Mahato, C., &Mahata, G. C. (2023). Dual-channel supply chain inventory model under two-warehouse setting with order volume-linked trade credit, all-units discount and partial backordering. *Soft Computing*, *27*(21), 15817-15852.https://doi.org/10.1007/s00500-023-08824-3

[6] Esmaeili, M., &Nasrabadi, M. (2021). An inventory model for single-vendor multi-retailer supply chain under inflationary conditions and trade credit. *Journal of Industrial and Production Engineering*, *38*(2), 75-88. https://doi.org/10.1080/21681015.2020.1845248

[7] Ghandehari, M., & Karimi-Lenji, M. (2022). An optimal inventory model for a deteriorating item with multivariate demand function. *International Journal of Mathematics in Operational Research*, *22*(1), 1-25. https://doi.org/10.1504/IJMOR.2022.123117

[8] Indrajitsingha, S. K., Samanta, P. N., &Misra, U. K. (2018). A fuzzy inventory model for deteriorating items with stock dependent demand rate. *International Journal of Logistics Systems and Management*, *30*(4), 538-555. https://doi.org/10.1504/IJLSM.2018.093588

[9] Jaggi, C. K., Cárdenas-Barrón, L. E., Tiwari, S., &Shafi, A. (2017). Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments. *Scientia Iranica*, *24*(1), 390-412. https://doi.org/10.24200/sci.2017.4042

[10] Jaggi, C. K., Tiwari, S., &Shafi, A. (2015). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers & Industrial Engineering*, *88*, 378-385. https://doi.org/10.1016/j.cie.2015.07.019

[11] Khakzad, A., &Gholamian, M. R. (2020). The effect of inspection on deterioration rate: An inventory model for deteriorating items with advanced payment. *Journal of cleaner production*, *254*, 120117. https://doi.org/10.1016/j.jclepro.2020.120117

[12] Kumar, A., & Chanda, U. (2018). Two-warehouse inventory model for deteriorating items with demand influenced by innovation criterion in growing technology market. *Journal of Management Analytics*, *5*(3), 198-212. https://doi.org/10.1080/23270012.2018.1462111

[13] Kumar, B. A., Paikray, S. K., Mishra, S., & Routray, S. (2020). A fuzzy inventory model of defective items under the effect of inflation with trade credit financing. In *Recent Advances in Intelligent Information Systems and Applied Mathematics* (pp. 804-821). Springer International Publishing.https://doi.org/10.1007/978-3-030-34152-7_62

[14] Kumar, S., Singh, S. R., Agarwal, S., & Yadav, D. (2023). Joint effect of selling price and promotional efforts on retailer's inventory control policy with trade credit, time-dependent holding cost, and partial backlogging under inflation. *RAIRO-Operations Research*, *57*(3), 1491-1522. https://doi.org/10.1051/ro/2023066

[15] Kumar, V., Rani, M., & Kumar, S. (2022, December). An inventory model for deteriorating items when demand depends on advertisement and stock with partial backlogging. In AIP Conference Proceedings (Vol. 2597, No. 1). AIP Publishing.

[16] Kumar, V., Sharma, A., Kumar, A., & Gupta, C. B. (2020). An inventory model for deteriorating items with multivariate demand and trade credit. Advances in Mathematics: Scientific Journal, 9(9), 7501-7514.

[17] Mahata, G. C., & De, S. K. (2016). An EOQ inventory system of ameliorating items for price dependent demand rate under retailer partial trade credit policy. *Opsearch*, *53*, 889-916.https://doi.org/10.1007/s12597-016-0252-y

[18] Mahato, F., Choudhury, M., &Mahata, G. C. (2023). Inventory models for deteriorating items with fixed lifetime, partial backordering and carbon emissions policies. *Journal of Management Analytics*, *10*(1), 129-190. https://doi.org/10.1080/23270012.2023.2179431

[19] Moradi, S., Gholamian, M. R., &Sepehri, A. (2023). An inventory model for imperfect quality items considering learning effects and partial trade credit policy. *Opsearch*, *60*(1), 276-325.https://doi.org/10.1007/s12597-022-00602-3

[20] Mousavi, S. M., Sadeghi, J., Niaki, S. T. A., &Tavana, M. (2016). A bi-objective inventory optimization model under inflation and discount using tuned Pareto-based algorithms: NSGA-II, NRGA, and MOPSO. *Applied soft computing*, *43*, 57-72. https://doi.org/10.1016/j.asoc.2016.02.014
[21] Pal, D., Manna, A. K., Ali, I., Roy, P., & Shaikh, A. A. (2024). A two-warehouse inventory model with credit policy and inflation effect. *Decision Analytics Journal*, *10*, 100406. https://doi.org/10.1016/j.dajour.2024.100406
[22] Pal, S., Mahapatra, G. S., &Samanta, G. P. (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness. *Economic modelling*, *46*, 334-345. https://doi.org/10.1016/j.econmod.2014.12.031
[23] Palanivel, M., Sundararajan, R., &Uthayakumar, R. (2016). Two-warehouse inventory model with non-instantaneously deteriorating items, stock-dependent demand, shortages and inflation. *Journal of Management Analytics*, *3*(2), 152-173. https://doi.org/10.1080/23307706.2015.1092099
[24] Panda, G. C., Khan, M. A. A., & Shaikh, A. A. (2019). A credit policy approach in a two-warehouse inventory model for deteriorating items with price-and stock-dependent demand under partial backlogging. *Journal of Industrial Engineering International*, *15*(1),147-170. https://doi.org/10.1007/s40092-018-0269-3
[25] Pervin, M., Mahata, G. C., & Kumar Roy, S. (2016). An inventory model with declining demand market for deteriorating items under a trade credit policy. *International Journal of Management Science and Engineering Management*, *11*(4), 243-251. https://doi.org/10.1080/17509653.2015.1081082
[26] Pervin, M., Roy, S. K., & Weber, G. W. (2019). Multi-item deteriorating two-echelon inventory model with price-and stock-dependent demand: A trade-credit policy. *Journal of Industrial & Management Optimization*, *15*(3), 1345.
[27] Qiu, R., Sun, Y., & Sun, M. (2022). A robust optimization approach for multi-product inventory management in a dual-channel warehouse under demand uncertainties. *Omega*, *109*, 102591. https://doi.org/10.1016/j.omega.2021.102591
[28] San-José, L. A., Sicilia, J., Cárdenas-Barrón, L. E., & González-de-la-Rosa, M. (2024). A sustainable inventory model for deteriorating items with power demand and full backlogging under a carbon emission tax. *International Journal of Production Economics*, *268*, 109098. https://doi.org/10.1016/j.ijpe.2023.109098
[29] Sarkar, A., Guchhait, R., & Sarkar, B. (2022). Application of the artificial neural network with multithreading within an inventory model under uncertainty and inflation. *International Journal of Fuzzy Systems*, *24*(5), 2318-2332.https://doi.org/10.1007/s40815-022-01276-1
[30] Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research*, *229*, 677-702.https://doi.org/10.1007/s10479-014-1745-9
[31] Sethy, N. N., Maharana, N., Nayak, A. K., &Swayamjyoti, S. (2020). Inclusion of two-warehouse production prototype for deteriorating inventory items in payments. *International Journal of Inventory Research*, *5*(4), 318-338. https://doi.org/10.1504/IJIR.2020.109785

[32] Shah, B. J., & Shroff, A. (2022). Inventory model for sustainable operations of fixed-life products: Role of trapezoidal demand and two-level trade credit financing. *Journal of Cleaner Production*, *380*, 135093. https://doi.org/10.1016/j.jclepro.2022.135093

[33] Shah, N. H., & Vaghela, C. R. (2018). Imperfect production inventory model for time and effort dependent demand under inflation and maximum reliability. *International Journal of Systems Science: Operations & Logistics*, *5*(1), 60-68. https://doi.org/10.1080/23302674.2016.1229076

[34] Shah, N. H., Keswani, M., Khedlekar, U. K., & Prajapati, N. M. (2024). Non-instantaneous controlled deteriorating inventory model for stock-price-advertisement dependent probabilistic demand under trade credit financing. *Opsearch*, *61*(1), 421-459.https://doi.org/10.1007/s12597-023-00701-9

[35] Sharma, A., Kumar, V., Kumar, A., & Gupta, C. B. (2022, December). An EOQ model for non-instantaneously deteriorating items under permissible delay in payments and inflation. In AIP Conference Proceedings (Vol. 2597, No. 1). AIP Publishing.

[36] Sharma, A., Kumar, V., Singh, S. R., & Gupta, C. B. (2024). Green inventory model with two-warehouse system considering variable holding cost, time dependent demand, carbon emissions and energy consumption. International Journal of Procurement Management, 19(2), 274-296.

[37] Sharma, M. K., & Mandal, D. (2024). Preservation technology investment: a catalyst for sustainable two-warehouse inventory model and SDG achievement. *International Journal of Mathematics in Operational Research*, *28*(1), 18-39. https://doi.org/10.1504/IJMOR.2024.138529

[38] Sharma, S., Kumar, V., & Tyagi, A. (2021). A production inventory model for deteriorating items with effect of price discount under the stock dependent demand. Reliability: Theory & Applications, 16(SI 2 (64)), 213-224.

[39] Singh, S., & Chaudhary, R. (2023). Effect of inflation on EOQ model with multivariate demand and partial backlogging and carbon tax policy. *Journal of Future Sustainability*, *3*(1), 35-58. http://dx.doi.org/10.5267/j.jfs.2022.11.004

[40] Sundararajan, R., Vaithyasubramanian, S., & Nagarajan, A. (2021). Impact of delay in payment, shortage and inflation on an EOQ model with bivariate demand. *Journal of Management Analytics*, *8*(2), 267-294. https://doi.org/10.1080/23270012.2020.1811165

[41] Tiwari, S., Cárdenas-Barrón, L. E., Goh, M., & Shaikh, A. A. (2018). Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. *International Journal of Production Economics*, *200*, 16-36. https://doi.org/10.1016/j.ijpe.2018.03.006

[42] Tiwari, S., Cárdenas-Barrón, L. E., Khanna, A., & Jaggi, C. K. (2016). Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment. *International Journal of Production Economics*, *176*, 154-169. https://doi.org/10.1016/j.ijpe.2016.03.016

[43] Tsao, Y. C., Lee, P. L., Chen, C. H., & Liao, Z. W. (2017). Sustainable newsvendor models under trade credit. *Journal of cleaner production*, *141*, 1478-1491. https://doi.org/10.1016/j.jclepro.2016.09.228

[44] Xu, C., Zhao, D., Min, J., & Hao, J. (2021). An inventory model for non-perishable items with warehouse mode selection and partial backlogging under trapezoidal-type demand. *Journal of the Operational Research Society*, *72*(4), 744-763. https://doi.org/10.1080/01605682.2019.1708822

[45] Yadav, A. S., Bansal, K. K., Kumar, J., & Kumar, S. (2019). Supply chain inventory model for deteriorating item with warehouse & distribution centres under inflation. *International Journal of Engineering and Advanced Technology*, *8*(2), 7-13.

[46] Yadav, A. S., Tyagi, B., Sharma, S., & Swami, A. (2017). Effect of inflation on a two-warehouse inventory model for deteriorating items with time varying demand and shortages. *International Journal of Procurement Management*, *10*(6), 761-775. https://doi.org/10.1504/IJPM.2017.087318