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Mathematical Analysis of Thermodynamic Model to Predict the Engine Parameters

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Abstract: To Predict the thermodynamic performance parameters SI engine two zone thermodynamic simulation model was developed. The Engine performance and thermodynamic parameters was predicted using first order mathematical ordinary differential equations such as peak pressure, burned gas temperature, unburned gas temperature, heat transfer, heat leakage, heat flux and Adiabatic flame temperature. The Fuel is specified by the way of C-H-O-N Values, the equilibrium state of combustion products was determined by olikara and borman method. The model was developed for Air cooled, single cylinder, 4 stroke SI Engine with variable compression ratio of 6-8. Curve fit Co-efficient are used to simulate air and fuel data along with residuals. Thermodynamic parameters are plotted with respect to crank angle. The objective of this work was to study the thermodynamic, performance parameters of SI Engine using various mathematical models.

Keywords: Spark ignition engine; Alternate fuels; simulation; Two Zone; Thermodynamic model

I. INTRODUCTION

The present trend is towards the development of comprehensive 3-D models, which describes the functioning of engines at a very high level of detail and accuracy; however, these require substantial computational power. Also, the need for precise experimental input makes the process significantly complicated and time consuming. There are several instances where theoretical methods, which are based on a limited set of experimental data, are preferred. From these considerations, the need for a simple, fast and accurate engine simulation model is quite evident. A two-zone, Zero-dimensional model was used to simulate the engine operations. The most important assumptions were that, a) The working medium was considered, in general, to be a mixture of 14 species (O_2, N_2, CO_2, H_2O, H_2, OH, NO, CO, O, H, N, Ar, NO_2, HO_2) and fuel vapor. b) All 14 species were considered as ideal gases. And c) The alternate fuels are limited to C-H-O-N species. Hence, this paper aims to combine the benefits of various known models to achieve this goal.

2. Zero-Dimensional Thermodynamic Model Formulation

The combustion chamber is divided into two zones consisting of unburned gas (mixture of fuel, air and residuals) and burned gas (mixture of 10 product species), each under uniform composition. Following assumptions are considered while developing the model. The pressure at any instant is assumed to be uniform throughout the cylinder. At any instant of time during combustion, the cylinder volume is divided into burned and unburned zones by an infinitesimally thin flame-front with a spherical shape. There is no heat transfer between burned and unburned zones. The burned gases are assumed to be in chemical equilibrium during combustion and for the main expansion stroke while the end of expansion stroke the mixture is assumed to be frozen. The zero-dimensional model includes the formulation of mass and energy balance.

II. THERMODYNAMIC MODEL

In the present model, a Zero-dimensional combustion model is employed. The combustion chamber is divided into two zones consisting of unburned gas (mixture of fuel, air and residuals) are burned gas (mixture of 14 product species), each under uniform composition. This model assumes that at any instant of time during the combustion, the cylinder volume is divided into burned and unburned zones byu an infinitesimally thin flame-front with a spherical shape. The burned gases are assumed to be in chemical equilibrium during combustion and form the main expansion stroke, while near the end of expansion stroke the mixture is assumed frozen [7], [10], [8]. A wiebe function specifies the fule burn rate and controls the rate at which mixtures from the unburned zone is converted to the burned zone [10]. Mass and energy conservation relations and equations of state from the principle governing equations. Also considering crank angle as the independent variable, we thus form the base of our thermodynamic model.

A. Mass and Energy Balance

The equation of state for an ideal gas is

$$PV = mRT \tag{1}$$

The rate of change of mass within any open system is the net flux of mass across the system boundaries. Hence for a control volume enclosing the air-fuel mixture, we have

$$\dot{m} = \sum_k \dot{m}_k \tag{2}$$

The first law of thermodynamics to an open system yields the energy equation as

$$\dot{E} = \dot{Q} - W + \sum_k \dot{m}_k \; h_k \tag{3}$$

Equations (2) and (3) can be written as

$$\frac{dm}{d\theta} = \sum_k \frac{dm_k}{d\theta} \tag{4}$$

$$\frac{d(mu)}{d\theta} = \frac{dQ}{d\theta} - p\frac{dV}{d\theta} + \sum_k h_k \frac{d\dot{m}_k}{d\theta} \tag{5}$$

Equation (5) neglects changes of kinetic and potential energy in the control volume.

B. Air and Combustion products Data

Gordon and McBride [8], proposed the following expressions that wee curve-fitted to the tabulated JANAF Thermchemical tables [11].

$$\frac{C_p}{R} = a_1 + a_2T + a_3T^2 + a_4T^3 + a_5T^4 \tag{6}$$

$$\frac{h}{RT} = a_1 + \frac{a_2}{2}T + \frac{a_3}{3}T^2 + \frac{a_4}{4}T^3 + \frac{a_5}{5}T^4 + \frac{a_6}{T} \tag{7}$$

$$\frac{S}{R} = a_1 InT + a_2T + \frac{a_3}{2}T^2 + \frac{a_4}{3}T^3 + \frac{a_5}{4}T^4 + a_7 \tag{8}$$

Where c_p is the specific heat at constant pressure, h is the specific enthalpy and s is the specific entropy.

The coefficients a_1 to a_7 are calculated over two different temperature ranges: 1)300<T<1000 K; and 2)1000<T<5000 K and can be sourced from [8].

The most complete models are based on the assumption that the unburned mixture is frozen in composition and the burned mixture is in equilibrium [10]/. The following are species of interest during combustion: CO_2, H_2O, N_2, O_2, CO, H_2, H, O, OH and NO [7].

C. Fuel Data

Heywood [10] has represented the thermodynamic properties of fuels (in vapor phase) using curves that slightly differ from (6),(7),(8).

$$\frac{C_p}{R} = a_1 + a_2T + a_3T^2 + a_4T^3 + a_5\frac{1}{T^2} \quad (9)$$

$$\frac{h}{RT} = a_1 + \frac{a_2}{2}T + \frac{a_3}{3}T^2 + \frac{a_4}{4}T^3 - a_5\frac{1}{T^2} + \frac{a_6}{T} \quad (10)$$

$$\frac{S}{R} = a_1 InT + a_2T + \frac{a_3}{2}T^2 + \frac{a_4}{3}T^3 - \frac{a_5}{2}\frac{1}{T^2} + a_7 \quad (11)$$

Data for calculating the coefficients can be obtained from [11], [5] or [8]. Reference [9], has discussed about the methods for calculating the thermodynamic properties of user-defined fuels.

The following relation is proposed for deriving the properties like specific heats and enthalpies for various species, with ξ referring to the property and *x* is the burnt mass fraction.

$$\xi_{mixture} = \sum_{k-1}^{N} x_k \, . \, \xi_k \quad (12)$$

D. Equivalence Ratio

When modeling with a single fuel, the equivalence ratio is given by [7]

$$\phi = \left(\frac{[F]}{Air}\right)_{Act.} \Big/ \left(\frac{[F]}{Air}\right)_{St.} \quad (13)$$

Where subscript Act. Refers to actual and St. alcohol with hydrogen fuel, the equivalence ratio changes to [16]

$$\phi = \left(\frac{[F]}{[Air] - \frac{[H]}{\left({}^{H}\!/_{Air}\right)_{St.}}}\right) \Big/ \left(\frac{[F]}{Air}\right)_{St.} \quad (14)$$

E. Practical Chemical Equilibrium

Under the assumption of atmospheric air composition (21%$_v$ Oxygen and 79%$_v$ Nitrogen), and provided $\phi < 3$, the only species that are important because of dissociation are O, H, OH, and NO [7]. The combustion reaction thus becomes

$$\varepsilon\emptyset C_\alpha H_\beta O_\gamma N_\delta + 0.21O_2 + 0.79N_2 \rightarrow x_1CO_2 + X_2H_2O + X_3N_2 + X_4O_2 + X_5CO + X_6H_2 + X_7H + X_8O + X_9OH + X_{10}NO \qquad (15)$$

Where x_1 to x_{10} represent mole fractions of the products.

Olikara and Borman [14] included two additional mole fractions in the products: N and Ar. They also made provision to include user specified quality of air. Depcik [6] modified the Olikara and Borman [14] model by including the following two reactions

$$ON + \frac{1}{2}O_2 \leftrightarrow NO_2 \qquad (16)$$

$$O_2 + \frac{1}{2}H_2 \leftrightarrow HO_2 \qquad (17)$$

The combustion reaction now changes to

$$X_{13}\left[C_nH_mO_lN_k + \frac{n+0.25m-0.51}{\emptyset}\begin{pmatrix} O_2 + Q_{N_2}N_2N_2 + \\ Q_{Ar}Ar + \\ Q_{co_2}CO_2 + \\ Q_{H_2O}H_2O \end{pmatrix}\right] \qquad (18)$$

$$\rightarrow X_1H + X_2O + X_3N + X_4H_2 + X_5OH + X_6CO + X_7NO + X_8O_2 + X_9H_2O + X_{10}CO_2 + X_{11}N_2 + X_{12}Ar + X_{14}NO_2 + X_{15}HO_2$$

Here x and Q denote mole fraction and quality respectively. For clarity, we define: $\psi = \left(\frac{n+0.25m-0.51}{\phi}\right)$

The atom balance for the various elements gives

$$C: x_6 + x_{10} = x_{13}\left(\psi Q_{CO_2} + n\right) \qquad (19)$$

$$H: x_1 + 2x_4 + x_5 + 2x_9 + x_{15} = x_{13}\left(2\psi Q_{H_2O} + m\right) \qquad (20)$$

$$O: x_2 + X_5 + x_6 + X_7 + 2_{X_8} + x_9 + 2X_{10} + 2X_{14} + 2X_{15} = 2X_{13}\left(\psi\left(1 + Q_{CO_2} + 0.5Q_{H_2O}\right) + 0.51\right) \qquad (21)$$

$$N: x_3 + x_7 + 2x_{11} + x_{14} = 2x_{13}\left(\psi Q_{N_2} + 0.5k\right) \qquad (22)$$

$$Ar: x_{12} = x_{13}\psi Q_{Ar} \qquad (23)$$

The constraint that the mole fraction of all the products adds up to unity requires that

$$\sum_{k=1}^{12} x_k + x_{14} + x_{15} = 1 \qquad (24)$$

To solve for the unknowns, we need equations that are provided by the criteria of equilibrium among the products, which are expressed by the following hypothetical relations.

$$\frac{1}{2}H_2 \leftrightarrow H \qquad K_1 = \frac{X_1 \cdot P^{0.5}}{X_4^{0.5}} \qquad (25a)$$

$$\frac{1}{2}0_2 \leftrightarrow O \qquad K_2 = \frac{X_2 \cdot P^{0.5}}{X_8^{0.5}} \qquad (25b)$$

$$\frac{1}{2}H_2 \leftrightarrow N \qquad K_3 = \frac{X_3 \cdot P^{0.5}}{X_{11}^{0.5}} \qquad (25c)$$

$$\frac{1}{2}H_2 + \frac{1}{2}O_2 \leftrightarrow OH \qquad K_5 = \frac{X_5}{X_4^{0.5} \cdot X_8^{0.5}} \qquad (25d)$$

$$\frac{1}{2}O_2 + \frac{1}{2}N_2 \leftrightarrow NO \qquad K_7 = \frac{X_7}{X_8^{0.5} \cdot X_{11}^{0.5}} \qquad (25e)$$

$$H_2 + \frac{1}{2}O_2 \leftrightarrow H_2O \qquad K_9 = \frac{X_9}{X_4 \cdot X_8^{0.5} \cdot P^{0.5}} \qquad (25f)$$

$$CO + \frac{1}{2}O_2 \leftrightarrow NO_2 \qquad K_{14} = \frac{X_{14}}{X_7 \cdot X_8^{0.5} \cdot P^{0.5}} \qquad (25g)$$

$$NO + \frac{1}{2}H_2 \leftrightarrow HO_2 \qquad K_{15} = \frac{X_{15}}{X_{15} \cdot X_4^{0.5} \cdot P^{0.5}} \qquad (25h)$$

$$O_2 + \frac{1}{2}H_2 \leftrightarrow HO_2 \qquad K_{15} = \frac{X_{15}}{X_{15} \cdot X_4^{0.5} \cdot P^{0.5}} \qquad (25i)$$

The equilibrium constant K, for the above are determined from the Gibbs free energy as

$$-\frac{\Delta G^{\circ}}{RT} = InK_p \qquad (26)$$

Where ΔG° is the standard-state Gibbs free energy.

The values of the equilibrium constants can be obtained from [11]. Equations (25a) to (25i) reduce to the following which is then solved using Newton-Raphson iteration scheme.

$$x_1 + 2X_4 + 2X_9 + X_{15} - \frac{\left(2\psi Q_{H_2O} + m\right)}{\psi Q_{CO_2} + n}(X_6 + X_{10}) = 0 \qquad (27)$$

$$x_2 + x_5 + x_6 + x_7 + x_8 + x_9 + 2x_{10} + 2x_{14} + x_{15} - \frac{2\left(\psi\left(1 + Q_{CO_2} + 0.5Q_{H_2O}\right) + 0.51\right)}{\left(\psi Q_{CO_2} + n\right)}(x_6 + x_{10}) = 0 \qquad (28)$$

$$x_3 + x_7 + 2x_{11} + x_{14} - \frac{2\left(\psi Q_{N_2} + 0.5k\right)}{\left(\psi Q_{CO_2} + n\right)}(x_6 + x_{10}) = 0 \quad (29)$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{14} + x_{15} + \frac{\psi Q_{Ar}}{\left(\psi Q_{CO_2} + n\right)}(x_6 + x_{10}) - 1 = 0 \quad (30)$$

F. Frozen Composition

Depcik model [6] is applicable only over the range of 600-4000k. we use the Ferguson model [7] that calculates the species of combustion in the low ranges of temperature from 300-600K. This model assumes that the only species prevalent in the exhaust are: CO_2, H_2O, N_2, Ar, CO, O_2 and H_2. The general reaction is given by

$$\epsilon \emptyset C_a H_\beta O_\gamma N_\delta + (WO_2 + XN_2 + YA_r + ZCO_2) \rightarrow x_1 CO_2 + x_2 H_2 O + x_3 N_2 + x_4 O_2 + x_5 CO + x_6 H_2 + x_7 Ar \quad (31)$$

Equation (31) is formulated for stoichiometric case, i.e. $(\emptyset = 1)$wherein $x_4 = x_5 = x_6 = 0$. For lean, i.e. $(\emptyset \leq 1), x_5 = x_6 = 0$ and for rich, i.e. $(\emptyset \leq 1), x_4 = 0$. Performing atom balance for stoichiometric case and solving for the coefficients give $C: \emptyset \varepsilon \alpha + Z = x_1 \quad \Longrightarrow$

$$x_1 = \emptyset \varepsilon \alpha + Z \quad (32)$$

$$H: \emptyset \varepsilon \beta = 2x_2 \quad \Longrightarrow \quad x_2 = \frac{\emptyset \varepsilon \beta}{2} \quad (33)$$

$$O: \emptyset \varepsilon \gamma + 2W + 2Z = 2x_1 + x_2 \quad (34)$$

$$N: \emptyset \varepsilon \delta + 2X = 2x_3 \quad \Longrightarrow \quad x_3 = \frac{\emptyset \varepsilon \delta + 2X}{2} \quad (35)$$

$$Ar: Y = X_7 \quad \Longrightarrow \quad x_7 = Y \quad (36)$$

$$\varepsilon = \frac{W}{\alpha + \frac{\beta}{4} - \frac{\gamma}{2}} \quad (37)$$

Hence the stoichiometric fuel-air ratio is

$$\left(\frac{[F]}{Air}\right)_{St.} = \frac{\varepsilon(12.011\alpha + 1.008\beta + 16.0\gamma + 14.01\delta)}{31.998W + 28.012X + 38.948Y + 44.009Z} \quad (38)$$

Equation (38) is used in (13) or (14) to calculate the equivalence ratio. Similar formulation can be used for lean and rich mixture cases.

G. Thermal properties

We consider the unburned and burnt mixture zones as separate open systems. Therefore, the specific internal energy, u and specific volume, v is expressed as

$$u = \frac{U}{m} = xu_b + (1 - x)u_u \quad (39)$$

$$v = \frac{V}{m} = xV_b + (1 - x)V_u \quad (40)$$

Subscripts b and u refers to burnt gas and unburned gas respectively.

Going by our assumption that the pressures of burnt and unburned gases are equal, v_b and v_u are functions of T_b, T_u and p . Hence

$$\frac{dv_b}{d\theta} \frac{\partial v_b dT_b}{\partial T_b d\theta} \frac{\partial v_b}{\partial T} \frac{dp}{d\theta} \quad (41)$$

$$\frac{dv_u}{d\theta} = \frac{\partial v_u dT_u}{\partial T_u d\theta} + \frac{\partial v_u}{\partial p} \frac{dp}{d\theta} \quad (42)$$

Back substituting the logarithmic derivatives from Depcik model [6] on (41) and (42), we have

$$\frac{dv_b}{d\theta} = \frac{v_b}{T_b} \frac{\partial Inv_b}{\partial InT_b} \frac{dT_b}{d\theta} + \frac{v_b}{p} \frac{\partial Inv_b}{\partial In\ P} \frac{dp}{d\theta} \quad (43)$$

$$\frac{dv_u}{d\theta} = \frac{v_u}{T_u} \frac{\partial Inv_u}{\partial InT_u} \frac{dT_u}{d\theta} + \frac{v_u}{p} \frac{\partial Inv_u}{\partial In\ P} \frac{dp}{d\theta} \quad (44)$$

Similarly, the internal energies of both the burnt and unburned gases, under the same pressure condition and including the logarithmic derivatives can be written as

$$\frac{dv_b}{d\theta} = \left(cp_b - \frac{pv_b}{T} \frac{\partial Inv_b}{\partial InT_b}\right) \frac{dT_b}{d\theta} - v_b \left(\frac{\partial Inv_b}{\partial InT_b} + \frac{\partial Inv_b}{\partial In\ P}\right) \frac{dp}{d\theta} \quad (45)$$

$$\frac{du_u}{d\theta} = \left(cp_u - \frac{pv_u}{T_u} \frac{\partial Inv_u}{\partial InT_u}\right) \frac{dT_u}{d\theta} - v_u \left(\frac{\partial Inv_u}{\partial InT_u} + \frac{\partial Inv_u}{\partial In\ P}\right) \frac{dp}{d\theta} \quad (46)$$

H. Trapped Mass in Control Volume

The trapped mass in control volume is defined at various periods by [4]

$$for\ \theta_{IVC} \geq \theta \geq -360^{\circ} CA\ (intake)\ \ m = \frac{V(\theta)}{v_u} \quad (47)$$

$$for\ \theta_{EVO} \geq \theta \geq \theta_{IVC}\ (valve - closed)\ \ m = m_{IVC} e^{-c_b(\theta - \theta_{IVC})/\omega} \quad (48)$$

$$for\ 360^{\circ}CA \geq \theta \geq \theta_{EVO}\ (Blowdown\ and\ exhaust)\ m = \frac{V(\theta)}{v_b} \tag{49}$$

IVC refers to intake valve open and EVO refers to exhaust valve open.

The volume of the cylinder at any crank angle instant is given by [7]

$$V(\theta) = V_c\left\{1 + \frac{r-1}{2}\left\{1 - cos\theta + \frac{1}{\varepsilon}[1 - (1 - \epsilon^2 sin^2\theta)^{0.5}]\right\}\right\} \tag{50}$$

V_c is clearance volume, r is the compression ratio and $\epsilon = \frac{\text{Stroke}}{\text{2x length of connecting rod}}$

I. Fuel Burning Rate Model

Many experiments show that the burning rate depends mostly on the combustion chamber shape and the position of the spark plug. The Wiebe function represent the mass fraction burned, x_b versus crank angle and defined as [10]

$$x_b(\theta) = 1 - exp\{-a[(\theta - \theta_0)/\Delta\theta]^{m+1}\} \tag{51}$$

The present simulation uses a=5 and m=2 [10]. This model is capable of representing many combustion chamber shapes with different positions of spark plugs by adjusting *a* and *m*. the constant a is responsible for maximum value of x_b which is unity [12]. Hence *a* may represent the positions and/or number of spark plugs. By varying *m*, he observed the starting point and end point of x_b are independent of m. hence even though, for a particular a, the amount of burned mass fraction x_b is independent of *m*, its rate closely depends on *m*. Hence as *m* increases, x_b is found to decrease and hence *m* may represent combustion chamber shape-factor.

J. Heat Transfer Model

Heat transfer into a thermodynamic system is expressed in terms of heat loss

$$\frac{dQ}{d\theta} = \frac{-Q_1}{\omega} = \frac{\dot{Q}_b - \dot{Q}_u}{\omega} \tag{52}$$

Where

$$\dot{Q}_b = h\sum_{i=h,p,l} A_{bi}(T_b - T_{wi}) \tag{53}$$

$$\dot{Q}_u = h\sum_{i=h,p,l} A_{ui}(T_u - T_{wi}) \tag{54}$$

Here A_{bi} and A_{ui} are the areas of burnt and unburned gases in contact with the combustion chamber component at temperature T_{wi} and subscripts h, p, l refers to cylinder head, piston crown and liner, respectively. We have the following relations [7]

$$A_{bi} = A_i x^{0.5} \tag{55}$$

$$A_{ui} = A_i(1 - x^{0.5}) \tag{56}$$

$$A_h = \frac{\pi b^2}{2} \text{ (Hemispherical cylinder head)} \tag{57}$$

$$A_p = \frac{\pi b^2}{4} \text{ (Flat piston crown)} \tag{58}$$

$$A_l = \frac{4V(\theta)}{b} \text{ (Linear surface area exposed to gases)} \tag{59}$$

Subscript *i* refers to intake.

k. Heat Transfer Correlation

In (53) and (54), h is the instantaneous heat transfer coefficient. Woschni proposed a correlation of the form [18]

$$\text{Nu} = 0.035\ \text{Re}^{0.8} \tag{60}$$

Where *Nu* is Nusselt number and *Re* is Reynolds number. Now, assuming a local average gas velocity in the cylinder, becomes

$$h = 0.82 b^{-0.2}(p.10^{-3}.c)^{0.8} T^{-0.53} \tag{61}$$

whre h is the heat transfer coefficient, p is pressure.

c = 6.18cm (for gas exchange process)

(62)

$$T x T_b (1x) T_u \tag{63}$$

L. ***Blowby Energy Loss*** Enthalpy loss due to blowby is expressed as [7]

$$h_1 = (1 - x^2) h_u + x^2 h_b \tag{64}$$

Which indicates that more leaking is due to the unburned gas compared with the burnt gas in the early stage of combustion.

M. Principle Governing Equations

Differentiating (40) with respect to crank angle and also considering (43) and (44), we have [7]

$$\frac{1}{m}\frac{dV}{d\theta}+\frac{VC_b}{m\omega}=x\frac{v_b}{T_b}\frac{\partial Inv_b}{\partial InT_b}\frac{dT_b}{d\theta}+(1-x)\frac{v_u}{T_u}\frac{\partial Inv_u}{\partial InT_u}\frac{dT_u}{d\theta}+\left[x\frac{v_b}{P}\frac{\partial Inv_b}{\partial InP}+(1-x)\frac{v_u}{P}\frac{\partial Inv_u}{\partial InP}\right]\frac{dP}{d\theta}+(v_b-v_u)\frac{dx}{d\theta} \quad (65)$$

Here C_b is the blowby coefficient and given by $C_b=\dot{m}_1/m \ and \ \dot{m}_1$ is the leakage due to blowby.

Expressing the heat loss of burnt and unburned gases as a function of the rate of change of specific entropy gives

$$-\dot{Q}_b=m\omega xT_b\frac{ds_b}{d\theta} \quad (66)$$

$$-\dot{Q}_u=m\omega(1-x)T_u\frac{ds_b}{d\theta} \quad (67)$$

Where

$$\frac{ds_b}{d\theta}=\left(\frac{c_{pb}}{T_b}\right)\frac{dT_b}{d\theta}-\frac{v_b}{T_b}\frac{\partial Inv_b}{\partial InT_b}\frac{dP}{d\theta} \quad (68)$$

$$\frac{ds_u}{d\theta}=\left(\frac{c_{pu}}{T_u}\right)\frac{dT_u}{d\theta}\ \frac{v_u}{T_u}\frac{\partial Inv_u}{\partial InT_u}\frac{dP}{d\theta} \quad (69)$$

Expressing the heat loss of burnt and unburned gases as a function of the rate of change of specific entropy by combining (53)-(56) and (66)-(69)

$$C_{pb}\frac{dT_b}{d\theta}-v_b\frac{\partial Inv_b}{\partial InT_b}\frac{dP}{d\theta}=\frac{-h\sum_{i=h,p,I}A_{bi}(T_b-T_{wi})}{m\omega} \quad (70)$$

$$C_{pu}\frac{dT_u}{d\theta}-v_u\frac{\partial Inv_u}{\partial InT_u}\frac{dP}{d\theta}=\frac{-h\sum_{i=h,p,I}A_{ui}(T_u-T_{wi})}{m\omega(1-x)} \quad (71)$$

Differentiating Equations (50) and (51) and incorporating with Equation (4), (39), (40), (43)-(46) and (52)-(64) into Equation (5), we have the following relations [7]

$$\frac{dP}{d\theta}=\frac{f_1+f_2+f_3}{f_4+f_5} \quad (72)$$

$$\frac{dT_b}{d\theta}=\frac{-h\sum_{i=h,p,I}A_{bi}(T_b-T_{wi})}{m\omega c_{pb}x}+\frac{v_b}{c_{pb}}\frac{\partial Inv_b}{\partial InT_b}\frac{dP}{d\theta}+\frac{h_u-h_b}{xc_{pb}}\left[\frac{dx}{d\theta}-(x-x^2)\frac{c_b}{\omega}\right] \quad (73)$$

$$\frac{dT_u}{d\theta}\frac{-h\sum_{i=h,p,I}A_{ui}(T_u-T_{wi})}{m\omega c_{pu}(1-x)} \quad (74)$$

Where

$$f_1=\frac{1}{m}\left(\frac{dV}{d\theta}+\frac{VC_b}{\omega}\right) \quad (75)$$

$$f_2 = \frac{h}{m\omega}\left[\begin{array}{l}\frac{v_b}{c_{pb}}\frac{\partial Inv_b}{\partial InT_b}\frac{\sum_{i=h,p,I} A_{bi}(T_b - T_{wi})}{T_b} + \\ \frac{v_u}{c_{pu}}\frac{\partial Inv_u}{\partial InT_u}\frac{\sum_{i=h,p,I} A_{ui}(T_u - T_{wi})}{T_u}\end{array}\right] \quad (76)$$

$$f_3 = (v_b - v_u)\frac{dx}{d\theta} - v_b\frac{\partial Inv_b}{\partial InT_b}\frac{h_u - h_b}{c_{pb}T_b}\left[\frac{dx}{d\theta} - (x - x^2)\frac{C_b}{\omega}\right] \quad (77)$$

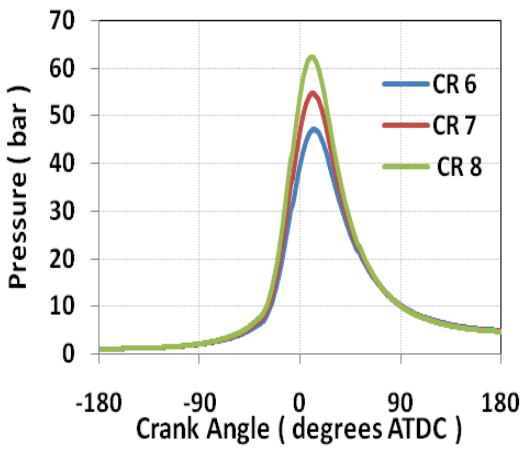
$$f_4 = x\left[\frac{v_b^2}{c_{pb}T_b}\left(\frac{\partial Inv_b}{\partial InT_b}\right)^2 + \frac{v_b}{p}\frac{\partial Inv_b}{\partial In\, p}\right] \quad (78)$$

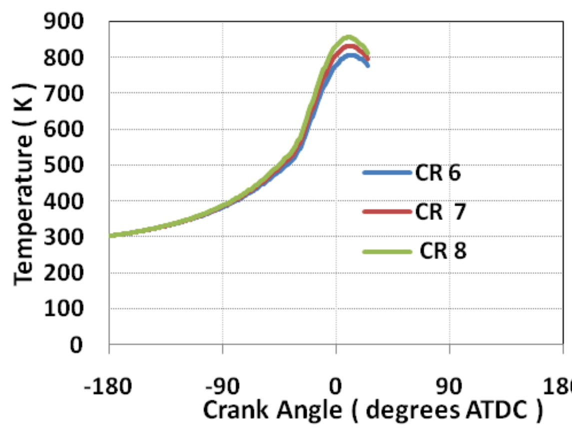
$$f_5 = (1 - x)\left[\frac{v_u^2}{c_{pu}T_u}\left(\frac{\partial Inv_u}{\partial InT_u}\right)^2 + \frac{v_u}{p}\frac{\partial Inv_u}{\partial In\, p}\right] \quad (79)$$

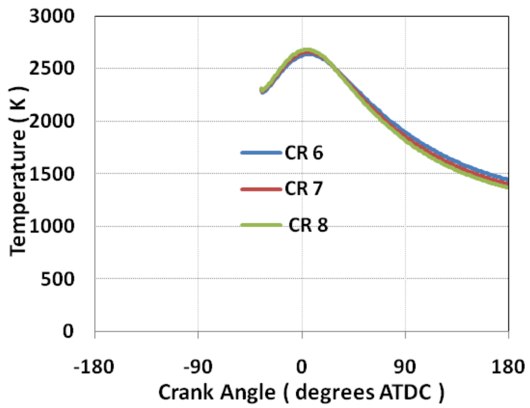
Equations (75)-(79) are functions of θ, P, T_b and T_u and are solved using the Runge-Kutta method.

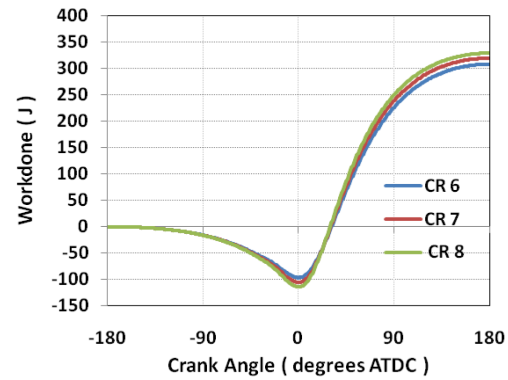
III. SIMULATION RESULTS

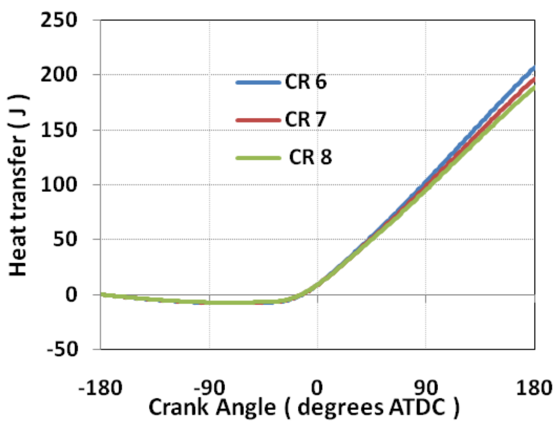
Having formulated the mathematical framework, we simulate pressure, Temperature, Work and Heat Transfer for an single cylinder, four stroke SI Engine running on Gasoline-Ethanol blend. We have assumed a bore of 0.1m, stroke of 0.08m and compression ratio vary from 6 to 8. The engine is assumed to operate at 2000 rpm, with an equivalence ratio of 0.8.

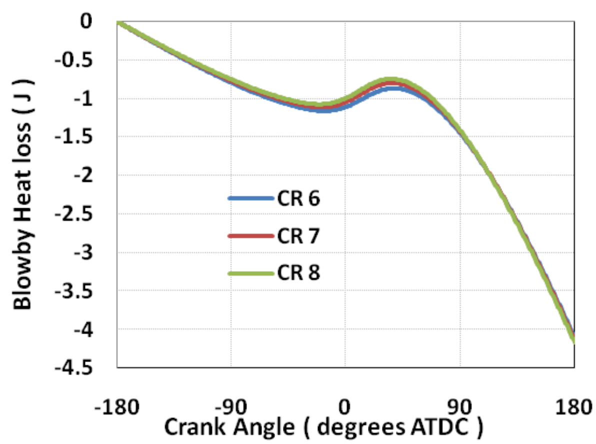


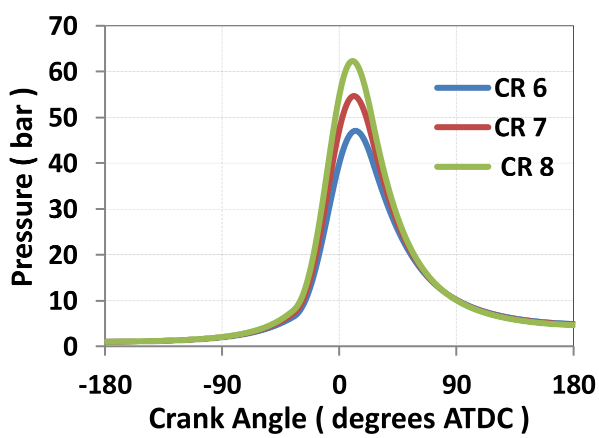



Temperature (K)
Crank Angle (degrees ATDC)
CR 6
CR 7
CR 8
3000
2500
2000
1500
1000
500
0
-180
-90
0
90
180


Workdone (J)
Crank Angle (degrees ATDC)
CR 6
CR 7
CR 8
400
350
300
250
200
150
100
50
0
-50
-100
-150
-180
-90
0
90
180


Heat transfer (J)
Crank Angle (degrees ATDC)
CR 6
CR 7
CR 8
250
200
150
100
50
0
-50
-180
-90
0
90
180


Blowby Heat loss (J)
Crank Angle (degrees ATDC)
CR 6
CR 7
CR 8
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-180
-90
0
90
180


Pressure (bar)
Crank Angle (degrees ATDC)
CR 6
CR 7
CR 8
70
60
50
40
30
20
10
0
-180
-90
0
90
180

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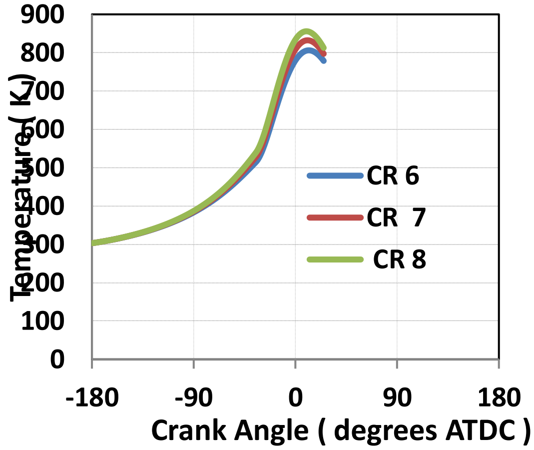

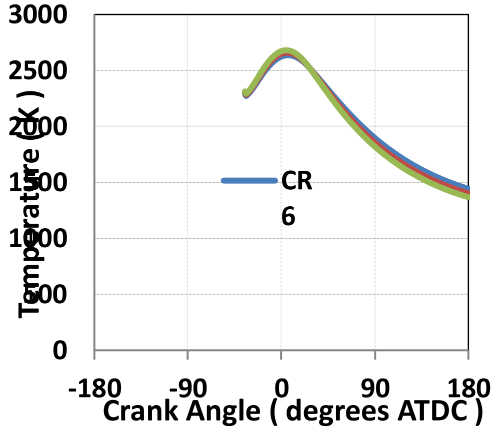

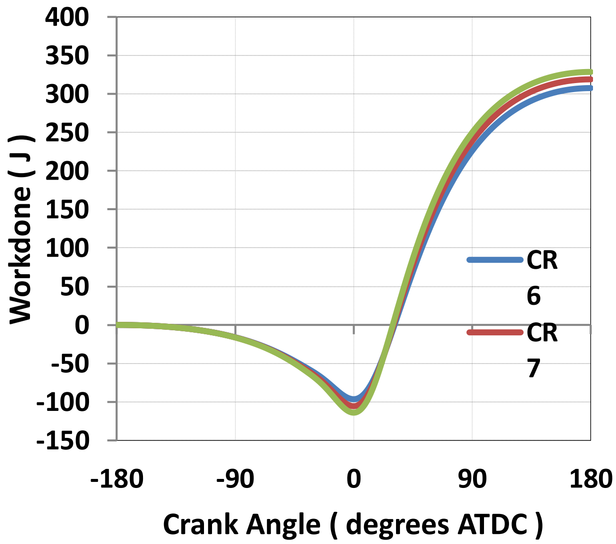

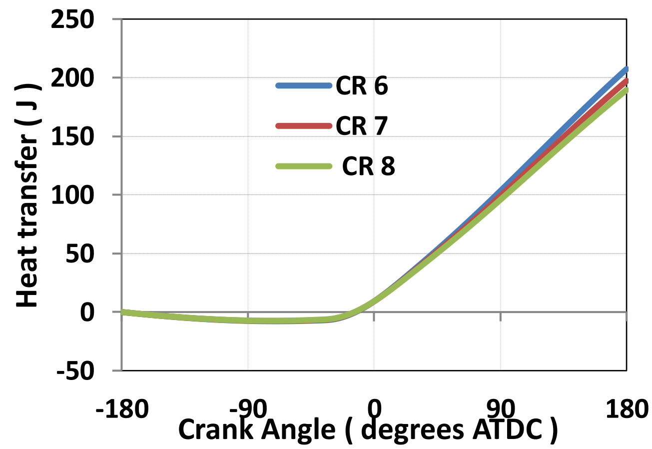

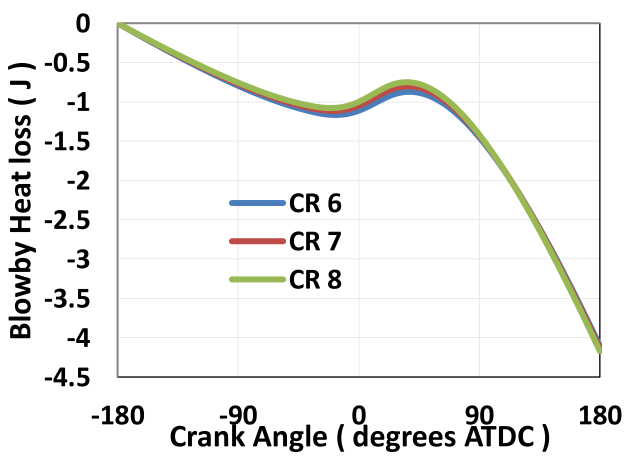

IV. CONCLUSION

The present work achieves its goal by being a simple, fast and accurate engine simulation model. The results obtained can be used as a first-degree approximation and is useful in numerous engineering applications including general design predictions. The model can predict an array of thermodynamic and combustion parameters and easily adapt to any combustion chamber shape. Due to its simplicity and computational efficiency, the model can also be used as a preliminary test on a wide range of alternate hydrocarbon fuels.

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