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Fermatean Fuzzy Transportation Problems for Finding an Optimal Solution based on a Score function with Decision Making

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Abstract:

A key domain of operational research is the transportation problem(TP), which is associate with moving items that are to start with various origins to their final purpose while keeping the overall transportation expenses to a low level. In this research, we provide a new technique for dealing with fermatean fuzzy transportation problems(FFTPs) based on the assumption that the decision-maker is unsure about the precise numerical values of transportation costs(TCs), demand and supply of commodities. Fermatean fuzzy numbers(FFNs) are used to illustrate the costs of commodities distribution, demand and transportation in the proposed strategy. To demonstrate the potential and efficacy of our study, numerical calculations were performed using our proposed technique and compared with other current methodologies.

Keywords: Fermatean Fuzzy Costs(FFC), Fermatean Fuzzy Transportation Problem (FTP), Score Function , Optimal Solution .

1. Introduction

The transportation problem (TP), which has correctly obtained considerable focus in the literature, is a substantial network-well organized linear programming (LP) problem that commonly arises in various settings. The primary obstacle is to ascertain the most economical cost of transportation for a product meeting demand at destinations while optimizing resources at origins. Transportation challenges can be influenced by a variety of factors, incorporating arrangement of tasks, production, investment, factory placement, management of stock, employee scheduling, and many others. Transportation problems are normally settled under the presumptions that the values and supply expense and request are designated precisely or in a transparent setting. But the responsible persons frequently don't have a thorough understanding of the numerical factors related to the transportation issue. The fuzzy TP easily emerges in these circumstances because the concomitant coefficients or components that state the problem can be explained as fuzzy sets. Hitchcock was the first to formulate the fundamental transportation problem [11]. The TPs problems can be tackled using the simplex method by modelling them as a typical linear programming problem. However, it was originally noted that the TPs very peculiar mathematical structure may allow the simplex method to be extremely efficient in terms of evaluating the required simplex method information.

The North-West Corner Rule (NWCR), the Matrix Minima Method (MMM), and Vogel's Approximation Method can all be used to find an Initial Basic Feasible Solution (IBFS) for the TP. To find a preliminary, straightforward and feasible answer, apply Vogel's Approximation Method (VAM) [18,25]. Others had to work hard to come up with modifications to Vogel's Approximation, which was utilized to generate preliminary solutions to TPs. Because of confusing decisions, a lack of evidence and other factors, there are numerous diverse situations in real life. Sometimes obtaining pertinent, precise data for the cost parameter is impossible. A probability distribution's chosen random variables don't always appropriately represent this kind of ambiguous data. [2,4,5,8,10,12-14,16,25] are obtaining initial basic feasible solution using a precise data, but precise data are not appearing natural way. The idea of fuzzy sets was put up by Zadeh [26] and it is an essential tool for conveying uncertainty. Chanas and Kuchta [6] proposed the best solution to the TP using fuzzy coefficients written as fuzzy integers and devised an algorithm to get the best answer. [27] Zimmerman showed that the solutions obtained by fuzzy LP are always optimum and efficient. Pandian and Natarajan [17] created a unique approach called the fuzzy zero point method to discover a fuzzy optimum solution for an FTP that takes into account TC, supply and demand.[3,24] proposed a new approach for solving fuzzy TP using generalized trapezoidal FNs.

Intuitionistic Fuzzy Sets (IFS), a generalization of the fuzzy set notion, were first suggested by Atanassov [1]. Using a single step approach, Nagoorgani and Poonalagu [9] offered a solution to the intuitionistic fuzzy LP problem. [7] Presented to address the challenge of intuitionistic FLP. TCs are uncertain, yet demand and supply are represented by triangular intuitionistic fuzzy numbers (TIFNs), according to Nagoor Gani and Abbas [10], who proposed a novel average approach for dealing with intuitionistic fuzzy transportation issues. Several authors have recently focused their attention on intuitionistic FTP [3-5, 7].Finding the least possible resolution for the uneven intuitionistic FTP is the tactical approach primary goal of this effort. In order to solve disproportionate intuitionistic FTP, a new strategy is showcased in this study that is both direct and easy to understand. The modification supply chain can be substituted with this. This method does not necessitate route tracking. With appropriate numerical examples, the algorithm of the methodology is described in profoundness. By using example problems, additional comparisons between the new approach and other current algorithms are advanced.

2. Objectives

The goal of this research is to develop a new method to solve Fermatean Fuzzy Transportation Problems (FFTPs), which are a key focus of operational research. Transportation problems deal with moving items from several starting points to their destinations while keeping costs as low as possible. However, in real-world scenarios, decision-makers often face uncertainty about the exact values of transportation costs, demand, and supply. Traditional methods struggle to handle this uncertainty effectively, which is where our work comes in.

This study introduces a new approach using Fermatean Fuzzy Numbers (FFNs). These numbers are better suited for dealing with uncertainty compared to other fuzzy systems, making them ideal for representing transportation costs, demand and supply. By using FFNs, our method provides a more flexible and accurate way to model transportation problems under uncertain conditions. To show how effective this new approach is, we applied it to real-world-like numerical examples. We also compared the results with those obtained using other existing methods. These comparisons help highlight the strengths of our method, such as its ability to deliver more accurate, adaptable and efficient solutions. The ultimate aim is to give decision-makers a powerful tool to manage transportation problems in uncertain environments. This tool can help optimize costs while ensuring reliable and practical solutions for real-world transportation challenges.

3. Methods

3.1. Preliminaries [9]

In this part, a few fundamental concepts of fermatean fuzzy sets (FFSs) are presented.

3.1.1. Let X be a universal set. A FFS is an object from the form $\tilde{F}=\{\langle x,\alpha_{\tilde{F}}(x),\beta_{\tilde{F}}(x)\rangle : x\in X\}$ where $\alpha_{\tilde{F}}(x):X\to[0,1]$ and $\beta_{\tilde{F}}(x):X\to[0,1]$ which satisfies the relation $0\le(\alpha_{\tilde{F}}(x))^3+(\beta_{\tilde{F}}(x))^3\le 1,\forall x\in X$. The number $\alpha_{\tilde{F}}(x)$ and $\beta_{\tilde{F}}(x)$ are the degree of membership and non-membership of the element $x\in X$ in the FFS $\tilde{F}$.

For any FFS $\tilde{F}$ and $x\in X$, the degree of indeterminacy is represented by $\pi_F(x)=\sqrt[3]{1-(\alpha_{\tilde{F}}(x))^3-(\beta_{\tilde{F}}(x))^3}$. It is to be noted that, for simplicity, we shall denote the object $\tilde{F}=\langle\alpha_{\tilde{F}},\beta_{\tilde{F}}\rangle$ instead of $\tilde{F}=\{\langle x,\alpha_{\tilde{F}}(x),\beta_{\tilde{F}}(x)\rangle : x\in X\}$.

3.1.2. Arithmetic operations [21,22]

Let $\tilde{F}_1=\left(a_{\ddot{F}_1},b_{\ddot{F}_2}\right)$ *and* $\tilde{F}_2=\left(a_{\ddot{F}_1},b_{\ddot{F}_2}\right)$, be two FFNs. Then the basic arithmetical operations of two FFS are defined as follows:

(i) Addition $\tilde{F}_1\oplus\tilde{F}_2=\left(\sqrt[3]{(a\tilde{F}_1)^3+(a\tilde{F}_2)^3}\right)-(a\tilde{F}_1)^3(a\tilde{F}_2)^3, b\tilde{F}_1 b\tilde{F}_2$

(ii) Multiplication $\tilde{F}_1\otimes\tilde{F}_2=\left(b\tilde{F}_1 b\tilde{F}_2,\sqrt[3]{(a\tilde{F}_1)^3+(a\tilde{F}_2)^3}\right)-(a\tilde{F}_1)^3(a\tilde{F}_2)^3$

(iii) Scalar Multiplication $\lambda\bullet\tilde{F}=\left(\sqrt[3]{1-(1-(a\tilde{F})^3)^{\lambda}},(b\tilde{F})^{\lambda}\right)$

3.1.3. Score function [19]

$$Type1\quad S_{1F}(\tilde{F})=\frac{1}{2}\left(1+a_{\ddot{F}}^3-b_{\ddot{F}}^3\right)$$

$$Type2\quad S_{2F}(\tilde{F})=\frac{1}{3}\left(1+2a_{\ddot{F}}^3-b_{\ddot{F}}^3\right)$$

$$Type3\quad S_{3F}(\tilde{F})=\frac{1}{2}\left(1+a_{\ddot{F}}^2-b_{\ddot{F}}^2\right)[a_{\ddot{F}}-b_{\ddot{F}}]$$

3.2. Mathematical Approach to FFTPs.

Traditional TP presume that all parameters have fixed or accurate values. However, in fact, all of the elements of transportation concerns may not be set or accurate owing to unavoidable changes in the economic environment. Several scholars [12-13, 21-22] have developed and solved FFTPs by treating transportation costs, supply and demand as ambiguous numbers. FTP, in which a decision maker is confused about the precise values of transportation characteristics such as cost, supply and demand can be described as follows

Minimize $\tilde{z}_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\tilde{c}_{ij}$

Subject to $\sum_{i=1}^{m} x_{ij} = \tilde{a}_i$ for i=1,2,3,……,m

$\sum_{j=1}^{n} x_{ij} = \tilde{b}_i$ for j=1,2,3,……,n

$x_{ij} \geq 0$ for i=1,2,3,……,m and for j=1,2,3,……,n (1)

where

$\tilde{a}_i$ Total available supply quantity.

$\tilde{b}_i$ The current requirement quantity.

x_{ij} The quantity of unit commodities moved from warehouses to outlet.

c_{ij} The shifting cost of one unit commodity.

$\tilde{z}_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\tilde{c}_{ij}$ (The total amount cost of transportation)

3.3. Theoretical representation of TP Fermat's environment of uncertainty

Now if we replace the fuzzy parameters $\tilde{c}_{ij}$, $\tilde{a}_i$ and $\tilde{b}_j$ of (1) by FF parameters then the mathematical model (1) reduces as follows

$$Minmize\left\langle \alpha_{z_0}, \beta_{\tilde{z}_0} \right\rangle = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\Theta\left\langle \alpha_{\tilde{c}_{ij}}, \beta_{\tilde{c}_{ij}} \right\rangle$$

Subject to $\sum_{j=1}^{n} x_{ij} = \left\langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i} \right\rangle$ for j = 1,2,3,…….,n

$\sum_{i=1}^{m} x_{ij} = \left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j} \right\rangle$ for i = 1,2,3,…….,m (2)

Where

$0 \leq (\alpha_{\tilde{z}_0})^3 + (\beta_{\tilde{z}_0})^3 \leq 1$

$0 \leq (\alpha_{\tilde{a}_i})^3 + (\beta_{\tilde{a}_i})^3 \leq 1$ for i = 1,2,3,…….,m

$$0 \le (\alpha_{\tilde{b}_j})^3 + (\beta_{\tilde{b}_j})^3 \le 1 \qquad \text{for } j = 1,2,3,\ldots\ldots,n$$

$$x_{ij} \ge 0, 0 \le (\alpha_{\tilde{c}_{ij}})^3 + (\beta_{\tilde{c}_{ij}})^3 \le 1 \qquad \text{for } i = 1,2,3,\ldots\ldots,m \text{ and } j = 1,2,3,\ldots\ldots,n$$

Now, problem (2) is a mathematical model of TP in FFE. It is to be noted that if $\sum_{i=1}^{m} \oplus \left\langle \alpha_{\tilde{a}_i}, \beta_{\tilde{a}_i} \right\rangle = \sum_{j=1}^{n} \oplus \left\langle \alpha_{\tilde{b}_j}, \beta_{\tilde{b}_j} \right\rangle$ then the FFTP is balanced, otherwise it is called a unbalanced FFTP. The symbol $\sum \oplus$ is denoted as summation in terms of fermatean addition sense.

3.4. Proposed Model for FFTPs

To determine the first basic viable solution and the optimal solution for the FFTP.

3.4.1. FF Vogel's Approximation Method (FFVAM)

The FFVAM is a renowned computational solution for TPs. In this post, we use FFVAM to solve the TP. The flowchart for FFVAM is presented in Figure 1.

FFVAM 1: In this problem, the cell cost, demand and supply are represented as FFNs and the selected TP is imbalanced before being converted into a balanced one with a fake row or column of FF zero costs.

FFVAM 2: Compute the FF cost of the penalty for the supplied transportation table, making a distinction between the least FF cost and the next lowest FF cost in each column and row.

FFVAM 3: choose the greatest penalty amount from all column and row differences discovered in FFVAM 2 and then choose the appropriate row or column.

FFVAM 4: For FFVAM, pick the lowest FF cost in the row or column indicated in FFVAM 3.

FFVAM 5: This FFVAM assigns the smallest FF value appropriate to the row and column values selected in FFVAM 4.

FFVAM 6: According to this cell, this column or row has been altered, or eliminated.

FFVAM 7: The same process used in FFVAM 2 through FFVAM 6 was used to the remaining unallocated cells until all requests and supplies were adjusted.

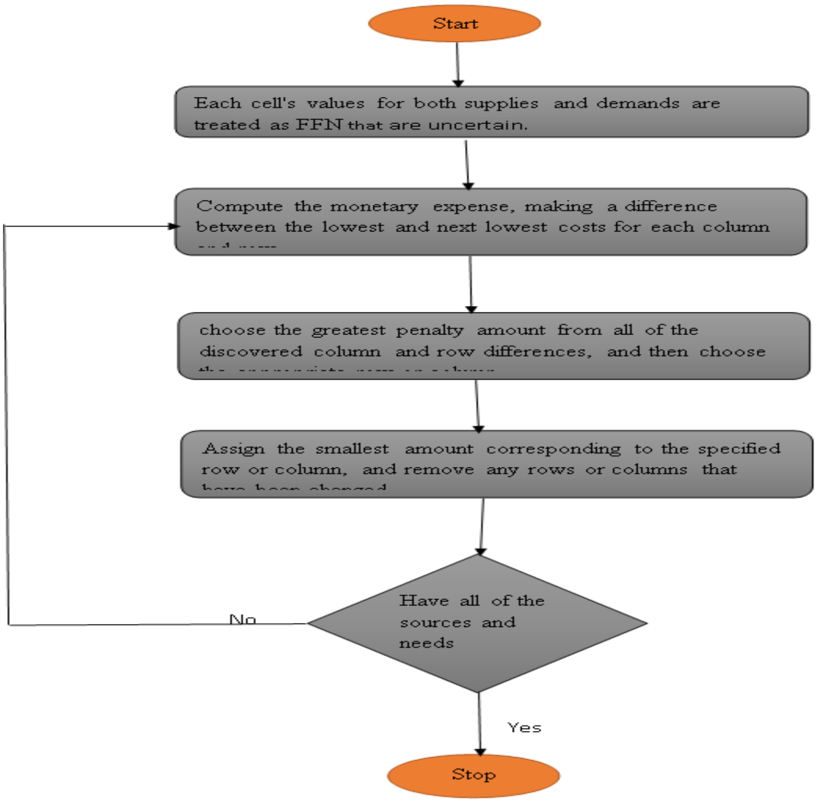

Fig_1: Flowchart of Fermatean Fuzzy Vogel's Approximation Method

3.4.2. FF Modified Distribution Method (FFMODI) [18,23]

The ideal result tests are always conducted based on the first basic viable solution of a TP, where the value of m + n − 1 represents the number of non-negative vacant cells, n is the number of columns and m is the number of rows. All allotted cells remain in a distinct status. This strategy is always used to achieve greater results than the first basic workable answer. The flowchart for the FFMODI is given in Figure 2.

FFMODI 1: Determine u_i and v_j using the equation $u_i + v_j = c_{ij}$ for each occupied cell.

FFMODI 2: Consider the value of u_i or v_j equal to 0 to any row or column, respectively, with having maximum no of allocation. If it is more than one then choose any one of them arbitrarily and calculate rest of all u_i and v_j for all rows and columns, respectively.

FFMODI 3: Use the equation $\tilde{z}_{ij} = c_{ij} - (u_i + v_j)$ to get the value of each unused cell.

Scenario I: Have a unique solution and it is an optimal solution if all $\tilde{z}_{ij} > 0$.

Scenario II: Have an alternative solution and it is optimal if all $\tilde{z}_{ij} \geq 0$.

Scenario III: It has no optimal solution, if $\tilde{z}_{ij} < 0$ for that type, then the case consider the next step for the optimal solution.

FFMODI 4: If scenario III happens, choose the most negative value of $\tilde{z}_{ij}$ and form a close loop with occupied cell and assign "+" and "–" sign alternately. Find out the minimum allocation with a negative sign. It will be added to the positive allocation cell and subtract from the negative allocation cell.

FFMODI 5: Determine and redirect the occupied cell to obtain a new set of essential possibilities.

FFMODI 6: Carry out FFMODI 1 through FFMODI 5 until an optimum solution is obtained.

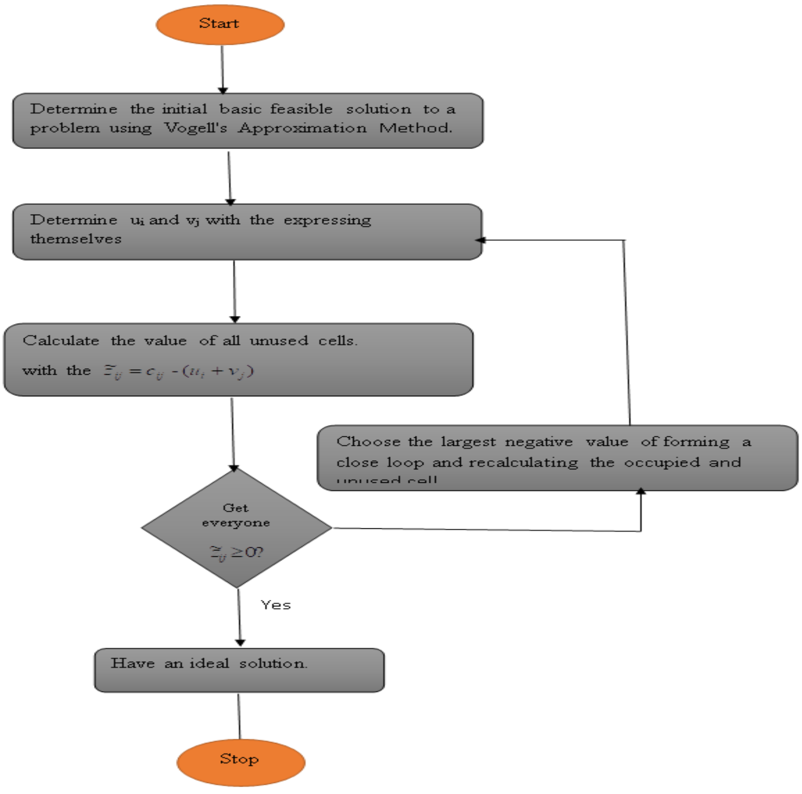

Fig_2: Flowchar of Fermatean Fuzzy MODI

3.4.3. FF Single Zero Method (FFSZM)

We have a reliable strategy for quickly locating the best solution when using the adopted approach for FFTP. The stages listed below can be used to summarize the FFSZM.

FFSZM 1. Initialization.

Considering the data provided, create the fuzzy transportation table. Create a balanced solution to an unbalanced, irrational or FTP.

FFSZM 2. Create Reduce table.

(a) Reduce rows one by one.

The fuzzy cost table is provided; find the largest element in each row and subtract it from each element in that row.

(b) Reduce columns one by one.

Find the largest element in each column of the reduced matrix you obtained from FFSZM 2(a) and then remove it from each element in that column.

FFSZM 3. Create an allocation opportunity costs.

(a). Determine the cost matrix's greatest unit transportation cost using FFSZM 2 or FFSZM 4.

(b). If there are cells that correspond to the ith row and jth column, choose a row single FFZ and/or a column single FFZ cell for allotment.

(c). Give the cell the absolute bare minimum of resources, adjust supply and demand, and remove the fulfilled row or column.

(d). If the i^{th} row and j^{th} column do not contain a row single FFZ or a column single FFZ cell, choose the next greatest unit transportation cost and proceed as described in FFSZM 3(a) through FFSZM 3(d).

FFSZM 4. Update the opportunity costs.

(a). After completing FFSZM 3, make sure that each row and column has at least one FFZ. Go to FFSZM 4 (b). If not, proceed to FFSZM 5.

(b). Find the lowest element and subtract it from each row (column) if there isn't a single FFZ for each.

FFSZM 5. Selection of the allocation cell.

Repeat FFSZM 3 and 4 as necessary to meet all demand at all destinations and all supply at all sources.

FFSZM 6. Calculate the fuzzy cost table at the starting point, calculate the overall fuzzy transport costs for the realistic allocation.

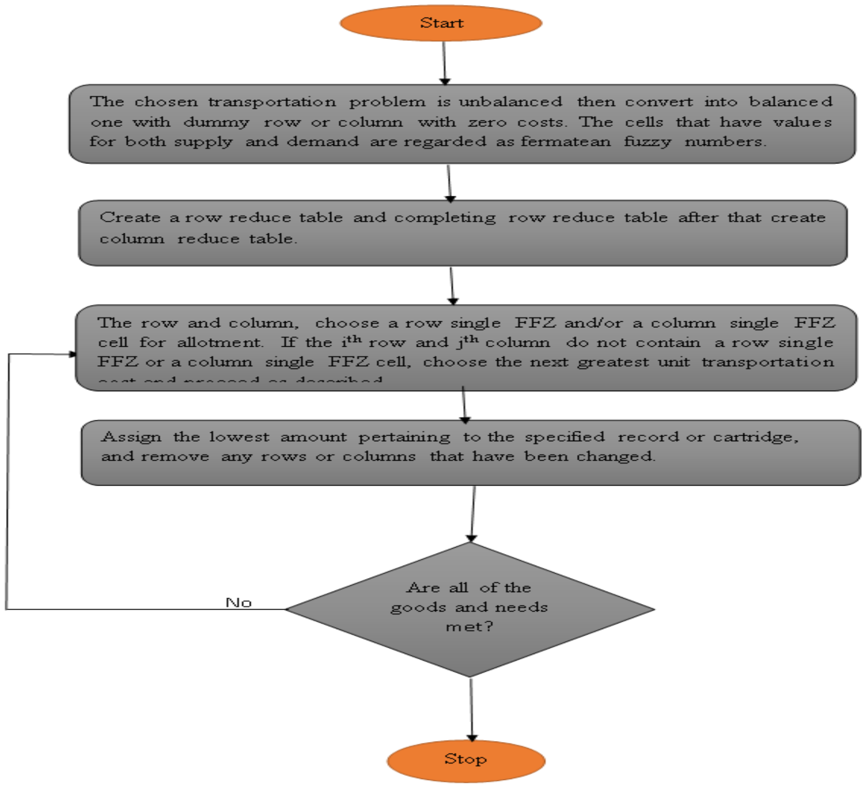

Fig_3: Flow diagram of of FFSZM

3.5. Analytical Example

3.5.1. Problem_1

Table 1 lists the three sources of product supply. Fermatean fuzzy numbers indicate the product's demand and supply from each source to each destination, as well as the demand at each of the three destinations, D1, D2, D3 and D4. To keep total shipping costs as low as possible, determine the best fuzzy transportation strategy for the merchandise.

Table_1 Data inputs for TP with Fermatean fuzzy rates, consumer demand, and accessibility

	D_1	**D_2**	**D3**	**D4**	**Supply (a_i)**
S_1	(0.1,0.9)	(0.2,0..8)	(0.1,0.8)	(0.1,0.9)	(0.7,0.1)
S_2	(0.01,0.99)	(0.3,0.9)	(0.3,0.8)	(0.1,0.7)	(0.8,0.1)
S_3	(0.1,0.8)	(0.4,0.8)	(0.4,0.9)	(0.2,0.9)	(0.9,0.1)
Demand (b_j)	(0.4,0.7)	(0.7,0.3)	(0.8,0.1)	(0.6,0.4)	

Table_2 Evaluation of score when $S_{1F}(\tilde{F}) = \frac{1}{2}\left(1 + a_{\tilde{F}}^2 - b_{\tilde{F}}^2\right)$ [19]

	D_1	**D_2**	**D3**	**D4**	**Supply (a_i)**
S_1	0.1000	0.2000	0.1850	0.1000	0.7400
S_2	0.0100	0.1400	0.2250	0.2600	0.8150.
S_3	0.8150	0.2600	0.1750	0.1150	0.9000
Demand (b_j)	0.3350	0.7000	0.8150	0.6050	

Since $\sum a_i = \sum b_j$ =2.4550 so the chosen problem is balanced FFTP

Table_3 Evaluation of score when FFNWCR

	D_1	**D_2**	**D3**	**D4**	**Supply (a_i)**
S_1	0.1000 **0.3350**	0.2000 **0.4050**	0.1850	0.1000	*
S_2	0.0100	0.1400 **0.2950**	0.2250 **0.5200**	0.2600	*
S_3	0.8150	0.2600	0.1750 **0.2950**	0.1150 **0.6050**	*
Demand (b_j)	*	*	*	*	

Minimum cost of FFNWCR is 0.3940

Table_4 Evaluation of score when FFLCM

	D_1	**D_2**	**D3**	**D4**	**Supply (a_i)**
S_1	0.1000	0.2000 **0.3150**	0.1850	0.1000 **0.6050**	*
S_2	0.0100 **0.3350**	0.1400 **0.4800**	0.2250	0.2600	*
S_3	0.8150	0.2600 **0/0850**	0.1750 **0.8150**	0.1150	*
Demand (b_j)	*	*	*	*	

Minimum cost of FFLCM is 0.3328

Table_5 Evaluation of score when FFVAM

	D_1	D_2	D3	D4	Supply (a_i)
S_1	0.1000	0.2000 **0.3150**	0.1850	0.1000 **0.6050**	*
S_2	0.0100	0.1400 **0.4800**	0.2250	0.2600	*
S_3	0.8150 **0.3350**	0.2600 **0.0850**	0.1750 **0.8150**	0.1150	*
Demand (b_j)	*	*	*	*	

Minimum cost of FFVAM is 0.3228

Table_6 Evaluation of score when FFMODI

	D_1	D_2	D3	D4	Supply (a_i)
S_1	0.1000	0.2000 **0.2200**	0.1850	0.1000 **0.5200**	*
S_2	0.0100 **0.3350**	0.1400 **0.4800**	0.2250	0.2600	*
S_3	0.8150	0.2600	0.1750 **0.8150**	0.1150 **0.0850**	*
Demand (b_j)	*	*	*	*	

Minimum cost of FFMODI is 0.3190

Table_7 Evaluation of score when FFSZM

	D_1	D_2	D3	D4	Supply (a_i)
S_1	0.1000	0.2000 **0.2200**	0.1850	0.1000 **0.5200**	*
S_2	0.0100 **0.3350**	0.1400 **0.4800**	0.2250	0.2600	*
S_3	0.8150	0.2600	0.1750 **0.8150**	0.1150 **0.0850**	*
Demand (b_j)	*	*	*	*	

Minimum cost of FFSZM is 0.3190

Table_8 Numerical Results

S.No.	FFNWCR	FFLCM	FFVAM	FFMODI	FFSZM
1	0.3940	0.3228	0.3228	0.3190	0.3190
2	1.2751	0.9294	0.9231	0.91760	0.9176
3	1.0470	0.7730	0.763	0.7630	0.7630
4	0.9440	0.9210	0.8210	0.8100	0.8100

4. Results and Discussion

In this section, we discuss a suitable example to illustrate our FFSZM. This example taken from [15]. It is important to note that, all the parameters of the FTP are satisfied the Pythagorean FS property and also satisfied FFS property. The research and the previously mentioned find result make it clear that the suggested approach is preferable than our way for handling with fermatean fuzzy transportation challenges and has the benefit that it produces the optimal results.

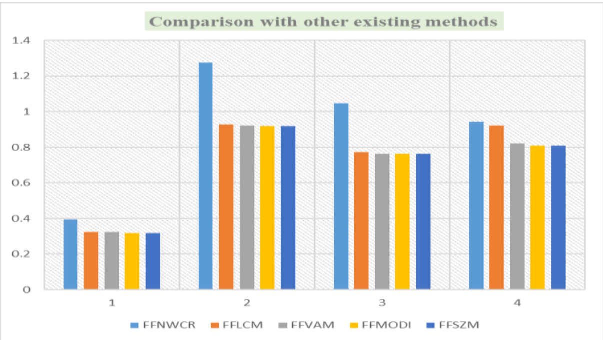

Figure_4: Comparison to other current techniques

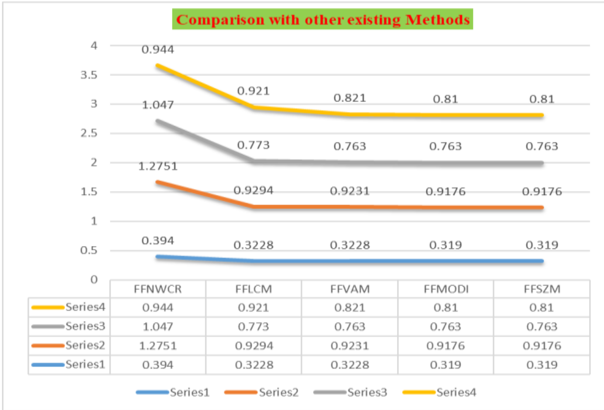

Figure_5: Comparison to other current techniques

Based on the findings from the Inquiries Figure_4 and Figure_5, it is evident that the proposed technique method outperforms existing methods and our approach for solving FFTP, as it produces an optimal solution.

Table_9 Comparison Results

Problems	Optimal solutions by Kumar [15]	Optimal solutions by [19,20]	Optimal solutions by [FFSZM]
1	0.3190	0.3190	0.3190
2	0.9176	0.9176	0.9176
3	0.7630	0.7630	0.7630
4	0.8100	0.8100	0.8100

Table 9 showed that Kumar [19,20] achieved the lowest cost in accurately evaluating the costs, and also the smallest range in terms of precise values. The Fermatean fuzzy costs, because there are no logistical challenges in the Fermatean fuzzy setting, are also noteworthy. Furthermore, it was stated that the goal of the numerical experiment was to tackle 3x4 size transportation challenges. However, these challenges can be broadly defined for any m x n size transportation issues and solved with the LINGO software.

There are numerous approaches to convert vague data into clear information data, and various strategies are discussed for dealing with fuzzy data, including, but not limited to, intuitionistic data, Pythagorean data, Fermatean data, and other uncertain data types. In this research, we introduce a new assessment metric for ranking Fermatean Fuzzy numbers, which aids in managing the uncertainty associated with Fermatean Fuzzy data within a well-defined context. Using fermatean fuzzy numbers to portray the product's supply, request, and transportation expenses, this study proposals a novel method for setting the FFTP using LINGO software. Therefore, issue with fermatean fuzzy transportation that occur in practical application can be solved using this methodology.

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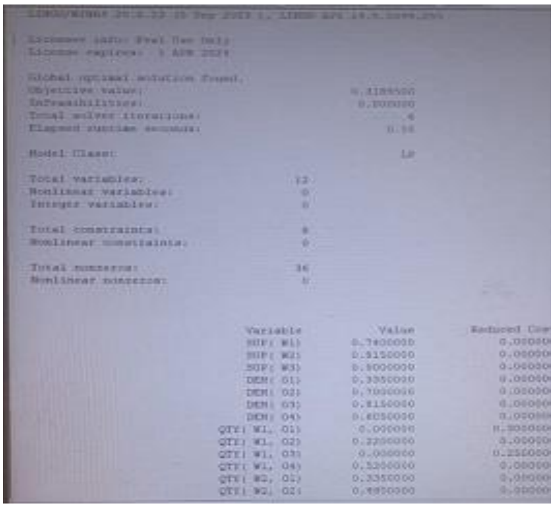
Appendix


Global optimal solution found.
Model Class: LP
Total variables: 12
Nonlinear variables: 0
Integer variables: 0
Nonlinear constraints: 0
Nonlinear nonzeros: 0

Problem_2

	D_1	D_2	D3	D4	Supply (a_i)
S_1	(0.2,0.5)	(0.2,0..4)	(0.3,0.7)	(0.6,0.5)	(0.7,0.1)
S_2	(0.3,0.5)	(0.2,0.9)	(0.7,0.1)	(0.4,0.7)	(0.8,0.1)
S_3	(0.7,0.1)	(0.2,0.3)	(0.5,0.1)	(0.6,0.4)	(0.9,0.1)
Demand (b_j)	(0.4,0.7)	(0.7,0.3)	(0.8,0.1)	(0.6,0.4)	

Problem_3

	D_1	D_2	D3	D4	Supply (a_i)
S_1	(0.4,0.7)	(0.3,0.7)	(0.6,0.5)	(0.2,0.8)	(0.7,0.3)
S_2	(0.3,0.5)	(0.4,0.8)	(0.3,0.7)	(0.6,0.2)	(0.9,0.1)
S_3	(0.4,0.7)	(0.4,0.6)	(0.3,0.1)	(0.5,0.1)	(0.9,0.1)
Demand (b_j)	(0.6,0.4)	(0.4,0.6)	(0.6,0.4)	(0.9,0.5)	

Problem_4

	D_1	D_2	D3	D4	Supply (a_i)
S_1	(0.2,0.3)	(0.6,0.7)	(0.3,0.7)	(0.6,0.5)	(0.9,0.1)
S_2	(0.7,0.1)	(0.2,0.9)	(0.6,0.7)	(0.4,0.7)	(0.7,0.3)
S_3	(0.8,0.4)	(0.3,0.7)	(0.2,0.5)	(0.7,0.1)	(0.7,0.3)
Demand (b_j)	(0.7,0.3)	(0.7,0.3)	(0.6,0.4)	(0.3,0.7)	