

## Finite Integral Formulas Involving $\overline{H}$ -Function and Multivariable Polynomial

**Naresh Bhati<sup>1</sup>, Meena Kumari Gurjar<sup>2</sup>, Anil Kumar Vishnoi<sup>3</sup>, and Rajneesh Kumar<sup>4</sup>**

<sup>1,2,3</sup> Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur, Rajasthan (India)

<sup>4</sup> Department of Mathematics, Kurukshetra University, Kurukshetra, INDIA

Email: nareshbhati1978@gmail.com; meenanetj@gmail.com; anilvishnoirahar@gmail.com;  
 rajneesh\_kuk@rediffmail.com

Corresponding author: meenanetj@gmail.com

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**Abstract:**

This study presents three finite integrals that include the product of multivariable polynomials and the  $\overline{H}$ -function. Additionally, some special cases have been identified that involve well-known functions like the H-function, G-function, and the generalized Wright hypergeometric function.

**Keywords:**  $\overline{H}$ -function, generalized Wright hypergeometric function, multivariable polynomial.

**MSC:** 33C45, 33C60.

### 1. Introduction

Inayat Hussain (1987) proposed a special function called the generalized fox H-function, also known simply as the  $\overline{H}$ -function. It can be represented as follows:

$$\overline{H}_{p,q}^{m,n} \left[ z \left| \begin{matrix} (a_i, \alpha_i, A_i)_{1,n} ; (a_i, \alpha_i)_{n+1,p} \\ (b_i, \beta_i, B_i)_{1,m} ; (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} \right. \right] = \frac{1}{2\pi\omega} \int_{-\infty}^{\infty} \overline{\phi}(s) z^s ds \quad \text{for } z \neq 0 \quad (1)$$

where 
$$\overline{\phi}(s) = \frac{\prod_{i=1}^m \Gamma(b_i - \beta_i s) \prod_{i=1}^n \Gamma(1 - a_i + \alpha_i s)^{A_i}}{\prod_{i=m+1}^q \Gamma(1 - b_i + \beta_i s)^{B_i} \prod_{i=n+1}^p \Gamma(a_i - \alpha_i s)}$$

and  $\omega = \sqrt{-1}$  (2)

This includes fractional powers of the gamma function. Let  $a_i (i=1,2,\dots,p)$  and  $b_i (i=1,2,\dots,q)$  be complex numbers and  $(\alpha_i)_{1,p}$  and  $(\beta_i)_{1,q}$  are real positive numbers greater than zero. The values of  $(A_i)_{1,n}; (B_i)_{m+1,q}$  can be non-integer numbers. The integers  $m, n, p$  and  $q$  are all assumed to be positive to keep things standardized.

In the standard definition of the H-function, the outline is taken to be the imaginary axis (where the real part of  $s$  is zero). This axis is slightly bent to steer clear of the singularities of the gamma

functions and to place those singularities on the correct sides. Other important restrictions on the parameters are the same as those given in Srivastava et al. (1982).

The integral in (1) that defines the H-function is based on information from Erdelyi and others from 1953.

$$|\Gamma(x + \omega y)| \square \sqrt{2\pi} |y|^{x-\frac{1}{2}} e^{-\frac{\pi|y|}{2}} \quad (|y| \rightarrow \infty) \quad (3)$$

Using the information from (3), we can find an approximation for  $|\Gamma(1 - a_i + \alpha_i s)^A|$  along the line  $\text{Re}(s) = x$ . We only need to make a small change to the conditions required for the contour integral (1) to converge absolutely.

$$\Lambda \equiv \sum_{i=1}^m |\beta_i| + \sum_{i=1}^n |\alpha_i A_i| - \sum_{i=m+1}^q |\beta_i B_i| - \sum_{i=n+1}^p |\alpha_i| > 0 \quad (4)$$

This condition clearly shows that the integrand in (1) decreases rapidly. The area where this is valid is given by  $|\arg(z)| < \frac{1}{2} \pi \Lambda$

$$(5)$$

where  $\Lambda$  is defined in (4).

We will need the following function, which is a special case of the  $\overline{H}$ -function, for what comes next.

$$\overline{H}_{n,q+1}^{1,n} \left[ z \left| \begin{matrix} (1 - a_i, \alpha_i; A_i)_{1,p} \\ (0, 1); (1 - b_i, \beta_i; B_i)_{1,q} \end{matrix} \right. \right] = {}_p \Psi_q \left[ \begin{matrix} (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q} \end{matrix} ; -z \right] \quad (6)$$

We will describe and represent a general type of multivariable polynomials like this:

$$S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} [t_1, t_2, \dots, t_r] = \sum_{\delta_1=0}^{v_1} \dots \sum_{\delta_r=0}^{v_r} \frac{(-v_1)_{u_1 \delta_1}}{\delta_1!} \dots \frac{(-v_r)_{u_r \delta_r}}{\delta_r!} A[v_1, \delta_1; \dots; v_r, \delta_r] t_1^{\delta_1} \dots t_r^{\delta_r} \quad (7)$$

where  $\beta_1, \dots, \beta_r = 0, 1, 2, \dots; \alpha_1, \dots, \alpha_r$  are arbitrary positive integers, the coefficients  $A[\beta_1, \delta_1; \dots; \beta_r, \delta_r]$  are arbitrary constants, real or complex. The expression includes numbers and variables and can produce several well-known polynomials in specific situations. These include Jacobi polynomials, Bessel polynomials, Laguerre polynomials, Brafman polynomials, and more.

Recently Tyagi et al. (2024) have studied the importance and applications of the  $\overline{H}$ -function, G-function in the theory of special functions and some applications.

We will need the following formulas given in Gradshteyn and Ryzhik [p. 326, Eqs. 7, 8, and 9] for our study.

$$\int_0^\infty \frac{x^n dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} = \frac{\sqrt{\pi} \Gamma(1+n)}{\sqrt{c} (\sqrt{ac} + b)^{n+1} 2^{n+1} \Gamma\left(n + \frac{3}{2}\right)} \quad [a \geq 0; c > 0; b + \sqrt{ac} > 0] \quad (8)$$

$$\int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} = \frac{\sqrt{\pi}\Gamma(1+n)}{\sqrt{a}(\sqrt{ac} + b)^{n+1} 2^{n+1}\Gamma\left(n + \frac{3}{2}\right)} \quad [a > 0; c \geq 0; b + \sqrt{ac} > 0] \quad (9)$$

$$\int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} = \frac{\Gamma\left(n + \frac{1}{2}\right)\sqrt{\pi}}{2^n\sqrt{2a}(b + \sqrt{ac})^{n+\frac{1}{2}}\Gamma(n+1)} \quad [a > 0; c > 0; b + \sqrt{ac} > 0] \quad (10)$$

## 2. Main Outcome

In this part, we introduced three new integrals:

### 2.1 First Integral:

$$\begin{aligned} & \int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\ & \quad \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \right. \\ & \quad \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \right] dx \\ & = \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\ & \quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \right. \\ & \quad \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \right] \end{aligned} \quad (11)$$

The above result will be converged under the conditions  $a \geq 0; c > 0; b + \sqrt{ac} > 0$  and  $\sigma_i, \lambda$  are positive numbers.

**Proof:** To prove the first integral (11), we first rewrite the  $\bar{H}$ -function occurring in its left side using the Mellin-Barnes contour integral shown in (1) and the polynomial from (7). Then, we swap the order of summation and integration. After simplifying a bit, we get the following result:

$$\begin{aligned} & \Rightarrow \frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \bar{\phi}(s) \sum_{\delta_1=0}^{v_1} \dots \sum_{\delta_r=0}^{v_r} \frac{(-v_1)_{u_1 \delta_1}}{\delta_1!} \dots \frac{(-v_r)_{u_r \delta_r}}{\delta_r!} A[v_1, \delta_1; \dots; v_r, \delta_r] t_1^{\delta_1} \dots t_r^{\delta_r} \int_0^\infty \frac{x^{n+\lambda s + \sum_{i=1}^r \sigma_i \delta_i} dx}{(ax^2 + 2bx + c)^{n+\lambda s + \sum_{i=1}^r \sigma_i \delta_i + \frac{3}{2}}} z^s ds \\ & \Rightarrow \frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \frac{\prod_{i=1}^m \Gamma(b_i - \beta_i s) \prod_{i=1}^n \Gamma(1 - a_i + \alpha_i s)^{A_i}}{\prod_{i=m+1}^q \Gamma(1 - b_i + \beta_i s)^{B_i} \prod_{i=n+1}^p \Gamma(a_i - \alpha_i s)} \sum_{\delta_1=0}^{v_1} \dots \sum_{\delta_r=0}^{v_r} \frac{(-v_1)_{u_1 \delta_1}}{\delta_1!} \dots \frac{(-v_r)_{u_r \delta_r}}{\delta_r!} \end{aligned}$$

$$\begin{aligned} & \times A[v_1, \delta_1; \dots; v_r, \delta_r] t_1^{\delta_1} \dots t_r^{\delta_r} \frac{\sqrt{\pi} \Gamma\left(1+n+\lambda s+\sum_{i=1}^r \sigma_i \delta_i\right)}{\sqrt{c}\left(2\sqrt{ac}+2b\right)^{n+\lambda s+\sum_{i=1}^r \sigma_i \delta_i+1} \Gamma\left(n+\lambda s+\sum_{i=1}^r \sigma_i \delta_i+\frac{3}{2}\right)} ds \\ \Rightarrow & \frac{\sqrt{\pi}}{2\pi\omega\sqrt{c}\left(2\sqrt{ac}+2b\right)^{n+1}} \int_{-\infty}^{\infty} \frac{\prod_{i=1}^m \Gamma(b_i-\beta_i s) \prod_{i=1}^n \Gamma(1-a_i+\alpha_i s)^{A_i} \Gamma\left(1+n+\sum_{i=1}^r \sigma_i \delta_i+\lambda s\right)}{\Gamma\left(n+\sum_{i=1}^r \sigma_i \delta_i+\frac{3}{2}+\lambda s\right) \prod_{i=m+1}^q \Gamma(1-b_i+\beta_i s)^{B_i} \prod_{i=n+1}^p \Gamma(a_i-\alpha_i s)} \times \\ & \sum_{\delta_1=0}^{v_1} \dots \sum_{\delta_r=0}^{v_r} \frac{(-v_1)_{u_1 \delta_1}}{\delta_1!} \dots \frac{(-v_r)_{u_r \delta_r}}{\delta_r!} A[v_1, \delta_1; \dots; v_r, \delta_r] \left\{ \frac{t_1}{\left(2\sqrt{ac}+2b\right)^{\sigma_1}} \right\}^{\delta_1} \dots \left\{ \frac{t_r}{\left(2\sqrt{ac}+2b\right)^{\sigma_r}} \right\}^{\delta_r} \left\{ \frac{z}{\left(2\sqrt{ac}+2b\right)^\lambda} \right\}^s ds \end{aligned}$$

Then interpreting with the help of (1) and (7) provides first integral.

$$\begin{aligned} \Rightarrow & \frac{\sqrt{\pi}}{\sqrt{c}\left(2\sqrt{ac}+2b\right)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{\left(2\sqrt{ac}+2b\right)^{\sigma_1}}, \frac{t_2}{\left(2\sqrt{ac}+2b\right)^{\sigma_2}}, \dots, \frac{t_r}{\left(2\sqrt{ac}+2b\right)^{\sigma_r}} \right] \\ & \times \bar{H}_{p+1, q+1}^{m, n+1} \left[ \frac{z}{\left(2\sqrt{ac}+2b\right)^\lambda} \left| \begin{matrix} \left\{ -n-\sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2, n+1}; (a_i, \alpha_i)_{n+2, p+1} \\ (b_i, \beta_i)_{1, m}; (b_i, \beta_i, B_i)_{m+1, q}; \left\{ -\frac{1}{2}-n-\sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \end{aligned}$$

**2.2. SECOND INTEGRAL:**

$$\begin{aligned} & \int_0^\infty \frac{x^{n+1} dx}{\left(ax^2+2bx+c\right)^{\frac{3}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{\left(ax^2+2bx+c\right)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{\left(ax^2+2bx+c\right)^{\sigma_r}} \right] \\ & \times \bar{H}_{p, q}^{m, n} \left[ \frac{zx^\lambda}{\left(ax^2+2bx+c\right)^\lambda} \left| \begin{matrix} (a_i, \alpha_i, A_i)_{1, n}; (a_i, \alpha_i)_{n+1, p} \\ (b_i, \beta_i)_{1, m}; (b_i, \beta_i, B_i)_{m+1, q} \end{matrix} \right. \right] dx \\ = & \frac{\sqrt{\pi}}{\sqrt{a}\left(2\sqrt{ac}+2b\right)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{\left(2\sqrt{ac}+2b\right)^{\sigma_1}}, \dots, \frac{t_r}{\left(2\sqrt{ac}+2b\right)^{\sigma_r}} \right] \\ & \times \bar{H}_{p+1, q+1}^{m, n+1} \left[ \frac{z}{\left(2\sqrt{ac}+2b\right)^\lambda} \left| \begin{matrix} \left\{ -n-\sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2, n+1}; (a_i, \alpha_i)_{n+2, p+1} \\ (b_i, \beta_i)_{1, m}; (b_i, \beta_i, B_i)_{m+1, q}; \left\{ -\frac{1}{2}-n-\sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \tag{12} \end{aligned}$$

The result will become stable if the following conditions are met:  $a > 0; c \geq 0; b + \sqrt{ac} > 0$  and  $\sigma_i, \lambda$  are positive integers.

**2.3. THIRD INTEGRAL:**

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \left| \begin{matrix} (a_i, \alpha_i, A_i)_{1,n} \\ (b_i, \beta_i, B_i)_{1,m} \end{matrix} ; \begin{matrix} (a_i, \alpha_i)_{n+1,p} \\ (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} \right. \right] dx \\
 & = \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+\frac{1}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \frac{t_2}{(2\sqrt{ac} + 2b)^{\sigma_2}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i ; \lambda ; 1 \right\} \\ (b_i, \beta_i, B_i)_{1,m} ; (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} ; \begin{matrix} (a_i, \alpha_i, A_i)_{2,n+1} ; (a_i, \alpha_i)_{n+2,p+1} \\ \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i ; \lambda ; 1 \right\} \end{matrix} \right. \right] \tag{13}
 \end{aligned}$$

The above result will be converged under the conditions  $a > 0; c > 0; b + \sqrt{ac} > 0$  and  $\sigma_i, \lambda$  are positive numbers.

We can prove the second (12) and third (13) integrals in a way similar to how the first integral (11) was proven, using results (9) and (10).

**3. SPECIAL CASES:**

**Corollary 3.1:** If we set  $A_i = B_i = 1$ , the  $\bar{H}$ -function simplifies to Fox's H-function [7, p. 10, eq. 2.1.1]. This means that equations (11), (12), and (13) will have the following results:

$$\begin{aligned}
 & \int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times H_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \left| \begin{matrix} (a_i, \alpha_i)_{1,p} \\ (b_i, \beta_i)_{1,q} \end{matrix} \right. \right] dx \\
 & = \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \frac{t_2}{(2\sqrt{ac} + 2b)^{\sigma_2}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times H_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i ; \lambda \right\} \\ (b_i, \beta_i)_{1,q} ; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i ; \lambda ; 1 \right\} \end{matrix} ; \begin{matrix} (a_i, \alpha_i)_{2,p+1} \end{matrix} \right. \right] \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times H_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| \begin{matrix} (a_i, \alpha_i)_{1,p} \\ (b_i, \beta_i)_{1,q} \end{matrix} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times H_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i)_{2,p+1} \\ (b_i, \beta_i)_{1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right] \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times H_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| \begin{matrix} (a_i, \alpha_i)_{1,p} \\ (b_i, \beta_i)_{1,q} \end{matrix} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+\frac{1}{2}}} S_{v_1, v_2, \dots, v_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times H_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i)_{2,p+1} \\ (b_i, \beta_i)_{1,q}; \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right] \tag{16}
 \end{aligned}$$

The conditions for (14), (15), and (16) can be easily understood based on the mentioned condition in (11), (12), and (13).

**Corollary 3.2:** Using the findings from (11), (12), and (13) for Hermite Polynomials, we can make specific adjustments. We set  $S_v^2(x) \rightarrow x^{\frac{v}{2}} H_v \left[ \frac{1}{2\sqrt{x}} \right]$  to represent the Hermite Polynomial function with

values for  $r=1; u_1 = \dots = u_r = 2; v_1 = \dots = v_r = v \ t_i = t; \sigma_i = \sigma \ A[v_i, \delta_i] = (-1)^\delta$ . As a result, we obtain the following outcomes:

$$\int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} \left\{ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right\}^{\frac{v}{2}} H_v \left[ \frac{1}{2} \sqrt{\frac{(ax^2 + 2bx + c)^\sigma}{x^\sigma t}} \right] \\ \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \right. \\ \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \right] dx \\ = \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \frac{(-1)^\delta (-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\ \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \right. \\ \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta; \lambda; 1 \right\} \right] \quad (17)$$

$$\int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} \left\{ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right\}^{\frac{v}{2}} H_v \left[ \frac{1}{2} \sqrt{\frac{(ax^2 + 2bx + c)^\sigma}{x^\sigma t}} \right] \\ \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \right. \\ \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \right] dx \\ = \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \frac{(-1)^\delta (-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\ \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \right. \\ \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta; \lambda; 1 \right\} \right] \quad (18)$$

$$\int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} \left\{ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right\}^{\frac{v}{2}} H_v \left[ \frac{1}{2} \sqrt{\frac{(ax^2 + 2bx + c)^\sigma}{x^\sigma t}} \right] \\ \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \right. \\ \left. (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \right] dx$$

$$\begin{aligned}
 &= \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \frac{(-1)^\delta (-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\
 &\quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \quad (19)
 \end{aligned}$$

The rules for (17), (18), and (19) are straightforward and come from the rules in (11), (12) and (13).

**Corollary 3.3:** Using the findings from equations (11), (12), and (13) for Laguerre Polynomials [4,5], we replace  $S_v^2(x) \rightarrow L_v^\rho[x]$  while setting the values of  $r=1; u_1 = \dots = u_r = 2; v_1 = \dots = v_r = v;$

$t_i = t; \sigma_i = \sigma; A[v_i, \delta_i] = \binom{v+\rho}{v} \frac{1}{(\rho+1)_\delta}$ , etc. This gives us the following results:

$$\begin{aligned}
 &\int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} L_v^\rho \left[ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right] \times \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \left| \begin{matrix} (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} \right. \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \binom{v+\rho}{v} \frac{1}{(1+\rho)_\delta} \frac{(-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\
 &\quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} L_v^\rho \left[ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right] \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \left| \begin{matrix} (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} \right. \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \binom{v+\rho}{v} \frac{1}{(1+\rho)_\delta} \frac{(-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\
 &\quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \quad (21)
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} L_v^\rho \left[ \frac{x^\sigma t}{(ax^2 + 2bx + c)^\sigma} \right] \bar{H}_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| \begin{matrix} (a_i, \alpha_i, A_i)_{1,n}; (a_i, \alpha_i)_{n+1,p} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q} \end{matrix} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} \sum_{\delta=0}^{\frac{v}{2}} \binom{v+\rho}{v} \frac{1}{(1+\rho)_\delta} \frac{(-v)_{2\delta}}{\delta!} \left\{ \frac{t^\delta}{(2\sqrt{ac} + 2b)^{\sigma\delta}} \right\} \\
 & \quad \times \bar{H}_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i, A_i)_{2,n+1}; (a_i, \alpha_i)_{n+2,p+1} \\ (b_i, \beta_i)_{1,m}; (b_i, \beta_i, B_i)_{m+1,q}; \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right] \tag{22}
 \end{aligned}$$

The rules for (20), (21), and (22) are clearly based on those in (11), (12), and (13).

**Corollary 3.4:** If we put  $A_i = 1 = B_i; \alpha_i = 1 = \beta_i$  in (1) then the  $\bar{H}$ -function reduces general type of G-

function [12]  $\bar{H}_{p,q}^{m,n} \left[ z \begin{matrix} (a_i, 1)_{1,n}; (a_i, 1)_{n+1,p} \\ (b_i, 1)_{1,m}; (b_i, 1)_{m+1,q} \end{matrix} \right] = G \left[ z \begin{matrix} (a_i, 1)_{1,p} \\ (b_i, 1)_{1,q} \end{matrix} \right]$ . Using the same assumptions from (11), (12), and (13), we arrive at this form:

$$\begin{aligned}
 & \int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{\frac{3}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] G_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| \begin{matrix} (a_i, 1)_{1,p} \\ (b_i, 1)_{1,q} \end{matrix} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{c}(2\sqrt{ac} + 2b)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times G_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \middle| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, 1)_{2,p+1} \\ (b_i, 1)_{1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right] \tag{23}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{\frac{3}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] G_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \middle| \begin{matrix} (a_i, 1)_{1,p} \\ (b_i, 1)_{1,q} \end{matrix} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right]
 \end{aligned}$$

$$\begin{aligned} & \times G_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, 1)_{2,p+1} \\ (b_i, 1)_{1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \tag{24} \\ & \int_0^\infty \frac{x^{\frac{n+1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] G_{p,q}^{m,n} \left[ \frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \left| \begin{matrix} (a_i, 1)_{1,p} \\ (b_i, 1)_{1,q} \end{matrix} \right. \right] dx \\ & = \frac{\sqrt{\pi}}{\sqrt{a} (2\sqrt{ac} + 2b)^{\frac{n+1}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\ & \times G_{p+1,q+1}^{m,n+1} \left[ \frac{z}{(2\sqrt{ac} + 2b)^\lambda} \left| \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, 1)_{2,p+1} \\ (b_i, 1)_{1,q}; \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} \right. \right] \tag{25} \end{aligned}$$

The conditions for (23), (24), and (25) are simply based on the conditions provided in (11), (12), and (13).

**Corollary 3.5:** If we put  $n = p; m = 1; q = q + 1; b_1 = 0, \beta_i = 1; a_i = 1 - a_i; b_i = 1 - b_i$  in (1) then the  $\bar{H}$ -function reduces generalized Wright hypergeometric function [16] i.e  $\bar{H}_{p,q+1}^{-1,p} \left[ z \left| \begin{matrix} (1 - a_i, \alpha_i; A_i)_{1,p} \\ (0, 1); (1 - b_i, \beta_i; B_i)_{1,q} \end{matrix} \right. \right] = {}_p\bar{\Psi}_q \left[ \begin{matrix} (a_i, \alpha_i; A_i)_{1,p}; -z \\ (b_i, \beta_i; B_i)_{1,q} \end{matrix} \right]$ . Using the same conditions from equations (11), (12), and (13), we arrive at this result:

$$\begin{aligned} & \int_0^\infty \frac{x^n}{(ax^2 + 2bx + c)^{\frac{n+3}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\ & \times {}_p\bar{\Psi}_q \left[ \begin{matrix} (a_i, \alpha_i; A_i)_{1,p}; -\frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \\ (b_i, \beta_i; B_i)_{1,q} \end{matrix} \right] dx \\ & = \frac{\sqrt{\pi}}{\sqrt{c} (2\sqrt{ac} + 2b)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \frac{t_2}{(2\sqrt{ac} + 2b)^{\sigma_2}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\ & \times {}_p\bar{\Psi}_q \left[ \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; -\frac{z}{(2\sqrt{ac} + 2b)^\lambda} \end{matrix} \right] \tag{26} \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+1} dx}{(ax^2 + 2bx + c)^{n+\frac{3}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times {}_p\overline{\Psi}_q \left[ \begin{matrix} (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q} \end{matrix} ; -\frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times {}_p\overline{\Psi}_q \left[ \begin{matrix} \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q}; \left\{ -\frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} ; -\frac{z}{(2\sqrt{ac} + 2b)^\lambda} \right] \tag{27}
 \end{aligned}$$

$$\begin{aligned}
 & \int_0^\infty \frac{x^{n+\frac{1}{2}} dx}{(ax^2 + 2bx + c)^{n+1}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{x^{\sigma_1} t_1}{(ax^2 + 2bx + c)^{\sigma_1}}, \dots, \frac{x^{\sigma_r} t_r}{(ax^2 + 2bx + c)^{\sigma_r}} \right] \\
 & \quad \times {}_p\overline{\Psi}_q \left[ \begin{matrix} (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q} \end{matrix} ; -\frac{zx^\lambda}{(ax^2 + 2bx + c)^\lambda} \right] dx \\
 &= \frac{\sqrt{\pi}}{\sqrt{a}(2\sqrt{ac} + 2b)^{n+\frac{1}{2}}} S_{\nu_1, \nu_2, \dots, \nu_r}^{u_1, u_2, \dots, u_r} \left[ \frac{t_1}{(2\sqrt{ac} + 2b)^{\sigma_1}}, \dots, \frac{t_r}{(2\sqrt{ac} + 2b)^{\sigma_r}} \right] \\
 & \quad \times {}_p\overline{\Psi}_q \left[ \begin{matrix} \left\{ \frac{1}{2} - n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\}; (a_i, \alpha_i; A_i)_{1,p} \\ (b_i, \beta_i; B_i)_{1,q}; \left\{ -n - \sum_{i=1}^r \sigma_i \delta_i; \lambda; 1 \right\} \end{matrix} ; -\frac{z}{(2\sqrt{ac} + 2b)^\lambda} \right] \tag{28}
 \end{aligned}$$

The truth of (26), (27), and (28) can be easily understood from the conditions in (11), (12), and (13).

#### 4. Conclusions

These integral formulas are versatile and helpful for many mathematical uses. These formulas can be changed into easier kinds, like the Fox H-function, the G function, and the generalized Wright hypergeometric function. These results provide a solid base for developing individual examples. It is concluded that each of these formulas field interesting new formula for certain multivariable hypergeometric function e.q. Lauricella hypergeometric function etc. The result obtained find potentially useful application in variety of areas involving special function and general class of polynomial.

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