

Co - Total 2 - ODED Number in Graphs

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Abstract:

Let D be a subset of V and $u \in D$ the out degree of u is defined by $|N(u) \cap (V - D)|$ and denoted by $od_D(u)$. A dominating set D of V is said to be co- total dominating set if the sub graph induced by $V - D$ has no isolated vertices Based on the concepts we introduce a new domination called co-total 2 - ODED number. Here the proposed domination number verified for some general and some named graphs.

Keywords: Two out degree, domination number, isolated vertices, co-total

1. Introduction

The graphs considered here are simple and without isolated vertices defined from [1-5]. In a graph $G=(V,E)$, $|V|=n$ and $|E|=m$. Let u be vertex and $N(u)$ denotes the open neighborhood and defined by $N(u) = \{v \in V(G) / uv \in E(G)\}$ The degree of a vertex u is denoted $deg(u)$ and defined by $deg(u) = |N(u)|$. If the vertex u of a graph is isolated vertex if $deg(u) = 0$.

O. Ore [10] and C. Berge [4] introduced the concept of domination number. A sub set D of V is said to dominating set if each vertex of V is adjacent to some vertex in D . The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. Kulli V.R., Jankiram B and Radha R Iyer [6] introduce co-total domination number $\gamma_{ct}(G)$. A dominating set D of V is said to be co- total dominating set if the sub graph induced by $V-D$ has no isolated vertices. The minimum number of vertices of co-total dominating set is called co-total domination number and it is represented by $\gamma_{ct}(G)$. Sahal. A and V. Mathad [11] introduce the 2- ODED number. A dominating set D of $V(G)$ is said to be 2- ODED set for any vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$ where $od_D(u) = |N(u) \cap (V - D)|$. The minimum represented of vertices in 2- ODED set is called 2- ODED number and it is denoted by $\gamma_{2oe}(G)$. Based on above domination number, here we introduce a new domination parameter called co - total 2- ODED ($\gamma_{ct2oe}(G) - set$) number.

2. Co-Total 2 - ODED Number

Definition 2.1 A sub set D of V is called co- total 2 - ODED set if D is dominating set and for any vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$ where $od_D(u) = |N(u) \cap (V - D)|$ and the

induced sub graph $\langle V - D \rangle$ has no vertices of degree zero. The minimum cardinality of a co-total 2 - ODED set is called co-total 2 - ODED number and it is represented by $\gamma_{ct2oe}(G)$.

Example 2.2

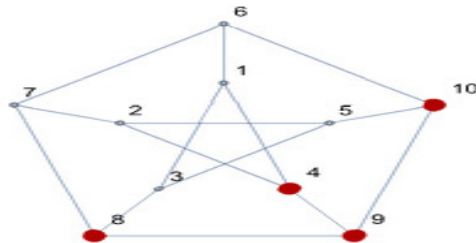


Figure 1. CO-total 2 - ODED number

Take $D = \{4, 8, 9, 10\}$ and $V - D = \{1, 2, 3, 5, 6, 7\}$

$$\begin{aligned} od_S(4) &= |N(4) \cap (V - D)| \\ &= |[1, 2, 9] \cap \{1, 2, 3, 5, 6, 7\}| = |\{1, 2\}| = 2 \end{aligned}$$

$$\begin{aligned} od_S(8) &= |N(8) \cap (V - D)| \\ &= |[3, 7, 9] \cap \{1, 2, 3, 5, 6, 7\}| = |\{3, 7\}| = 2 \end{aligned}$$

$$\begin{aligned} od_S(9) &= |N(9) \cap (V - D)| \\ &= |[4, 8, 10] \cap \{1, 2, 3, 5, 6, 7\}| = |\{\emptyset\}| = 0 \end{aligned}$$

$$\begin{aligned} od_S(10) &= |N(10) \cap (V - D)| \\ &= |[5, 6, 9] \cap \{1, 2, 3, 5, 6, 7\}| = |\{5, 6\}| = 2 \end{aligned}$$

Then any $u, v \in D, |od_D(u) - od_D(v)| \leq 2$

So $D = \{4, 8, 9, 10\}$ form 2- ODED set of G and the sub graph induced by $\langle V - D \rangle$ has no vertices of degree zero. Hence D is a co-total 2- ODED set with minimum cardinality. Hence $\gamma_{ct2oe}(G) = 3$

Observation 2.3 A graph G with p vertices, then $2 \leq \gamma_{ct2oe}(G) \leq n$.

Observation 2.4

1. For any complete graph: $\gamma_{ct2oe}(K_n) = 2$
2. For any star $K_{1,n}$, then $\gamma_{ct2oe}(K_{1,n-1}) = n$
3. For any complete bipartite graph $K_{p,q}$, is $\gamma_{ct2oe}(K_{p,q}) = \begin{cases} 2 & \text{if } |p - q| \leq 2 \\ p + q - 2 & \text{otherwise} \end{cases}$
4. For any cycle C_n , then $\gamma_{ct2oe}(C_n) = n - 2$
5. For the Path P_n , $\gamma_{ct2oe}(P_n) = n - 2, n \geq 2$
6. For the double star $S_{r,t}$, $\gamma_{ct2oe}(S_{r,t}) = r + t$

3. Co-total 2 - ODE D number for different graphs

Here begin the investigation of the co-total 2- ODED number by computing its values for some general of graphs.

Theorem: 3.1 For any combo graph $\gamma_{ct2oe}(P_n^+) = n$

Proof: Let $V(P_n^+) = \{v_1, v_2, v_3 \dots v_n, v_{n+1}, v_{n+2}, v_{n+3}, \dots, v_{2n}\}$. Here $\{v_1, v_2, v_3 \dots v_n\}$ be the vertices of the path $\{v_{n+1}, v_{n+2}, v_{n+3}, \dots, v_{2n}\}$ be the vertices of degree one is incident with each vertex of the path. Let $D = \{v_{n+1}, v_{n+2}, v_{n+3}, \dots, v_{2n}\}$ be minimal dominating set and $V - D = \{v_1, v_2, v_3 \dots v_n\}$. Each vertices D has a neighborhood $V - D$ and $od_D(v_{n+i}) = |N(v_{n+i}) \cap (V - D)| = 1$ for all $i=1,2,3,\dots,n$. then $|od_D(v_{n+i}) - od_D(v_{n+j})| \leq 2$. Then D is 2- ODED set and $\langle V - D \rangle$ has no vertices of degree zero. Then D is minimal co-total 2- ODED set . Then $\gamma_{ct2oe}(P_n^+) = |D| = n$.

Theorem 3.2 For any crown graph $C_n^+, \gamma_{ct2oe}(C_n^+) = n$

Proof: Let $V(C_n^+) = \{v_1, v_2, v_3 \dots v_n, v_{n+1}, v_{n+2} \dots v_{2n}\}$. Here $\{v_1, v_2, v_3 \dots v_n\}$ be the vertices of cycle C_p and $\{v_{n+1}, v_{n+2}, \dots, v_{2n}\}$ be the vertices of degree zero which adjacent to $v_1, v_2, v_3 \dots v_n$ respectively. Let us take $D = \{v_{n+1}, v_{n+2}, \dots, v_{2n}\}$ be dominating set with minimal cardinality and $V - D = \{v_1, v_2, v_3 \dots v_n\}$ then $od_D(v_{n+i}) = |N(v_{n+i}) \cap (V - D)| = |\{v_i\} \cap \{v_1, v_2, v_3 \dots v_n\}| = |\{v_i\}| = 1$. Then $|od_D(v_i) - od_D(v_j)| = 0 < 2$.

Hence D is a 2 - ODED set and the sub graph induced by $V - D$ has no vertices of degree zero. Then D is minimal co- total 2- ODED set . So $\gamma_{ct2oe}(C_n^+) = |D| = n$.

Theorem:3.3 For any triangular snake graph $\gamma_{ct2oe}(nC_3) = n - 1$

Proof: The graph G contains $2n - 1$ vertices and $n - 1$ triangles. The upper vertices labeled from v_1 to v_{n-1} and the lower vertices are labeled form v_n to v_{2n-1} . Let $D = \{v_1, v_2, \dots, v_{n-1}\}$ and $V - D = \{v_n, v_{n+1}, v_{n+2}, v_{n+3} \dots v_{2n-2}, v_{2n-1}\}$.

The vertex v_1 is adjacent to v_n, v_{n+1} . The vertex v_2 is adjacent to v_{n+1}, v_{n+2} . The vertex v_{n-1} is adjacent to v_{2n-2}, v_{2n-1} . Hence the set $\{v_{n+1}, v_{n+3} \dots v_{2n-2}\}$ is minimum dominating set. Now $od_D(v_1) = |N(v_1) \cap (V - D)| = |\{v_n, v_{n+1}\}| = 2$

$od_D(v_2) = |N(v_2) \cap (V - D)| = |\{v_{n+1}, v_{n+2}\}| = 2$ and $od_D(v_{n-1}) = |N(v_{n-1}) \cap (V - D)| = |\{v_{2n-2}, v_{2n-1}\}| = 2$.

Hence $|od_D(u) - od_D(v)| \leq 2$ for all $u, v \in D$, and D is minimum 2 - ODE dominating set and Clearly $\langle V - D \rangle$ has no vertices of degree zero. Hence D is minimum co- total 2- ODE dominating set. Hence $\gamma_{ct2oe}(G) = n - 1$.

Theorem 3.4 For any double triangular snake graph $\gamma_{ct2oe}(D(nC_3)) = n + 1$

Proof: Let $V(D(nC_3)) = \{u_1, u_2, u_3 \dots u_{n+1}, v_1, v_2, v_3 \dots v_n, w_1, w_2, w_3 \dots w_n\}$. Here $\{u_1, u_2, u_3 \dots u_{n+1}\}$ be the vertices of path P_n . From path P_n join u_i and u_{i+1} to a new edges v_i

by edges $u_i v_i$ and $u_{i+1} v_i$, for $i=1,2,3, \dots, n$ and join u_i and u_{i+1} to a new edges w_i by edges $u_i w_i$ and $u_{i+1} w_i$ for $i=1,2,3, \dots, n$

Take the vertices of path P_n , $D = \{u_1, u_2, u_3, \dots, u_{n+1}\}$ and $V - D = \{v_1, v_2, v_3, \dots, v_n, w_1, w_2, w_3, \dots, w_n\}$. Clearly D is a dominating $V - D$. Now $od_D(u_i) = |N(u_i) \cap (V - D)| = |\{v_i, w_i\}| = 2$ for $i = 1, n + 1$ and $od_D(u_i) = |N(u_i) \cap (V - D)| = |\{v_{i-1}, v_i, w_{i-1}, w_i\}| = 4$ for $i = 2, \dots, n$. Hence $|od_D(u_i) - od_D(u_j)| \leq 2$ for all $u_i, u_j \in D$, and D is minimum 2 - ODED set and clearly $\langle V - D \rangle$ has no vertices of degree zero. So D is minimum co-total 2- ODED set. Hence $\gamma_{ct2oe}(D(nC_3)) = n + 1$

Definition 3.5 The square of a given graph G denoted by G^2 has the same number vertices as of G and has a vertices are adjacent in G^2 if they are at distance of one or two apart in G .

Theorem 3.6 For any square of bistar graph $\gamma_{ct2oe}(B_{p,q}^2) = p + q$

Proof: Consider Bistar $B_{p,q}$, with vertices $\{u, v, u_1, u_2, u_3, \dots, u_p, v_1, v_2, v_3, \dots, v_q\}$ where u_i, v_i are pendant vertices which is adjacent u and v respectively and u and v adjacent.

Take $D = \{u_1, u_2, u_3, \dots, u_p, v_1, v_2, v_3, \dots, v_q\}$ and $V - D = \{u, v\}$. Clearly D is a dominating $V - D$. Now $od_D(u_i) = |N(u_i) \cap (V - D)| = |V - D| = 2$ and $od_D(v_i) = |N(v_i) \cap (V - D)| = |V - D| = 2$. Hence $|od_D(u_i) - od_D(u_j)| = 0 \leq 2$ and D is minimum co-total 2 - ODED set. Clearly $\langle V - D \rangle$ has no vertices of degree zero. So D is a minimum co-total 2 - ODED set. Hence $\gamma_{ct2oe}(B_{p,q}^2) = 2$.

Theorem 3.7 For any path P_n , $\gamma_{ct2oe}(P_n^2) = \begin{cases} 2 \lfloor \frac{n}{7} \rfloor + 1 & \text{if } n \equiv 0 \text{ or } 6 \pmod{7} \\ 2 \lfloor \frac{n}{7} \rfloor & \text{otherwise} \end{cases}$

Proof: Let $V(P_n^2) = \{v_1, v_2, v_3, \dots, v_n\}$ be the vertex set where $\deg(v_1) = \deg(v_n) = 2$, $\deg(v_2) = \deg(v_{n-1}) = 3$ and $\deg(v_i) = 4$ for all $i = 1, 2, 3, \dots, n - 2$

Case: 1 $n \equiv 0 \text{ or } 6 \pmod{7}$

If $n \equiv 0 \pmod{7}$ we have $D = \{(v_{7i+2}, v_{7i+4}) \cup \{v_{n-1}\}\}$ for $0 \leq i \leq \lfloor \frac{n}{7} \rfloor - 1$ and

If $n \equiv 6 \pmod{7}$ we have $D = \{(v_{7i+2}, v_{7i+4}) \cup \{v_n\}\}$ for $0 \leq i \leq \lfloor \frac{n}{7} \rfloor - 1$

Now $od_D(v_{7i+2}) = |N(v_{7i+2}) \cap (V - D)| = 3$,

$od_D(v_{7i+4}) = |N(v_{7i+4}) \cap (V - D)| = 4$, $od_D(v_{n-1}) = |N(v_{n-1}) \cap (V - D)| = 2$ and $od_D(v_n) = |N(v_n) \cap (V - D)| = 2$

Hence $|od_D(u) - od_D(v)| \leq 2$. For any $u, v \in D$.

Here $\langle V - D \rangle$ has no zero degree vertices then D is co-total 2 - ODED set also $D - \{u\}$ is no zero degree vertices then D is a minimal co-total 2- ODED set. $\gamma_{t2oe}(P_n^2) = 2 \lfloor \frac{n}{7} \rfloor + 1$

Case: 2 $n \not\equiv 0 \text{ or } 6 \pmod{7}$

If $n \equiv 1$ or 2 or 3 or $4 \pmod{7}$ we have $D = \{(v_{7i+2}, v_{7i+4}) \cup \{v_{n-2}, v_n\}\}$ for $0 \leq i \leq \lfloor \frac{n}{7} \rfloor - 1$ and

If $n \equiv 5 \pmod{7}$ we have $D = \{(v_{7i+2}, v_{7i+4})\}$ for $0 \leq i \leq \lfloor \frac{n}{7} \rfloor$

Now $od_D(v_{7i+2}) = |N(v_{7i+2}) \cap (V - D)| = 3$,

$od_D(v_{7i+4}) = |N(v_{7i+4}) \cap (V - D)| = 3$, $od_D(v_n) = |N(v_n) \cap (V - D)| = 2$ and $od_D(v_{n-2}) = |N(v_{n-2}) \cap (V - D)| = 2$

Hence $|od_D(u) - od_D(v)| \leq 2$. For any $u, v \in D$.

Here $\langle V - D \rangle$ has no vertices of degree zero then D is a co-total 2 - ODED set then $\gamma_{ct2oe}(P_n^2) = 2 \lfloor \frac{n}{7} \rfloor$.

Definition 3.8 The semi total point graph $T_2(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$, whose vertices are adjacent if they adjacent vertices of G or one is a vertex of G and another edge of G incident with it.

Theorem 3.9 For any cycle C_n , $\gamma_{ct2oe}(T_2(C_n)) = n - 1$ for $n \geq 3$

Proof: Let vertex set of $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(C_n) = \{e_1, e_2, \dots, e_n\}$. Now, $V(T_2(C_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$ be vertices of $T_2(C_n)$. Let $D = \{e_1, e_2, \dots, e_{n-3}, e_n, v_{n-1}\}$ be the minimal dominating set of $T_2(C_n)$ then $V-D = \{v_1, v_2, \dots, v_{n-2}, v_n, e_{n-2}, e_{n-1}\}$. Clearly, the vertices $v_1, v_2, \dots, v_{n-2}, v_n$ in $V-D$ forms a path and by the definition of semi total point graph the vertices e_{n-2}, e_{n-1} is adjacent to v_{n-2}, v_n in $V-D$. Thus $\langle V-D \rangle$ has no isolated vertices. Except v_{n-1} all other elements in D is exactly adjacent to two vertices in $V-D$, so the out degree of these elements in D is two. For v_{n-1} in D , it is exactly adjacent with four vertices, hence the out degree is four. Thus, for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. So, D is the minimum co-total 2 - ODED set. Now, $|D| = n - 1$. Hence, $\gamma_{ct2oe}(T_2(C_n)) = n - 1$ for $n \geq 3$.

Theorem 3.10 For any path P_n , $\gamma_{ns2oe}(T_2(P_n)) = n - 1$ for $n \geq 2$

Proof: Let P_n be a path for $n \geq 2$, Here $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{e_1, e_2, \dots, e_{n-1}\}$. Now, $V(T_2(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\}$ be vertices of $T_2(P_n)$. Let $D = \{e_1, e_2, \dots, e_{n-1}\}$ be the minimal dominating set of $T_2(P_n)$. Then $V-D = \{v_1, v_2, \dots, v_n\}$. The induced subgraph of $\langle V-D \rangle$ is the given P_n which has no isolated vertices. Hence two out degree of any vertex in D is 2. Clearly for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. Therefore, D is the minimum co-total 2- ODED set. Hence, $\gamma_{ct2oe}(T_2(P_n)) = n - 1$ for $n \geq 2$.

Theorem 3.11 For all Combo Graph P_n^+ , $\gamma_{ct2oe}(T_2(P_n^+)) = 2n - 1$ for $n \geq 2$

Proof: Let P_n^+ be a Combo Graph for $n \geq 2$, Here $V(P_n^+) = \{v_1, v_2, \dots, v_{2n}\}$ and $E(P_n^+) = \{e_1, e_2, \dots, e_{2n-1}\}$. Now, $V(T_2(P_n^+)) = \{v_1, v_2, \dots, v_{2n}, e_1, e_2, \dots, e_{2n-1}\}$ is the vertex set of $T_2(P_n^+)$. Let $D = \{e_1, e_2, \dots, e_{2n-1}\}$ be the minimal dominating set of $T_2(P_n^+)$. Then $V-D = \{v_1, v_2, \dots, v_{2n}\}$. The subgraph induced by $V-D$ is the given graph P_n^+ which has no isolated vertices and also each vertex in D is exactly adjacent to two vertices in $V-D$. Now, $od_D(u) = 2$ for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. Therefore, D is the minimum co-total 2 - ODED set. Hence $\gamma_{ct2oe}(T_2(P_n^+)) = 2n - 1$ for $n \geq 2$.

Theorem 3.12 For all Fan graph F_n , $\gamma_{ct2oe}(T_2(F_n)) = 2n - 1$ for $n \geq 2$

Proof: Let vertex set $V(F_n) = \{v, v_1, v_2, \dots, v_n\}$ and edge set $E(F_n) = \{e_1, e_2, \dots, e_{2n-1}\}$. Now, $V(T_2(F_n)) = \{v, v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{2n-1}\}$ is the vertex set of $T_2(F_n)$.

Let $D = \{e_1, e_2, \dots, e_{2n-1}\}$ be the minimal dominating set of $T_2(F_n)$. Then $V-D = \{v, v_1, v_2, \dots, v_n\}$. The induced subgraph of $V-D$ is the given graph F_n which has no isolated vertices and also each vertex in D is exactly adjacent to two vertices in $V-D$. Now, for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. Therefore, D is the minimum co-total 2 - ODED set. Hence $\gamma_{ct2oe}(T_2(F_n)) = 2n - 1$ for $n \geq 2$.

Theorem 3.13 For any star graph $K_{1,n}$, $\gamma_{ns2oe}(T_2(K_{1,n})) = n$ for $n \geq 1$

Proof: Let $K_{1,n}$ be a star graph for $n \geq 1$ with vertex set $V(K_{1,n}) = \{v_1, v_2, \dots, v_{n+1}\}$ and edge set $E(K_{1,n}) = \{e_1, e_2, \dots, e_n\}$. Now, $V(T_2(K_{1,n})) = \{v_1, v_2, \dots, v_{n+1}, e_1, e_2, \dots, e_n\}$ be the vertices of $T_2(K_{1,n})$. Let $D = \{e_1, e_2, \dots, e_n\}$ then $V-D = \{v_1, v_2, \dots, v_{n+1}\}$. The induced subgraph of $V-D$ is $K_{1,n}$, hence it has no vertices of degree zero. Also each vertices in D is exactly adjacent to two vertices in $V-D$. Hence for any vertex $v \in D$ then $od_D(v) = 2$. Clearly, for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. So, D is a minimum co-total 2 - ODED set. Hence $\gamma_{ct2oe}(T_2(K_{1,n})) = n$ for $n \geq 1$.

Theorem 3.14 For any triangular snake graph nC_3 , $\gamma_{ns2oe}(T_2(nC_3)) = 2n$ for $n \geq 1$

Proof: Let nC_3 be a triangular snake graph for $n \geq 1$, with vertex set $V(nC_3) = \{v_1, v_2, \dots, v_{2n+1}\}$ and edge set $E(nC_3) = \{e_1, e_2, \dots, e_{3n}\}$. Now, $V(T_2(nC_3)) = \{v_1, v_2, \dots, v_{2n+1}, e_1, e_2, \dots, e_{3n}\}$ be the vertex set of $T_2(nC_3)$. Let $D = \{e_{2n+1}, \dots, e_{3n}, v_{n+2}, \dots, v_{2n+1}\}$ be the minimal dominating set of $T_2(nC_3)$ then $V-D = \{v_1, v_2, \dots, v_{n+1}, e_1, e_2, \dots, e_{2n}\}$. Clearly, the vertices v_1, v_2, \dots, v_{n+1} in $V-D$ forms a path and by the definition of semi total point graph the vertices e_1, e_2, \dots, e_{2n} is adjacent to v_1, v_2, \dots, v_{n+1} in $V-D$. Thus $\langle V-D \rangle$ has no isolated vertices. The vertices e_{2n+1}, \dots, e_{3n} in D is adjacent exactly two vertices in $V-D$, so the out degree is two. The remaining vertices v_{n+2}, \dots, v_{2n+1} of D is adjacent exactly four vertices in $V-D$, hence the out degree is four. Thus, for any vertex $u, v \in D$, $|od_D(u) - od_D(v)| \leq 2$. So, D is the minimum co-total 2 - ODED set. Now, $|D| = 2n$. Hence $\gamma_{ct2oe}(T_2(nC_3)) = 2n$ for $n \geq 1$

4. Conclusion

In the next paper we study the bounds of co-total 2- ODED number and find the above number for some new family of graphs. Also we like to extend the study to find the limitations and applications of the co-total 2- ODED number.

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