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# A Fermatean Fuzzy Environment is used in Transportation Problems to Extend a New Score Function

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**Abstract:**

Since the economy and environment are in a state of instability. In a transportation difficulty, it is no longer possible to determine supply, demand, and transportation costs. This work aims to explore transportation, including supply, demand, and transportation costs expressed as FFNs. However, Pythagorean fuzzy numbers or generalized fuzzy numbers are used as the parameters in each of these recommendations. It is simpler to deal with uncertain data when making decisions because of the innovative idea of Fermatean fuzzy sets. For the first time, we have overcome the transportation problem with Fermatean fuzzy parameters. Our method was created to address the Fermatean fuzzy parameter transportation problem, and it was successfully handled by using a well-known method. The best result can then be obtained by performing arithmetic operations on Fermatean fuzzy numbers. Specifically, to illustrate the suggested approach, Our solution to a numerical instance and then presented and compared the results with the current literature. The approval of the effort along with its possibility future developments have been discussed.

**Keywords:** Pythagorean fuzzy numbers, Fermatean fuzzy numbers, Score function, Accuracy function, Transportation problem, QM windows solver.

## 1. Introduction

In the dynamics of our fiercely competitive industry, figuring out the best strategy to create and deliver items to customers in the most efficient way, Among the many difficulties faced by the transport network's administration is adjusting to technological advancements, Increasing output, internationalization, shifting consumer expectations, market dynamics, and security concerns. However, given the current situation, addressing these issues becomes tiresome.A considerable structure is included in the transport system to handle these kinds of challenges and ensure that various types of items are delivered on time.

A difficulty with transportation was initially developed by Hitchcock in (1941) [1]. The optimization requirement was proposed by Tjalling, C.Koopmans in (1949) [2]. The transportation algorithms was introduced by Dantzig, G. B. (1963) [3]. Transportation plays a major role in real-world scenarios. Korukoglu and Balli(2011) [4] presented an enhanced version of Vogel's approximation technique for problems related to transportation. The basic question in the transportation problem is to discover the least total transportation cost of a commodity to satisfy requests at destinations using available supplies at origins. In a traditional transportation problem, it is assumed that the values of supply and demands, as well as the costs of transportation, are precisely known. In a traditional transportation problem, it is assumed that the values of supply and demand, as well as the costs of transportation, are precisely known. Stated differently, the decision-makers are unable to precisely know the parameter data, judgment uncertainty, high information cost, or the data or outcome of a real-world transportation problem. Furthermore, these parameters may be more stable since imprecision might arise from a variety of factors, such as incomplete or inaccurate information, a lack of confidence in one's judgment, a high information cost, or the result of specific flexibility required by a source firm or demand marketplace.

A fuzzy transportation problem naturally arises because the imprecision built into the parameters might not be of the probabilistic kind. This kind of uncertainty can be managed with fuzzy parameters. The notion of fuzzy sets, which was introduced by Zadech in (1965)[5], has been widely used in a variety of disciplines, including engineering, management, and economics, and is recognized as a useful instrument for resolving ambiguity and vagueness. Fuzzy sets theory has been developed over the last few decades by a large number of researchers using various techniques and unique innovations. Thereafter, Bellman,R.E., (1970)[6], introduced the concept of decision making problems involving uncertainty. Subsequently, Bellman,R.E (1970)[7], presented the notion of decision-making issues containing uncertainty. Fuzzy linear programming techniques are applied to the linear vector maximum problem by Zimmermann(1978) [8] in order to demonstrate the efficacy of these solutions. They examine the implications of combining different goal functions to achieve the optimal solution utilizing different approaches. Additionally, they offer insightful information on how well fuzzy linear programming works when solving multi-objective problems and give advice on which methods to use to reach the best possible compromise.

Thereafter, S. Chanas et al. (1984)[9] a fuzzy supply and demand model with crisp cost was presented to handle the transportation problem. After that, a number of specialists developed the utilizing a multi-objective (MOTP) model or a single objective transportation problem, that considers different fuzzy contents. Chanas, S. & Kuchata, D.(1996)[10], investigated the TP's form with ambiguous parameters.

After that, specialists from many fields created the transportation problem using the MOTP model, which is multi-objective and takes into account several fuzzy contexts or a single target TP. Tada & Ishii(1996)[11], Kaur & kumar(2011)[12], Considering the MOTP, Li & Lai (2000)[13] implemented into the MOTP a crude concession programming method, A multi-objective transportation facility localization problem was presented by Das, S.K., & Roy, S.K. (2019)[14]. To create an actual transportation network, Das, S.K., et al.(2020)[15], integrated a type-2 intuitionistic ambiguity in two dimensions. Ghosh et al.(2021)[16] created a MOSTP in the intuitionistic fuzzy context that was fixed. Midya et al.(2021)[17] created a multistage, fixed-charge MOSTP in a fuzzy, intuitionistic environment with a green supply network. Prabha(2021)[18], created a geometric mean approach to solve TP in fuzzy Pythagorean contexts. Kundu et al.(2014)[19], Singh & Yadav (2016)[20], Gupta & Anupum(2017)[21], Arora(2018)[22], and Hashmi et al.( 2019)[23] had contributed to inaccurate factors and concentrated on one objective TP, Ahmad & Adhami(2019)[24] seen as non-linearity in MOTPs, resulting in nonlinear membership for each goal function within a system of neutrosophies.

Atanassov(1986)[25] introduced the idea of the Intuitionistic Fuzzy Set (IFS) that included both membership grade $\rho$ further to non-membership grade $\tau$ performs with a margin of hesitation $\pi$ in the way that follows $\rho + \tau \leq 1$ and $\rho + \tau + \eta = 1$ However, there are some situations in which the sum of the membership grade and non-membership grade can be greater than one. Consequently, Senapati, T & Yager, R. R. (2013)[26] Pythagorean Fuzzy Set theory was recently introduced. (PFS), a progression of IFS. The following are some actual uses in Pythagorean fuzzy environments that have been suggested by different academics: Zhang & Xu (2014)[27] introduced a refined rank preference technique and created new creative PFS operating rules in 2014. Furthermore, Mohd & Abdullah (2017)[28] introduced the Pythagorean fuzzy analytic hierarchy method to establish the assessment criterion's weight. And after that, Kumar et al. ( 2019)[29] suggested models for TP optimization in a fuzzy Pythagorean setting. Jeyalakshmi et al.( 2021)[30] outlined the Monalisha technique for solving TP using PFS parameters.

However, in practical implementations, there can be circumstances, like 0.9 and 0.6 supporting and opposing membership in the fuzzy sets, respectively. Since, It defies the requirements of the PFS and IFS limitations. Consequently, Senapati,T., & Yager, R. R. (2020)[31], Senapati,T., & Yager, R. R.( 2020)[32] contrasted FFS with PFSs and IFSs and suggested the concept of FFS to handle these types of situations in decision-making. Senapati,T., & Yager, R. R.(2019)[33]. And after that, Laxminarayan Sahoo(2021)[34] conventional TP is first solved in FFTP, and the optimal solution is subsequently found by employing the QM Windows solver to solve the problem. Thus, Laxminarayan (2021)[35] established a three-point grading system for the FFS and applied the TOPSIS method to tackle a problem involving many criteria in a fuzzy Fermatean environment. Score functions for the defuzzification of FFS were the primary focus of this research study. An approach to transportation problems in Fermatean fuzzy surroundings is presented in this study. The suggested approach uses an orthopair fuzzy set to study the product's supply and demand as well as transportation costs. $\langle \rho, \tau \rangle$ that accomplishes that connection $0 \leq \rho^3 + \tau^3 \leq 1$ and our method of converting the TP into crisp TP problems was to use score functions. The connection $0 \leq \rho^3 + \tau^3 \leq 1$ is considered because, regarding every TP parameter in the Fermatean fuzzy setting, Support for membership and opposition to membership degree add up to a cube that is either equal to or less than one. Next, the QM Window solver was used to solve the converted TP issue in order to determine the best solutions. The suggested approach is finally demonstrated by solving a numerical example, the computed results of which are presented and contrasted with the body of current literature.

The remaining portions of the paper are arranged as follows: under Section 2 Given below are some fundamental definitions of Pythagorean and Fermatean fuzzy sets. In Section, the TP mathematical model is shown 3 and 4. The Part 6, The solution methodology that has been suggested is examined. In section, the outcomes of the fuzzy transportation problem are examined 7. The conclusions are presented in Section 8.

**2. Preliminaries**

This section provides some basic explanations of Fermatean fuzzy sets.

**Table. 1. This section provides some basic explanations of Fermatean fuzzy sets.**

| Authors and References | Year | Significance |
|---|---|---|
| Lid.Xu[37] | 1988 | A fuzzy multi-objective programming technique for |
| Yager,R.R.[38] | 1988 | In multi criteria decision making, order-weighted averaging aggregation operators |
| Shyi-Ming Chen[39] | 2012 | Interval-valued intuitionistic fuzzy sets as the foundation for multi criteria fuzzy decision making |
| Guiwu WEI et al.[40] | 2013 | The applications of fuzzy power aggregation operators to multiple attribute group decision-making |
| Yager, R.R.[41] | 2014 | Multicriteria Decision-Making: Pythagorean |
| Yingdong He et al.[42] | 2014 | Using intuitionistic fuzzy geometric interaction |
| Yingdong He [42] | 2015 | Making decisions with generalised intuitionistic fuzzy power interaction averaging operators |
| Harish Garg et al.[43] | 2015 | Multi-criteria decision making utilising entropy in a |
| Harish Garg[44] | 2016 | Using Einstein's t-norm, generalised intuitionistic fuzzy interactive geometric interaction operators |
| Shu-Ping Wan et al.[53] | 2016 | Using interval-valued fuzzy preference relations in group decision making:an intuitionistic fuzzy programming |
| V.LakshmanaGomathi | 2017 | An intuition-based fuzzy multicriteria system based on |
| Harish Garg[48] | 2017 | Utilizing generalized intuitionistic fuzzy soft sets and group-based approaches for decision-making |
| Harish Garg[49] | 2017 | Pythagorean fuzzy aggregation operators based on confidence levels and their application in decision- |
| Harish Garg[50] | 2018 | Some methods for strategic decision-making problems with immediate probabilities in Pythagorean fuzzy |
| Feng Feng et al.[51] | 2018 | An Alternative Perspective on Multiattribute Decision-making techniques associated with generalized |
| Abhishek Guleria & Rakesh Kumar Bajaj[52] | 2018 | On Pythagorean fuzzy soft matrices, their operations, \\ and medicinal diagnosis applications |
| Muhammad Sajjad Ali khan et al.[57] | 2018 | Multi-criteria group decision-making using Pythagorean fuzzy Einstein prioritized aggregation operators |
| Senapati,T., Yager,R.R.[58] | 2019 | Applications of Fermatean Fuzzy WPM and a Few new Operations over Fermatean fuzzy numbers |
| Harish Garg[44] | 2019 | Pythagorean fuzzy aggregation operators based on neutrality operations and its applications |
| Murat Kirisci[56] | 2019 | Comparing Intuitionistic Fuzzy Parametrized Fuzzy Soft Set and Riesz Summability in Medical decision-making |
| Shu-ping Wan et al.[53] | 2020 | Using interval-valued Atanassov intuitionistic fuzzy programming, an approach for group decision making |
| Shu-Ping Wan, & Jiu-Ying Dong[54] | 2021 | Hybrid multi-criteria group decision making using an interval-valued intuitionistic fuzzy mathematical |
| Xiaolu Zhang, & Zeshul Xu[55] | 2021 | Expanding TOPSIS for Using Pythagorean Fuzzy Sets in Multiple Criteria Decision Making |
| Fang Zhou & Ting-Yu Chen[60] | 2021 | A novel generalized distance measure paradigm and an extended Pythagorean fuzzy VIKOR approach with risk |

| | | |
|---|---|---|
| Muhammad Akram et al.[61] | 2022 | Fermatean fuzzy soft expert knowledge for collective |
| Muhammad Akram et al.[62] | 2022 | A novel framework for group decision-making using 2-tuple linguistic complicated q-rung image fuzzy sets as a |
| Muhammad Akram & Zohra Niaz[63] | 2022 | 2-A Double Linguistic fermatean fuzzy decision-making method using COCOSO and CRITIC |

**2.1. Definition**

A Fermatean fuzzy set on a universal basis X is represented as:

$$\hat{F} = \{\langle x, \rho_{\hat{F}}(x), \rangle, \tau_{\hat{F}}(x) : x \in X\}$$

Where $\rho_{\hat{F}}(x): X \to [0,1]$ and , $\tau_{\hat{F}}(x): X \to [0,1]$ with $0 \le (\rho_{\hat{F}}(x))^3 + (\tau_{\hat{F}}(x))^3 \le 1, \forall x \in X$.

Further, in $\hat{F}$, the degree of membership and non-membership of $x \in X$ are represented by $\rho_{\hat{F}}(x)$ and $\tau_{\hat{F}}(x)$. A Fermatean fuzzy set $\hat{F}$'s degree of uncertainty is expressed as follows:

$$\vartheta_{\hat{F}}(x) = \sqrt[3]{1 - (\rho_{\hat{F}}(x))^3 - (\tau_{\hat{F}}(x))^3}$$

Then set $\hat{F} = \{\langle x, \rho_{\hat{F}}(x), \rangle, \tau_{\hat{F}}(x) : x \in X\}$ is indicated as $\hat{F} = \langle \rho_{\hat{F}}, \tau_{\hat{F}} \rangle$ due to explicitness.

**2.2. Definition**

A few fundamental ideas regarding FFSs are briefly discussed in this section. Let us consider three FFS $\hat{F} = \langle \rho_{\hat{F}}, \tau_{\hat{F}} \rangle$, $\hat{F}_1 = \langle \rho_{\hat{F}_1}, \tau_{\hat{F}_1} \rangle$ and $\hat{F}_2 = \langle \rho_{\hat{F}_2}, \tau_{\hat{F}_2} \rangle$, regarding universal set X and $\eta > 0$. These are the definitions of the elementary operations on the FFS.

1. Addition: $\hat{F}_1 \oplus \hat{F}_2 = \langle \sqrt[3]{(\rho_{\hat{F}_1})^3 + (\tau_{\hat{F}_2})^3 - (\rho_{\hat{F}_1})^3 (\tau_{\hat{F}_2})^3}, \tau_{\hat{F}_1} \tau_{\hat{F}_2} \rangle$
2. Multiplication: $\hat{F}_1 \otimes \hat{F}_2 = \langle \rho_{\hat{F}_1} \rho_{\hat{F}_2}, \sqrt[3]{(\tau_{\hat{F}_1})^3 + (\tau_{\hat{F}_2})^3 - (\tau_{\hat{F}_1})^3 (\tau_{\hat{F}_2})^3} \rangle$
3. Scalar Multiplication: $\eta \odot \hat{F} = \langle \sqrt[3]{1 - (1 - \rho_{\hat{F}}^3)^\eta}, (\tau_{\hat{F}})^\eta \rangle$
4. Exponent: $\hat{F}^\eta = \langle (\tau_{\hat{F}})^\eta, \sqrt[3]{1 - (1 - \rho_{\hat{F}}^3)^\eta} \rangle$
5. Union: $\hat{F}_1 \cup \hat{F}_2 = \langle Max(\rho_{\hat{F}_1}, \rho_{\hat{F}_2}), Min(\tau_{\hat{F}_1}, \tau_{\hat{F}_2}) \rangle$
6. Intersection: $\hat{F}_1 \cap \hat{F}_2 = \langle Min(\rho_{\hat{F}_1}, \rho_{\hat{F}_2}), Max(\tau_{\hat{F}_1}, \tau_{\hat{F}_2}) \rangle$
7. Component: $\hat{F}^c = \langle \rho_{\hat{F}_1}, \tau_{\hat{F}_2} \rangle$

**Example 1.**

Let $\hat{F} = \langle 0.6, 0.3 \rangle$, $\hat{F}_1 = \langle 0.7, 0.6 \rangle$, $\hat{F}_2 = \langle 0.3, 0.8 \rangle$ be three FFS and η=2 be a scalar then:-

1. Addition: $\hat{F}_1 \oplus \hat{F}_2 = \langle 0.7, 0.6 \rangle + \langle 0.3, 0.8 \rangle = \langle 0.711, 0.110 \rangle$ (By definition 2.2)

2. Multiplication: $\hat{F_1} \otimes \hat{F_2} = \langle 0.7,0.6\rangle + \langle 0.3,0.8\rangle = \langle 0.21,0.6282\rangle$ (By definition 2.2)

3. Scalar Multiplication: $\eta \odot \hat{F} = 2 \odot \langle 0.6,0.3\rangle = \langle 0.7276,0.09\rangle$ (By definition 2.2)

4. Exponent: $\hat{F}^{\eta} = \langle (0.6)^2, \sqrt[3]{1-(1-0.3^3)^2}\rangle = \langle 0.36,0.3\rangle$ (By definition 2.2)

5. Union: $\hat{F_1} \cup \hat{F_2} = \langle Max(0.7,0.3), Min(0.6,0.8)\rangle = \langle 0.7,0.8\rangle$ (By definition 2.2)

6. Intersection: $\hat{F_1} \cap \hat{F_2} = \langle Min(0.7,0.3), Max(0.6,0.8)\rangle = \langle 0.3,0.6\rangle$ (By definition 2.2)

7. Component: $\hat{F}^{C} = \langle 0.6,0.3\rangle^{C} = \langle 0.3,0.6\rangle$ (By definition 2.2)

**3. Fermatean fuzzy score function**

Examining FFS $\hat{F} = \langle \rho_{\hat{F}}, \tau_{\hat{F}}\rangle$, subsequently score function of $\hat{F}$ suggested as $S_F(\hat{F}) = (\rho^3_{\hat{F}} - \tau^3_{\hat{F}})$.

In this instance. Certain score functions have been defined by us $S_F(\hat{F}) \in [-1,1]$. Nevertheless, we created a few score functions $S_F(\hat{F})$. $S_F(\hat{F}) \in [0,1]$ that are listed below:

(i). Type1 $S_{1F}(\hat{F}) = \frac{1}{2}(1 + \rho^3_{\hat{F}} - \tau^3_{\hat{F}})$

(ii). Type 2 $S_{2F}(\hat{F}) = \frac{1}{3}(1 + 2\rho^3_{\hat{F}} - \tau^3_{\hat{F}})$

(iii). Type 3 $S_{3F}(\hat{F}) = \frac{1}{2}(1 + \rho^2_{\hat{F}} - \tau^2_{\hat{F}})|\rho_{\hat{F}} - \tau_{\hat{F}}|$

**3. 1. Fermatean fuzzy set Score function**

Examining an FFS $\hat{F} = \langle \rho_{\hat{F}}, \tau_{\hat{F}}\rangle$. Next, the procedure that follows $\hat{F}$ provides an $S_F(\hat{F})$ explanation and representation of the score function.

$$S_F(\hat{F}) = (\rho^3_{\hat{F}} - \tau^3_{\hat{F}})$$

**Property 2.1.** Consider an FFS $\hat{F} = \langle \rho_{\hat{F}}, \tau_{\hat{F}}\rangle$ then $S_F(\hat{F}) \in [0,1]$ then $S_{1F}(\hat{F}) \in [0,1]$, $S_{2F}(\hat{F}) \in [0,1]$ and $S_{3F}(\hat{F}) \in [0,1]$.

**Proof.** For any Fermatean fuzzy set $\rho^3_{\hat{F}} \geq 0$ and $\tau^3_{\hat{F}} \leq 1$

$\Rightarrow 1 - \tau^3_{\hat{F}} \geq 0.$

Hence ,$1 + \rho^3_{\hat{F}} - \tau^3_{\hat{F}} \geq 0$ and certainly $S_{1F}(\hat{F}) \geq 0$.

Again, $\rho^3_{\hat{F}} + \tau^3_{\hat{F}} \leq 1$

$\Rightarrow 1 + \rho^3_{\hat{F}} + \tau^3_{\hat{F}} \leq 2$

$\Rightarrow 1 + \rho^3_{\hat{F}} - \tau^3_{\hat{F}} \leq 2$ as $\tau^3_{\hat{F}} \geq 0$

So, $\frac{1+\rho^3_{\hat{F}}-\tau^3_{\hat{F}}}{2} \leq 1$ and $S_F(\hat{F}) \leq 1$.

For type 1 score function, hence $S_{1F}(\hat{F}) \in [0,1]$, $S_{2F}(\hat{F}) \in [0,1]$ and $S_{3F}(\hat{F}) \in [0,1]$. Especially, if $\hat{F} = \langle 0,1\rangle$, then $S_{1F}(\hat{F}) = 0$, $S_{2F}(\hat{F}) = 0$, $S_{3F}(\hat{F}) = 0$. Again if if $\hat{F} = \langle 0,1\rangle$, then $S_{1F}(\hat{F}) = 1$, $S_{2F}(\hat{F}) = 1$, $S_{3F}(\hat{F}) = 1$.

**3.2. The FFS's Accuracy Function.**

Let us take an FFS. $S_F(\hat{F})$ and indicated in the following aspect.

$$S_F(\hat{F}) = (\rho^3_{\hat{F}} + \tau^3_{\hat{F}})$$

**3.3. The Ranking function of Fermatean fuzzy sets.**

Let us examine these two FFSs. $\hat{F}_1 = \langle \rho_{\hat{F}_1}, \tau_{\hat{F}_1} \rangle$ and $\hat{F}_2 = \langle \rho_{\hat{F}_2}, \tau_{\hat{F}_2} \rangle$, the ranking rules of $\hat{F}_1$ and $\hat{F}_2$ are given an interpretation in the following the manner:

$S^*_F(\hat{F}_1) \geq S^*_F(\hat{F}_2)$ then $Q^*_F(\hat{F}_1) > Q^*_F(\hat{F}_2)$ iff $\hat{F}_1 > \hat{F}_2$

$S^*_F(\hat{F}_1) \leq S^*_F(\hat{F}_2)$ then $Q^*_F(\hat{F}_1) > Q^*_F(\hat{F}_2)$ iff $\hat{F}_1 < \hat{F}_2$

$S^*_F(\hat{F}_1) = S^*_F(\hat{F}_2)$ then $Q^*_F(\hat{F}_1) < Q^*_F(\hat{F}_2)$ iff $\hat{F}_1 = \hat{F}_2$

**1. Example.**

Let $\hat{F}_1 = \langle 0.7, 0.6 \rangle$ and $\hat{F}_2 = \langle 0.5, 0.4 \rangle$ be two FFSs, we have the subsequent procedure, utilizing the type 1 score function

$$S^*_F(\hat{F}_1) = \frac{1}{2}(1 + \rho^3_{\hat{F}} - \tau^3_{\hat{F}}).$$

$$S^*_F(\hat{F}_1) = \frac{1}{2}(1 + 0.7^3 - 0.6^3) = 0.5635$$

$$S^*_F(\hat{F}_2) = \frac{1}{2}(1 + 0.5^3 - 0.4^3) = 0.5305$$

$$S^*_F(\hat{F}_1) \geq S^*_F(\hat{F}_2) \Rightarrow \hat{F}_1 > \hat{F}_2$$

**2. Example.**

Let $\hat{F}_1 = \langle 0.7, 0.5 \rangle$ and $\hat{F}_2 = \langle 0.9, 0.3 \rangle$ be two FFSs, we have the subsequent procedure, utilizing the type 2 score function $S^*_F(\hat{F}_2) = \frac{1}{3}(1 + 2\rho^3_{\hat{F}} - \tau^3_{\hat{F}})$.

$$S^*_F(\hat{F}_1) = \frac{1}{3}(1 + 2(0.7)^3 - 0.6^3) = 0.3630$$

$$S^*_F(\hat{F}_2) = \frac{1}{2}(1 + 2(0.8)^3 - 0.7^3) = 0.5603$$

$$S^*_F(\hat{F}_1) \leq S^*_F(\hat{F}_2) \Rightarrow \hat{F}_1 < \hat{F}_2$$

**4. Mathematical formulation.**

Analyze a TP extensive u source points and v incurable points, where $b_j > 0$ units are needed by the $j^{th}$incurable point and $a_i > 0$ units are transmitted by the $i^{th}$ source point. A unit shipping cost, $c_{ij}$, exists for each link $(i,j)$ from source point $i^{th}$ to incurable j. Finding the cost of the coefficient of solution for sending the given data is the work, which must be completed in order to meet specifications and reduce overall transport costs. $x_{ij}$ indicates the number of groups that need to be transferred from source i to incurable j. The mathematical expression for the standard TP is as follows:

$$\text{Minimize } Z_0 = \sum_{i=1}^{u} \sum_{j=1}^{v} C_{ij} X_{ij} \quad (1)$$

Subject to:

$\sum_{j=1}^{v} X_{ij} \leq a_i$, for i $= 1,2,\ldots,$ u

$\sum_{j=1}^{v} X_{ij} \leq b_j$, for i $= 1,2,\ldots,$ v

$x_{ij} \geq 0, \forall\, i, j$

When $a_i$, $b_j$, $c_{ij}$ or even when Fermatean fuzzy parameters are accepted for any of them separately. According, FFTP is ordinary TP in (1). With $0 \leq (\rho)^3 + (\tau)^3 \leq 1$, the FFS have the form $\langle \rho, \tau \rangle$. The Fermatean fuzzy transportation problem can be formulated in this manner:

$$\text{Min } \langle \rho_{x^\wedge_0}, \tau_{x^\wedge_0} \rangle = \sum_{i=1}^{u} \sum_{j=1}^{v} \langle \rho_{z^\wedge_0}, \tau_{z^\wedge_0} \rangle \quad (2)$$

Subject to :

$\sum_{j=1}^{v} x_{ij} \leq \langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle$, for $i = 1, 2, \ldots, u$

$\sum_{i=1}^{v} x_{ij} \leq \langle \rho_{b^\wedge_i}, \tau_{b^\wedge_i} \rangle$, for $j = 1, 2, \ldots, v$

Where $0 \leq (\rho_{z^\wedge_0})^3 + (\tau_{z^\wedge_0})^3 \leq 1$,

$0 \leq (\rho_{a^\wedge_i})^3 + (\tau_{a^\wedge_i})^3 \leq 1$, i=1,2,…,u

$0 \leq (\rho_{b^\wedge_j})^3 + (\tau_{b^\wedge_j})^3 \leq 1$, j=1,2,…,v

$x_{ij} \geq 0$, and $0 \leq (\rho_{c^\wedge_{ij}})^3 + (\tau_{c^\wedge_{ij}})^3 \leq 1$,

i=1,2,…,u j=1,2,…,v

The computational model of a TP in Fermatean fuzzy surroundings,or FFTP, is now problem(2). It should be noted that the FFTP is considered balanced, or imbalanced FFTP, if $\sum_{i=1}^{u} \oplus \langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle = \sum_{i=1}^{u} \oplus \langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle$. Fermatean additives are used to describe the structure $\sum\oplus$ as an integration.

### 4.1. The proposed method algorithm for solving FFTP.

In various fuzzy circumstances, the literature now in publication offers a range of methods for expressive both the optimal and initial feasible solutions for the transportation problem (TP). Intuitionistic fuzzy, Pythagorean fuzzy etc. The linear programming methodology, Vogel's approximation method, row, column, and matrix minima, as well as the north-west corner. Moreover, there are several algorithmic evolution approaches that aim to address the same issue. All the parameters in a Fermatean fuzzy environment are represented as FFSs, but no researcher has considered the TP inside this setting. Consequently, we solved the TP in the Fermatean fuzzy environs in this study using the well-known QM-Solver. The suggested algorithm involves the following steps:

1). As well as Fermatean fuzzy supply and demand, ascertain the score function value for each of these situations.

2). Determine the entire supply and total demand to confirm the transportation issue.

3). If supply and demand are equal throughout, go to step 7; if not, go to the next step. This suggests a balanced approach to the transportation issue.

4). To improve the balance, add a dummy variable in step 7 if the supply and demand are not equal.

5). The transportation issue should be expressed as a linear programming problem (LPP).

6). Use the QM for Windows solver to discover the best solution to the balanced transportation problem.

**Table 2: Enter data concerning supply, demand, and transportation costs.**

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Supply |
|---|---|---|---|---|---|
| $Z_1$ | ⟨0.1,0.8⟩ | ⟨0.3,0.7⟩ | ⟨0.2,0.7⟩ | ⟨0.3, 0.7⟩ | ⟨0.8,0.2⟩ |
| $Z_2$ | ⟨0.2,0.8⟩ | ⟨0.1,0.9⟩ | ⟨0.7,0.1⟩ | ⟨0.8, 0.1⟩ | ⟨0.7,0.2⟩ |
| $Z_3$ | ⟨0.6,0.3⟩ | ⟨0.4,0.6⟩ | ⟨0.7,0.2⟩ | ⟨0.8,0.2⟩ | ⟨0.9,0.1⟩ |
| Demand | ⟨0.3, 0.8⟩ | ⟨0.6,0.4⟩ | ⟨0.8, 0.2⟩ | ⟨0.6,0.5⟩ | |

## 5. The analysis and findings with a numerical example.

We provide an appropriate example in this part to demonstrate our suggested method for solving the problem. Examine an FFTP where Table 4 provides all of the parameters. $\langle \rho_{a\hat{}_i}, \tau_{a\hat{}_i}\rangle$. supplies i $=$ 1,2,3 requests for each j=1,2,3,4, $\langle \rho_{b\hat{}_i}, \tau_{b\hat{}_i}\rangle$, and expenses $\langle \rho_{c\hat{}_{ij}}, \tau_{c_{ij}}\rangle$. We regard $i = 1,2,3$: $j = 1,2,3,4$ to be FFNs. In this case, there are four destinations: $H_1$, $H_2$, $H_3$, and $H_4$ and three origins: $Z_1$, $Z_2$ and $Z_3$. The following LPP(3), for which Table 4 contains all of the input data, was solved before we could solve the FFTP.

$$\text{Minimize } S_{qF}\langle \rho_{z\hat{}_0}, \tau_{z\hat{}_0}\rangle \sum_{i=1}^{3}\sum_{j=1}^{4} S_{qF}\langle \rho_{c\hat{}_{ij}}, \tau_{c_{ij}}\rangle \odot x_{ij} \quad (3)$$

Subject to

$$\sum_{j=1}^{4} x_{ij} \leq S_{qF}(\langle \rho_{a\hat{}_i}, \tau_{a\hat{}_i}\rangle), \quad i = 1,2,3,$$

$$\sum_{i=1}^{3} x_{ij} \leq S_{qF}(\langle \rho_{\hat{b}_i}, \tau_{\hat{b}_i}\rangle), \quad j = 1,2,3,4$$

Where $0 \leq (\rho_{z\hat{}_0})^3 + (\tau_{z\hat{}_0})^3 \leq 1$,

$$0 \leq (\rho_{a\hat{}_i})^3 + (\tau_{a\hat{}_i})^3 \leq \quad i = 1,2,3$$

$$0 \leq (\rho_{b^\wedge_j})^3 + (\tau_{b^\wedge_j})^3 \leq \quad j = 1,2,3,4$$

$x_{ij} \geq 0,$ and $0 \leq (\rho_{c^\wedge_{ij}})^3 + (\tau_{c^\wedge_{ij}})^3 \leq 1,$

i =1,2,3, j=1,2,3,4

LPP(4), LPP(5) and LPP(6) are our three LPPs, using group 1(q=1), and group 2(q=2), and group(q=3) score functions and Table 3 in LPP(3). Theses are mentioned as follows:

Minimize $S_{qF}\langle \rho_{z^\wedge_0}, \tau_{z^\wedge_0} \rangle \sum_{i=1}^{3} \sum_{j=1}^{4} S_{qF}\langle \rho_{c^\wedge_{ij}}, \tau_{c_{ij}} \rangle \odot x_{ij}$ (4)

Subject to

$\sum_{j=1}^{4} x_{ij} \leq S_{1F}(\langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle), \quad i = 1,2,3,$

$\sum_{i=1}^{3} x_{ij} \geq S_{1F}(\langle \rho_{b^\wedge_i}, \tau_{b^\wedge_i} \rangle), \quad j = 1,2,3,4$

Where $0 \leq (\rho_{z^\wedge_0})^3 + (\tau_{z^\wedge_0})^3 \leq 1,$

$$0 \leq (\rho_{a^\wedge_i})^3 + (\tau_{a^\wedge_i})^3 \leq \quad i = 1,2,3$$

$$0 \leq (\rho_{b^\wedge_j})^3 + (\tau_{b^\wedge_j})^3 \leq \quad j = 1,2,3,4$$

$x_{ij} \geq 0,$ i=1,2,3, j=1,2,3,4

Minimize $S_{qF}\langle \rho_{z^\wedge_0}, \tau_{z^\wedge_0} \rangle \sum_{i=1}^{3} \sum_{j=1}^{4} S_{qF}\langle \rho_{c^\wedge_{ij}}, \tau_{c_{ij}} \rangle \odot x_{ij}$ (5)

Subject to

$\sum_{j=1}^{4} x_{ij} \leq S_{2F}(\langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle), \quad i = 1,2,3,$

$\sum_{i=1}^{3} x_{ij} \geq S_{2F}(\langle \rho_{b^\wedge_i}, \tau_{b^\wedge_i} \rangle), \quad j = 1,2,3,4$

Where $0 \leq (\rho_{z^\wedge_0})^3 + (\tau_{z^\wedge_0})^3 \leq 1,$

$$0 \leq (\rho_{a^\wedge_i})^3 + (\tau_{a^\wedge_i})^3 \leq \quad i = 1,2,3$$

$$0 \leq (\rho_{b^\wedge_j})^3 + (\tau_{b^\wedge_j})^3 \leq \quad j = 1,2,3,4$$

$x_{ij} \geq 0,$ i=1,2,3, j=1,2,3,4

Minimize $S_{qF}\langle \rho_{z^\wedge_0}, \tau_{z^\wedge_0} \rangle \sum_{i=1}^{3} \sum_{j=1}^{4} S_{qF}\langle \rho_{c^\wedge_{ij}}, \tau_{c_{ij}} \rangle \odot x_{ij}$ (6)

Subject to

$\sum_{j=1}^{4} x_{ij} \leq S_{3F}(\langle \rho_{a^\wedge_i}, \tau_{a^\wedge_i} \rangle), \quad i = 1,2,3,$

$$\sum_{i=1}^{3} x_{ij} \geq S_{3F}\left(\langle \rho_{b^\wedge_i}, \tau_{b^\wedge_i} \rangle\right), \; j = 1,2,3,4$$

Where $0 \leq (\rho_{z^\wedge_0})^3 + (\tau_{z^\wedge_0})^3 \leq 1,$

$$0 \leq (\rho_{a^\wedge_i})^3 + (\tau_{a^\wedge_i})^3 \leq \quad i = 1,2,3$$

$$0 \leq (\rho_{b^\wedge_j})^3 + (\tau_{b^\wedge_j})^3 \leq \quad j = 1,2,3,4$$

$x_{ij} \geq 0$, i=1,2,3, j=1,2,3,4

**Table 3: Enter data concerning supply, demand, and transportation costs.**

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Supply |
|---|---|---|---|---|---|
| $Z_1$ | 0.1633 | 0.2370 | 0.2243 | 0.2370 | 0.7520 |
| $Z_2$ | 0.1680 | 0.0910 | 0.5616 | 0.6743 | 0.6675 |
| $Z_3$ | 0.5945 | 0.4240 | 0.6675 | 0.7520 | 0.8640 |
| Demand | 0.2575 | 0.5760 | 0.7520 | 0.5455 | |

Using the recommended algorithm, the type-1 FFTP has the following as its optimal solution: $x_{13}$ = 0.2065, $x_{14}$=0.5455, $x_{21}$=0.2575, $x_{22}$=0.410, $x_{32}$=0.166, $x_{33}$=0.5455. Consequently, The type-1 FFTP has a minimum expense of 0.6907.

**Table 4: Enter data concerning supply, demand, and transportation costs.**

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Supply |
|---|---|---|---|---|---|
| $Z_1$ | 0.1633 | 0.2370 | 0.2213 | 0.2370 | 0.6720 |
| $Z_2$ | 0.1680 | 0.0910 | 0.5616 | 0.6743 | 0.5593 |
| $Z_3$ | 0.4683 | 0.6880 | 0.5593 | 0.6720 | 0.1523 |
| Demand | 0.1806 | 0.4560 | 0.6720 | 0.4356 | |

Using the recommended algorithm, the type-1 FFTP has the following as its optimal solution: $x_{13}$ = 0.597, $x_{14}$ = 0.075, $x_{21}$ = 0.1033, $x_{22}$ = 0.456, $x_{31}$ = 0.0773, $x_{33}$ = 0.075. Consequently, The type-2 FFTP has a minimum expense of 0. 31983.

**Table 5: Enter data concerning supply, demand, and transportation costs.**

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Supply |
|---|---|---|---|---|---|
| $Z_1$ | 0.1295 | 0.1200 | 0.1375 | 0.1200 | 0.4800 |
| $Z_2$ | 0.1200 | 0.1000 | 0.4440 | 0.5705 | 0.3625 |
| $Z_3$ | 0.1905 | 0.0800 | 0.3625 | 0.4800 | 0.7200 |
| Demand | 0.1125 | 0.1200 | 0.4800 | 0.0555 | |

Using the recommended algorithm, the type-3 FFTP has the following as its optimal solution: $x_{13}$ = 0.4245, $x_{14}$ = 0.555, $x_{21}$ = 0.1125, $x_{32}$ = 0.120, $x_{33}$ = 0, Consequently, The type-3 FFTP has a minimum expense of 0. 2887.

**Table 6: The LPP(4), LPP(5), and LPP(6) numerical outcomes**

| Method | Model | Score function | Optimal Solutions | Minimum Cost |
|---|---|---|---|---|
| Our proposed Method | LPP(4) | $S_{1F}(\hat{F}) = \frac{1}{2}(1 + \rho_{\hat{F}}^3 - \tau_{\hat{F}}^3)$ | $X_{13}$=0.2065, $X_{14}$=0.5455, $X_{21}$=0.2575, $X_{22}$=0.410, $X_{32}$=0.166, $X_{33}$=0.5455, | $S_{1F}(\hat{F})$ =0.6907 |
| | LPP(5) | $S_{2F}(\hat{F}) = \frac{1}{3}(1 + 2\rho_{\hat{F}}^3 - \tau_{\hat{F}}^3)$ | $X_{13}$=0.0.597, $X_{14}$=0.075, $X_{21}$=0.1033, $X_{22}$=0.456, $X_{31}$=0.0773, $X_{33}$=0.075, | $S_{2F}(\hat{F})$ =0.2887 |
| | LPP(6) | $S_{3F}(\hat{F}) = \frac{1}{2}(1 + \rho_{\hat{F}}^2 - \tau_{\hat{F}}^2)\lvert\rho_{\hat{F}} - \tau_{\hat{F}}\rvert$ | $X_{13}$=0.4245, $X_{14}$=0.555, $X_{21}$=0.1125, $X_{22}$=0.120, $X_{31}$=0.120, $X_{33}$=0.555, | $S_{3F}(\hat{F})$ =0.1082 |

**Table: 7 Comparison of numerical results for our results and proposed results by L. Shaoo**

| Method | Model | Score function | Optimal Solutions | Minimum Cost |
|---|---|---|---|---|
| Our proposed Method | LPP(4) | $S_{1F}(\hat{F}) = \frac{1}{2}(1 + \rho_{\hat{F}}^3 - \tau_{\hat{F}}^3)$ | $X_{13}$=0.2065, $X_{14}$=0.5455, $X_{21}$=0.2575, $X_{22}$=0.410, $X_{32}$=0.166, $X_{33}$=0.5455, | $S_{1F}(\hat{F})$ =0.6907 |
| | LPP(5) | $S_{2F}(\hat{F}) = \frac{1}{3}(1 + 2\rho_{\hat{F}}^3 - \tau_{\hat{F}}^3)$ | $X_{13}$=0.0.597, $X_{14}$=0.075, $X_{21}$=0.1033, $X_{22}$=0.456, $X_{31}$=0.0773, $X_{33}$=0.075, | $S_{2F}(\hat{F})$ =0.2887 |
| | LPP(6) | $S_{3F}(\hat{F}) = \frac{1}{2}(1 + \rho_{\hat{F}}^2 - \tau_{\hat{F}}^2)\lvert\rho_{\hat{F}} - \tau_{F}\rvert$ | $X_{13}$=0.4245, $X_{14}$=0.555, $X_{21}$=0.1125, $X_{22}$=0.120, $X_{31}$=0.120, | $S_{3F}(\hat{F})$ =0.1082 |

| | | | | |
|---|---|---|---|---|
| | | | $X_{33}$=0.555, | |
| Proposed method by L. Sahoo | LPP(4) | $S_{1F}(\hat{F}) = \frac{1}{2}(1 + \rho^3_{\hat{F}} - \tau^3_{\hat{F}})$ | $X_{13}$=0.2065, $X_{14}$=0.5455, $X_{21}$=0.2065, $X_{22}$=0.5455, $X_{32}$=0.756, $X_{33}$=0.108, | $S_{1F}(\hat{F})$ =0.6907 |
| | LPP(5) | $S_{2F}(\hat{F}) = \frac{1}{3}(1 + 2\rho^3_{\hat{F}} - \tau^3_{\hat{F}})$ | $X_{12}$=0.0198, $X_{14}$=0.473, $X_{21}$=0.361, $X_{22}$=0.395, $X_{33}$=0.6743, $X_{34}$=0.108, | $S_{2F}(\hat{F})$ =0.2887 |
| | LPP(6) | $S_{3F}(\hat{F}) = \frac{1}{2}(1 + \rho^2_{\hat{F}} - \tau^2_{\hat{F}})\lvert\rho_{\hat{F}} - \tau_{\hat{F}}\rvert$ | $X_{14}$=0.4245, $X_{21}$=0.555, $X_{22}$=0.1125, $X_{33}$=0.5705, | $S_{3F}(\hat{F})$ =0.1082 |

The numerical outcomes of solving LPP using the QM window solver are shown in Table 6. Table 7 presents the comparison results. The Fermatean fuzzy cost of 0.6907, 0.2887, and 0.1082 is visible. In a Fermatean fuzzy environment, the score function thus provides the ideal solution to a fuzzy transportation problem. Additionally, Table 7 shows that the scoring function values are contrasted with Laxminarayan Sahoo.

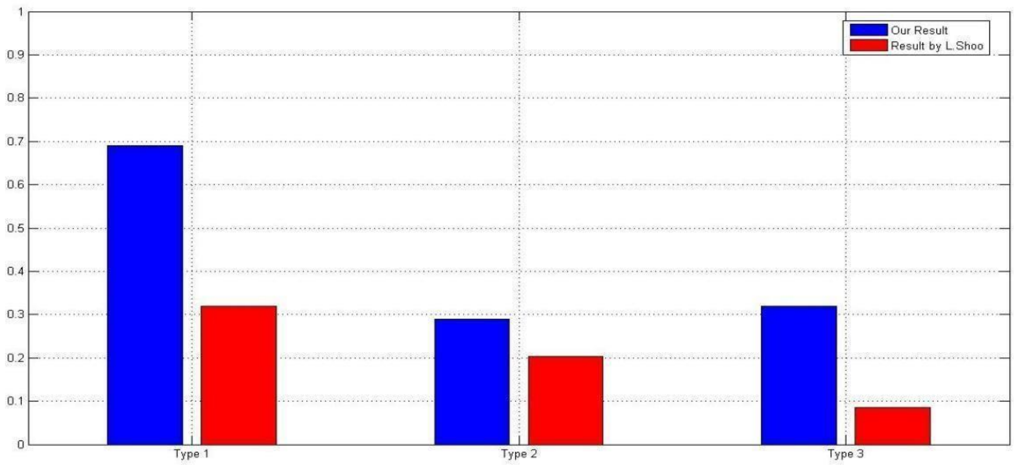


**Figure. 1 Comparison results for our results and Proposed results by L. Shaoo**

Graph 1 additionally demonstrates that our suggested method produces remarkable results for three different forms of FFTP in a Fermatean fuzzy scenario when compared to Laxminarayan Sahoo's strategy. Consequently, we claimed that our method of addressing the transportation problem in an ambiguous environment is unique. In addition, the Fermatean fuzzy transportation problem is what we have for our numerical experiment. It implies that our recommended strategy is a fresh approach to handling uncertainty in Fermatean fuzzy circumstances.

**6. Conclusion.**

The parameters relating to the transportation problem involve Fermatean fuzzy sets. The transportation issue, where costs, supply, and needs are not well defined in real-world situations, frequently dominates decision-making. Fermatean fuzzy parameters provide a more practical solution than accurate parameters in transportation problems since uncertainty is a common occurrence in real-life situations. Computing results show that Fermatean fuzzy sets are more appropriate than PFSs and can handle higher levels of uncertainty. It is now possible to ensure that the score function value for any Fermatean fuzzy sets falls inside the unit interval by presenting a new definition of the score function for Fermatean fuzzy sets. The suggested approach covered here is incredibly straightforward, easy to use, and effective for addressing practical decision-making issues in fields including operations, engineering, and management science. Future research and additional decision-making topics involving Fermatean fuzzy parameters.

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