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Finite Range Probability Distribution for Reliability Theory and Theoretical Physics

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Abstract:

The aim of this paper is to develop a probability distribution which can be used in the reliability theory and in theoretical Physics. Some basic properties of the distribution have been discussed and their mathematical forms have been evaluated.

Keywords: Finite Range distribution

1. INTRODUCTION:

Development of new probability distributions definitely increases the family of probability distributions and reduces the risk of using approximately near distribution. There are many research workers who have developed probability distribution in recent times, such as, Mukherjee and Islam (1983), Siddiqui et al(1992,1994,1995,2016).

This distribution will be useful when observations are presented in the form of percent increment or in decreasement.

Probability is the language of statistical mechanics. It is also fundamental to the understanding of quantum mechanics. In statistical mechanics the physical problems concern large groups of particles, like molecules in a gas. It is not possible to track every single particle’s motion; statistical mechanics uses probability distributions to describe the average behavior of the system. Probability is the backbone of thermodynamics. It enables us understand the likelihood of a system transitioning between different states and how it evolves over time. Unlike classical mechanics, in quantum mechanics particles do not have fixed properties. Their behavior is described by probability distributions. The probability amplitude of finding a particle in a certain state gives us the likelihood of that outcome.

In a radioactive decay process, unstable nuclei transform into more stable ones by emitting particles. The Poisson distribution enables us to model the number of decays that could happen in a given time period, given the average rate of decay. Reif (2009) discussed Statistical Physics in detail, Roe (2012) discussed the role of the theory of probability in experimental Physics. Kuzemsky (2016) discussed the use of the theory of probability, Michael (2021) discussed the probability related ideas across the theory of Physics.

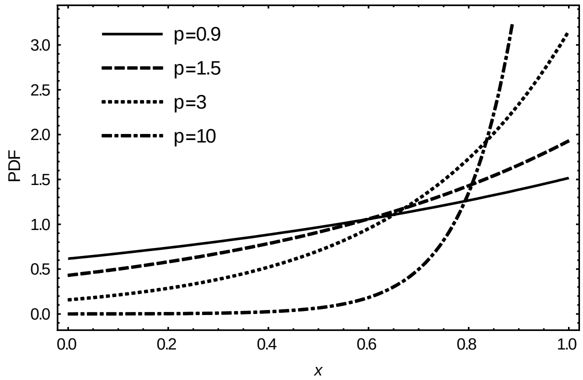
2. Proposed probability distribution

The probability density function of the proposed distribution is;

$f(x) = \frac{p}{e^p-1} e^{xp}$, $\quad 0 < x \leq 1,\ p < 1$ … (1)

Graph of pdf of the proposed distribution

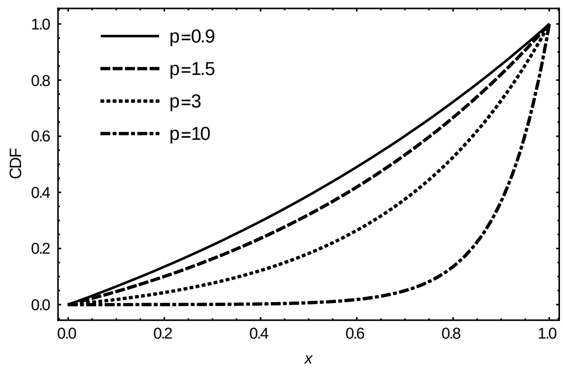
Where 'p' is the parameter of the distribution, both variable and the parameter are having similar range.



And the cumulative distribution function is;

$F(x) = \frac{e^{xp}-1}{e^p-1}, 0 < x \leq 1,\ p < 1$ … (2)

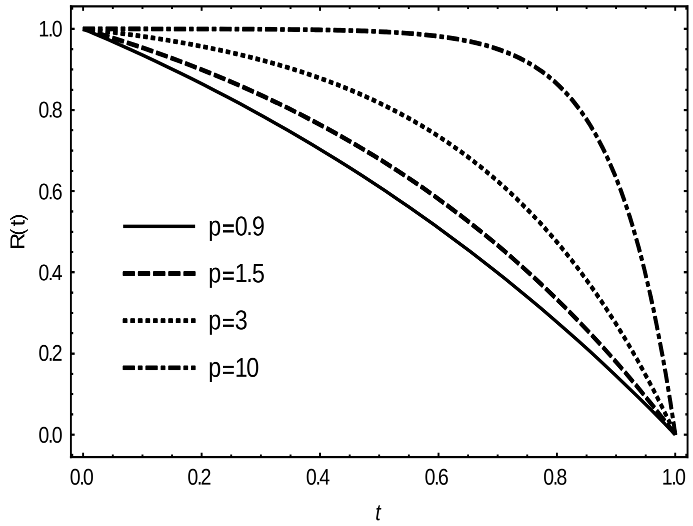
Graph of cumulative distribution function



Reliability function

$$R(t) = 1 - F(t) = 1 - \frac{e^{tp}-1}{e^p-1} = \frac{e^p - e^{tp}}{e^p-1}, \quad t > 0$$

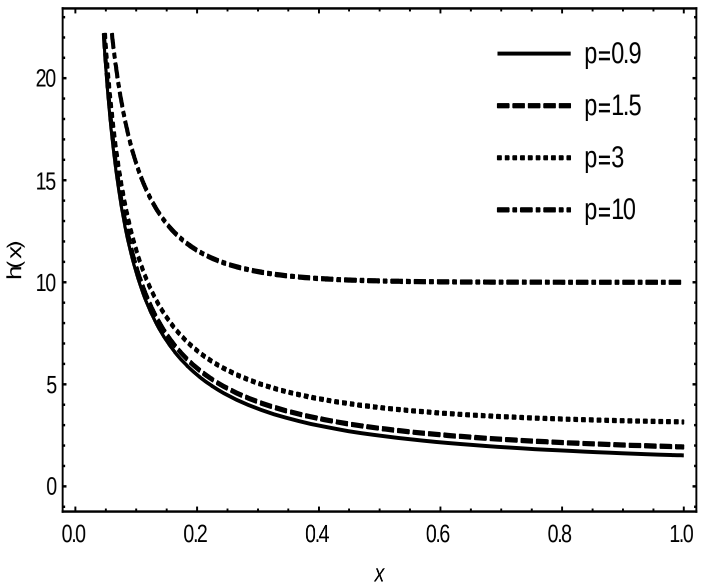
Graph of the Reliability function



Hazard Rate Function

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{pe^{tp} - 1}{e^{p} - 1}}{\frac{e^{tp} - 1}{e^{p} - 1}} = \frac{pe^{tp}}{e^{tp} - 1} \; t > 0$$

Graph of the HAZARD RATE function



3. CHARACTERIZATION OF THE DISITRIBUTION: MOMENTS GENERATING AND CHARACTERISTIC FUNCTIONS

3.1 Moments Generating Function

$$M_{x(t)} = E(e^{tx})$$

$$= \int_0^1 e^{tx} f(x)dx$$

$$M_{x(t)} = \frac{p(e^{t+p}-1)}{(e^p-1)(t+p)} \quad \ldots (3)$$

3.2 Characteristic Function

The characteristic function (c.f.) of the model can be obtained as below:

$$\phi_{x(t)} = E(e^{itx})$$

$$= \int_0^1 e^{itx} f(x)dx$$

$$\phi_{x(t)} = \frac{p(e^{it+p}-1)}{(e^p-1)(it+p)} \quad \ldots (4)$$

4. BASIC PARAMETERS OF THE DISTRIBUTION

4.1. r^{th} Moment About Origin

The r^{th} moment about origin is given by

$$\mu_r' = E(X^r)$$

$$\mu_r' = \frac{\Gamma(r+1,-p)-\Gamma(r+1,0)}{(-p)^r(e^p-1)} \quad \ldots (5)$$

This in turn gives the following results:

4.2 Mean

$$\boldsymbol{E(X) = \int_0^1 x\, f(x) dx}$$

$$\boldsymbol{E(X) = \int_0^1 x \frac{p}{e^p-1} e^{xp} dx}$$

$$\boldsymbol{E(X) = \frac{(e^p(p-1)+1)}{p(e^p-1)}} \quad \boldsymbol{\ldots.(6)}$$

$$\boldsymbol{\mu_1' = E(X) = \frac{e^{2p}}{e^p-1}}$$

$$\boldsymbol{\mu_2' = E(X) = \frac{2e^{2p}}{p^2(e^p - 1)}}$$

4.3 Variance

Since variance, $V(X) = E(X^2) - (E(X))^2 = (\mu_2' - \mu_1')^2$

$$\boldsymbol{V(X)} = \frac{2(e^p-1)-p^2}{p^2(e^p-1)^2} \qquad \text{.....(7)}$$

4.4 Median

To obtain the median we proceed as follows

$$\int_0^{Me} (1-p)\, x^{-p}\, dx = \frac{1}{2}$$

$$Me^{1-p} = \frac{1}{2}$$

Taking Log on both sides; we get

$$(1-p)\ln Me = \ln\frac{1}{2}$$

$$Me = \frac{e^{-3.010}}{1-p} \qquad \text{... (8)}$$

5. ESTIMATION OF PARAMETER

6.1 Maximum Likelihood Estimator of Parameter

$L = \prod_{i=1}^{n} \frac{p.e^{xp}}{e^p - 1}$

$= \frac{p^n}{(e^p-1)} e^{p\sum_{i=1}^{n} x_i}$

$$\boldsymbol{ln\, L = n\, ln\, p - n\, ln(e^p - 1) + p\sum_{i=1}^{n} x_i}$$

$$\boldsymbol{\hat{p} = \frac{n(e^p - 1)}{ne^p + (e^p - 1)\sum_{i=1}^{n} x_i}}$$

.....(9)

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