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# An Innovative Distance Measure for Picture Fuzzy Sets and Applications in Medical Diagnosis and Pattern Analysis

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**Abstract:**

In this paper, we proposed a novel distance measure and applied on picture fuzzy sets which have the various applications in uncertain problems that cannot be solved easily by fuzzy set, intuitionistic fuzzy sets, fermatean fuzzy sets, Pythagorean fuzzy sets, ortho-pair fuzzy sets easily. We can explore these sets with the help of problems in the field of personnel selection, pattern analysis, medical diagnosis, or human voting etc. This field type of problem required the answer in the form of yes, no, refusal and abstain. The distance measures play an important role for comparing the picture fuzzy sets. If we take a look in the background of literature then we find a lot of works have been done on distance measures for picture fuzzy sets. Unreasonable results in most of the problems have been found, all of these distance measures. So, we are suggesting a new distance measures for picture fuzzy sets in this paper which is more effective and useful compare to the already existed distance measures. We also illustrating its importance, classification and medical problems and view on performance with existed measures.

**Keywords**: Distance measure, fuzzy set, picture fuzzy set, intuitionistic fuzzy set, medical diagnosis, pattern analysis.

## 1. Introduction

Zadeh introduced the theory of fuzzy sets (FSs), emphasizing the notion of degrees of membership and non-membership. The applications of fuzziness has been extended to learning theory, algorithms, formal languages, automata, probability theory and it deals the imprecise, uncertainties and vagueness problems in an efficient way [1], [2]. It is also explained that fuzzy parametric function and nonfuzzy mappings are treated as special classes of fuzzy functions and mappings. Specially in situation of real life, due to imprecise nature of medical issues and uncertain information collected for decision making needs the concept of fuzzy[3], [4], [5], [6], [7]. In fuzzy set theory (FST), each element is given a membership value that ranges between 0 and 1. However sometimes it is not possible to assign for a membership function, therefore, it is more practical to assign an interval value. Fuzzy sets theory (FST) has been applied in different fields such as transmission system, medical diagnosis, facial pattern recognition, cluster point and decisions making process etc. The element's value in a fuzzy set (FSs) cannot be chosen independently, hence the concept of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov. [8]. Atanassov described intuitionistic fuzzy sets as having every element assigned a membership value and non-membership value, both of which lies in the closed interval [a, b] where 0$\leq a, b \leq 1$ and their sum must be less than or equal to one. This restriction of the sum of membership and non-membership values bound the scope of (IFSs) and opened the way for Yager [5] to the extended concept of Pythagorean fuzzy sets (PFSs). In a Pythagorean fuzzy set (PFSs), every member has a membership and non-membership value lies in the closed interval a, b] where 0$\leq a, b \leq$1 its property their square sum equal or less than to 1. Although Pythagorean fuzzy sets (PFSs) are much quicker compare to fuzzy sets (FSs) and Intuitionistic fuzzy sets (IFSs) as they cannot solve the conditions in which the square sum of membership grades exceeds one. Then, the next concept of generalized ortho pair fuzzy sets was introduced by Yager [6], and he named it q-rung orthopair fuzzy set (q-ROPFSs). Q-ring ortho-pair fuzzy set (q-ROPFSs) is characterized by each member having a membership value and a non-membership value both of which lies in the closed interval [0,1] using the power sum equal or less than 1. Indeed, these extensions of fuzzy set (FSs) are insufficient to consider for the neutrality degree of a member which play a vital role in various decision-making problems, often referred as voting, personnel selection, pattern analysis medical diagnosis and cluster point, etc. To fill this gap, picture fuzzy set (PFSs) was an extension of fuzzy set (FSs), which was suggested by Coung and Kreinovich [9], [10]. In a picture, fuzzy set (PFSs), every element is represented by three values referred by a degree by positive membership, a degree by negative membership, and a degree of neutral membership. In a picture fuzzy set, every member having membership value, a non-membership value and a neutrality value lies between the closed interval [0,1] with all sums equal or less than 1. Coung[9] also defined the picture fuzzy sets (PFSs) properties or operations. After that, Son[11], [12], [13] applied picture fuzzy sets (PIFSs) to solve clustering problems. Nguyen et al.[14] applied picture fuzzy sets into geographic data clustering applications. Some applications of picture fuzzy sets in databases and studies on some fuzzy logic operators for picture fuzzy sets were introduced by [13], [15]. Atanassov [16] has presented new results on intuitionistic fuzzy sets (IFSs), opening the door for further work. In another study, Atanassov [17] introduced new types of operations for intuitionistic fuzzy sets (IFSs). Later, Atanassov [18] defined several operators for interval-valued intuitionistic fuzzy sets, which are various applicable in various application to solve the real life problems. Atanassov[19] pro Meredith's axiom, axiom, which is valid for the intuitionistic fuzzy propositional calculus. Atanassov and Gargov have generalized the notion of intuitionistic fuzzy sets in the spirit of ordinary interval-valued fuzzy sets (IVFSs). In this paper, basic preliminaries of (IVIFSs) theory are determined. The relation of equivalencebetween two picture fuzzy sets and their applications in clustering were discussed in [20]. Additionally, [21] introduced methods for comparing two picture fuzzy sets (PFSs), as well as distance and dissimilarity measure operators for picture fuzzy sets (PFSs).

In this section we will discuss about the distance measures. Some distance measures in picture fuzzy sets(PIFSs) are introduced by Dutta [22] and Son[22] introduced few generalized distance measures using (PIFSs) for application in Clustering analysis. Joshi [23], [24], [25], [26] dealing with comparative study of distance measures on picture fuzzy sets. In strategic decision-making, Wei [27] explored similarity measures for picture fuzzy sets (PIFSs) using cosine and cotangent functions. Peng [28] introduces an algorithm for picture fuzzy sets (PIFSs) and applying this algorithm into the decision-making process[29], [30], [31]. Wei [32] contributes cosine, weighted cosine, weighted set-theoretic similarity measures for (PIFSs), describing their applications in pattern recognition problems. Picture Fuzzy Set (PFSs) serves as an extension of conventional fuzzy sets (FS) and intuitionistic fuzzy sets

(IFSs) introduced by Atanassov in 1986. Within the framework pf PFSs, elements exhibit positive, negative, neutral and refusal degrees, each elucidating the diverse levels of importance attributed to a member within a given set [33]. This extension provides a nuanced representation that captures a richer spectrum of information, allowing for a more comprehensive characterization of membership relationships[34].

Several significant factors motivated us to conduct this research which are following as:

- There is several (PIFSs) distance measure that do not satisfy all the necessary fundamental conditions
- Many existing (PIFSs) distance measures deliver unreasonable results when computing distances between different (PIFSs).
- Existing distance measures for (PIFSs) unable to identifying unknown patterns in problems related to pattern recognition.
- In view of these factors, this paper proposes a novel distance measure for (PIFSs) which explores its application in classification and medical problems
- This paper contributes including and introducing an innovative distance measure for (PIFSs) along with its properties.
- Proving, the proposed measure through numerical problems, that the proposed measure showing the limitations of existing distance measures.
- Illustrating how the proposed measure can be utilized in pattern analysis, medical diagnosis and comparing its efficiency to the existing measures.

The paper is divided mainly into the 6 section that follows. Preliminary is given in Section 2. In Section 3 a carefully reviewed covered the existed measures (PIFSs). In Section 4, a new distance measure for (PIFSs) is proposed together with its properties. Numerical problems are used to comparing this proposed measure with the previous ones. Section 5 discusses how the proposed measure is applied to medical and classification problems. Finally, Section 6 concludes the paper along with some suggestion for future study.

## 2. Preliminaries

Definition 2.1 Let $U$ be an universal set having elements such as $a_i$, then we define a fuzzy set (FS) $F_1$ in $U$ as $F_1 = \left\{\left(a_j, \mu_{F_1}(a_j)\right) \middle| a_j \in U\right\}$, where $\mu_{F_1}(a_j)$ is the degree of membership of $a_i$ in $U$ such that $0 \leq \mu_{F_1}(a_j) \leq 1$, here membership degree 0 denotes no presence and membership, degree 1 means the element is completely part of the set, while values between 0 and 1 indicate the element is only partially included in the set. [35].

Definition 2.2 In fuzzy set (FS) theory, intuitionistic fuzzy set (IFSs) consider more than one uncertainty membership and non-membership degrees.

The intuitionistic fuzzy set (IFSs) [35] $F_1$ $in$ $U$ may be referred as:
$F_1 = \left\{\left(a_j, \mu_{F_1}(a_j), \nu_{F_1}(a_j)\right) \middle| a_j \in U\right\}$, where $\mu_{F_1}(a_j)$ and $\nu_{F_1}(a_j)$ denotes the degree of membership and non-membership functions of an element $a_j$ in $U$ such that

$0 \le \mu_{F_1}(a_j) + \nu_{F_1}(a_j) \le 1$. Also, the intuitionistic fuzzy index known as hesitancy degree of $a_i\ in\ U$ is given by $\mu_{F_1}(a_j) + \nu_{F_1}(a_j) + \pi_{F_1}(a_j) = 1$.

Definition 2.3 For two intuitionistic fuzzy sets (IFSs) [8], [35], [36] $F_1 and\ F_2$ in $U$ with $\mu_{F_1}(a_j)\ and\ \nu_{F_1}(a_j)$ are the degree of membership and non-membership of the elements in set $F_1$, and $\mu_{F_2}(a_j)\ and\ \nu_{F_2}(a_j)$ are the membership and non-membership degrees of the elements in set $F_2$., then the following operations are hold.

a) $F_1 \subseteq F_2\ iff\ \mu_{F_1}(a_j) \le \mu_{F_2}(a_j)\ and\ \nu_{F_1}(a_j) \ge \nu_{F_2}(a_j)$,
b) $F_1 = F_2\ iff\ F_1 \subseteq F_2\ and\ F_2 \subseteq F_1$,
c) $F_1 \cup F_2 = \left\{a_j, max\left(\mu_{F_1}(a_j),\ \mu_{F_2}(a_j)\right), min\left(\nu_{F_1}(a_j),\ \nu_{F_2}(a_j)\right) \middle| a_j \in U\right\}$,
d) $F_1 \cap F_2 = \left\{a_j, min\left(\mu_{F_1}(a_j),\ \mu_{F_2}(a_j)\right), max\left(\nu_{F_1}(a_j),\ \nu_{F_2}(a_j)\right) \middle| a_j \in U\right\}$,
e) $(F_1)^c = \{(a_j,\ \mu_{F_1}(a_j), \nu_{F_1}(a_j)\ | a_j \in U\}$,
f) $F_1 . F_2 = \{a_j, \mu_{F_1}(a_j) . \mu_{F_2}(a_j),\ \nu_{F_1}(a_j) + \nu_{F_2}(a_j) - \nu_{F_1}(a_j) . \nu_{F_2}(a_j)\}$.

Definition 2.4 A picture fuzzy set (PFS) [9], [10] $F_1 \in U$ is defined as

$F_1 = \left\{\left(a_j, \mu_{F_1}(a_j), \nu_{F_1}(a_j), \gamma_{F_1}(a_j)\right) \middle| a_j \in U\right\}$, where $\mu_{F_1}(a_j), \nu_{F_1}(a_j)$ and $\gamma_{F_1}(a_j)$ denotes the degree of membership (positive), non-membership (negative) and neutrality respectively of $a_j \in U$ holds the following conditions

$0 \le \mu_{F_1}(a_j) + \nu_{F_1}(a_j) + \gamma_{F_1}(a_j) \le 1$ and

refusal degree $\varphi_{F_1} = 1 - \mu_{F_1}(a_j) - \nu_{F_1}(a_j) - \gamma_{F_1}(a_j)$ for all $a_j \in U$.

Definition 2.5 For two picture fuzzy set (PFS) [9], [10] $F_1 and\ F_2 \in U$ following operations holds

a) $F_1 \subseteq F_2\ iff\ \mu_{F_1}(a_j) \le \mu_{F_2}(a_j), \nu_{F_1}(a_j) \ge \nu_{F_2}(a_j)\ and\ \gamma_{F_1}(a_j) \le \gamma_{F_2}(a_j)$,
b) $F_1 = F_2\ iff\ F_1 \subseteq F_2\ and\ F_2 \subseteq F_1$,
c) $F_1 \cup F_2 =$
$\left\{a_j, max\left(\mu_{F_1}(a_j),\ \mu_{F_2}(a_j)\right), min\left(\nu_{F_1}(a_j),\ \nu_{F_2}(a_j)\right), min\left(\gamma_{F_1}(a_j), \gamma_{F_2}(a_j)\right) \middle| a_j \in U\right\}$,
d) $F_1 \cap F_2 =$
$\left\{a_j, min\left(\mu_{F_1}(a_j),\ \mu_{F_2}(a_j)\right), max\left(\nu_{F_1}(a_j),\ \nu_{F_2}(a_j)\right), min\left(\gamma_{F_1}(a_j), \gamma_{F_2}(a_j)\right) \middle| a_j \in U\right\}$,
e) $(F_1)^c = \{(a_j,\ \mu_{F_1}(a_j), \nu_{F_1}(a_j), \gamma_{F_1}(a_j)\ | a_j \in U\}$,

Definition 2.6 A picture fuzzy set (PFSs) [32], [33] then measure of distance of a function $m: PFS(U) \times PFS(U) \to [0, 1]$such that

$m_1.\ 0 \le m(F_1, F_2) \le 1$

$m_2.\ m(F_1, F_2) = m(F_2, F_1)$

$m_3.\ m(F_1, F_2) = 0\ iff\ F_1 = F_2$

$m\ _4.\ m(F_1, F_2) \le m(F_1, F_3)\ and\ m(F_2, F_3) \le m(F_1, F_3)$, where $F_1 \subseteq F_2 \subseteq F_3$.

## 3. Existing distance measure

This section provides a summary of the existing measures for Picture Fuzzy Sets (PIFSs), as described in the paper [37]. There are several distance measures that uses a various range of methodologies and technique. These measures have been applied to quantify the dissimilarity or similarity between multiple fields. These measures play a central role in diverse applications, contributing to the fields such as data analysis, pattern recognition, classification, medical diagnostic and cluster theory. Within the current context of distance measure, researchers as well as users are always exploring and improving methods to increase their efficiency and applications in various areas.

Hamming Distance

$$M_{d_1}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\left(\Delta\mu_j + \Delta\nu_j + \Delta\gamma_j + \left|\varphi_{F_1}(a_j) - \varphi_{F_2}(a_j)\right|\right)$$

Euclidean Distance

$$M_{d_2}(F_1, F_2) = \left[\frac{1}{4n}\sum_{j=1}^{n}\left(\left(\Delta\mu_j\right)^2 + \left(\Delta\nu_j\right)^2 + \left(\Delta\gamma_j\right)^2 + \left|\varphi_{F_1}(a_j) - \varphi_{F_2}(a_j)\right|^2\right)\right]^{\frac{1}{2}}$$

Hausdorff Distance

$$M_{d_3}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n} max\left(\Delta\mu_j, \Delta\nu_j, \Delta\gamma_j, \left|\varphi_{F_1}(a_j) - \varphi_{F_2}(a_j)\right|\right)$$

$$M_{d_4}(F_1, F_2) = \left[\frac{1}{4n}\sum_{j=1}^{n} max\left(\left(\Delta\mu_j\right)^2, \left(\Delta\nu_j\right)^2, \left(\Delta\gamma_j\right)^2, \left|\varphi_{F_1}(a_j) - \varphi_{F_2}(a_j)\right|^2\right)\right]^{\frac{1}{2}}$$

Where,

$$\Delta\mu_j = \left|\mu_{F_1}(a_j) - \mu_{F_2}(a_j)\right|$$

$$\Delta\nu_j = \left|\nu_{F_1}(a_j) - \nu_{F_2}(a_j)\right|$$

$$\Delta\gamma_j = \left|\gamma_{F_1}(a_j) - \gamma(a_j)\right|$$

$\Delta\delta_j = \left|\pi_{F_1}(a_j) - \pi_{F_2}(a_j)\right|$ for all $1 \leq j \leq n$,

$$\varphi_j^{F_1} = \left|\mu_{F_1}(a_j) + \nu_{F_1}(a_j) + \gamma_{F_1}(a_j)\right|$$

$\varphi_j^{F_2} = \left|\mu_{F_2}(a_j) + \nu_{F_2}(a_j) + \gamma_{F_2}(a_j)\right|$ for all $1 \leq j \leq n$,

The generalized normalized Hausdorff distance based on picture fuzzy set follws as:

$M_d\,(F_1, F_2) = \left[\frac{1}{4n}\sum_{j=1}^{n} max\left(\left(\Delta\mu_j\right)^\lambda + \left(\Delta\nu_j\right)^\lambda + \left(\Delta\gamma_j\right)^\lambda + \left|\varphi_{F_1}(a_j) - \varphi_{F_2}(a_j)\right|^\lambda\right)\right]^{\frac{1}{\lambda}}$ where $\lambda > 0$.

Son[33] mentioned the generalized picture distance measures Hausdorff-Hamming and Hausdorff Euclidean used these measures in clustering analysis for $(\lambda = 1)$ and $(\lambda = 2)$ given as:

$$M_{dd} = \frac{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^\lambda+\Delta\nu_j^\lambda+\Delta\gamma_j^\lambda}{3}+max\left(\Delta\mu_j^\lambda,\Delta\nu_j^\lambda,\Delta\gamma_j^\lambda\right)\right)\right]^{\frac{1}{\lambda}}}{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^\lambda+\Delta\nu_j^\lambda+\Delta\gamma_j^\lambda}{3}+max\left(\Delta\mu_j^\lambda,\Delta\nu_j^\lambda,\Delta\gamma_j^\lambda\right)\right)\right]^{\frac{1}{\lambda}}+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|\right]^{\frac{1}{\lambda}}+1}$$

For $\lambda = 1$

$$M_{d_5} = \frac{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j+\Delta\nu_j+\Delta\gamma_j}{3}+max\left(\Delta\mu_j,\Delta\nu_j,\Delta\gamma_j\right)\right)\right]}{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j+\Delta\nu_j+\Delta\gamma_j}{3}+max\left(\Delta\mu_j,\Delta\nu_j,\Delta\gamma_j\right)\right)\right]+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|\right]+1}$$

For $\lambda = 2$

$$M_{d_6} = \frac{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^2+\Delta\nu_j^2+\Delta\gamma_j^2}{3}+max\left(\Delta\mu_j^2,\Delta\nu_j^2,\Delta\gamma_j^2\right)\right)\right]^{\frac{1}{2}}}{\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^2+\Delta\nu_j^2+\Delta\gamma_j^2}{3}+max\left(\Delta\mu_j^2,\Delta\nu_j^2,\Delta\gamma_j^2\right)\right)\right]^{\frac{1}{2}}+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|\right]^{\frac{1}{2}}+1}$$

Son[33] also discussed the extended normalized picture Hausdorff-Hamming and the Hausdorff Euclidean taken as:

$$M_{ddd}\left(F_1,F_2\right) = \frac{\left[\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^\lambda+\Delta\nu_j^\lambda+\Delta\gamma_j^\lambda}{3}+max\left(\Delta\mu_j^\lambda,\Delta\nu_j^\lambda,\Delta\gamma_j^\lambda\right)\right)\right]^{\frac{1}{\lambda}}}{\left[\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^\lambda+\Delta\nu_j^\lambda+\Delta\gamma_j^\lambda}{3}+max\left(\Delta\mu_j^\lambda,\Delta\nu_j^\lambda,\Delta\gamma_j^\lambda\right)\right)\right]^{\frac{1}{\lambda}}+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\frac{1}{n}\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|^{\lambda}\right]^{\frac{1}{\lambda}}+1}$$

For $\lambda = 1$

$$M_{d_7}\left(F_1,F_2\right) = \frac{\frac{1}{n}\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j+\Delta\nu_j+\Delta\gamma_j}{3}+max\left(\Delta\mu_j,\Delta\nu_j,\Delta\gamma_j\right)\right)\right]}{\frac{1}{n}\left[\sum_{j=1}^{n}\left(\frac{\Delta\mu_j+\Delta\nu_j+\Delta\gamma_j}{3}+max\left(\Delta\mu_j,\Delta\nu_j,\Delta\gamma_j\right)\right)\right]+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\frac{1}{n}\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|\right]+1}$$

For $\lambda = 2$

$$M_{d_8}\left(F_1,F_2\right) = \frac{\left[\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^2+\Delta\nu_j^2+\Delta\gamma_j^2}{3}+max\left(\Delta\mu_j^2,\Delta\nu_j^2,\Delta\gamma_j^2\right)\right)\right]^{\frac{1}{2}}}{\left[\frac{1}{n}\sum_{j=1}^{n}\left(\frac{\Delta\mu_j^2+\Delta\nu_j^2+\Delta\gamma_j^2}{3}+max\left(\Delta\mu_j^2,\Delta\nu_j^2,\Delta\gamma_j^2\right)\right)\right]^{\frac{1}{2}}+\left[\max_j\left(\varphi_j^{F_1},\varphi_j^{F_2}\right)+\frac{1}{n}\sum_{j=1}^{n}\left|\varphi_j^{F_1}-\varphi_j^{F_2}\right|\right]^{\frac{1}{2}}+1}$$

Din and Thao [21] in his article applied the distance measures in pattern analysis which are given below,

$$M_{d_9}\left(F_1,F_2\right) = \frac{1}{3n}\sum_{j=1}^{n}\left(\Delta\mu_j+\Delta\nu_j+\Delta\gamma_j\right)$$

$$M_{d_{10}}\left(F_1,F_2\right) = \frac{1}{n}\left[\sum_{j=1}^{n}\left(\Delta\mu_j^2+\Delta\nu_j^2+\Delta\gamma_j^2\right)\right]^{\frac{1}{2}}$$

$$M_{d_{11}}(F_1, F_2) = \frac{1}{n}\sum_{j=1}^{n} max\left(\Delta\mu_j,\ \Delta\nu_j,\ \Delta\gamma_j\right)$$

$$M_{d_{12}}(F_1, F_2) = \frac{1}{n}\left[\sum_{j=1}^{n} max\left(\Delta\mu_j^2,\ \Delta\nu_j^2,\ \Delta\gamma_j^2\right)\right]^{\frac{1}{2}}$$

Dutta[22] proposed the new distance measures given below and applied these distance measures in the field of medical diagnosis.

Hamming Distance Measure:

$$M_{d_{13}}(F_1, F_2) = \frac{1}{2}\sum_{j=1}^{n}\left(\Delta\mu_j + \Delta\nu_j + \Delta\gamma_j + \Delta\delta_j\right)$$

Normalized Hamming Distance Measure:

$$M_{d_{14}}(F_1, F_2) = \frac{1}{2n}\sum_{j=1}^{n}\left(\Delta\mu_j + \Delta\nu_j + \Delta\gamma_j + \Delta\delta_j\right)$$

Euclidean Distance Measure:

$$M_{d_{15}}(F_1, F_2) = \left[\frac{1}{2}\sum_{j=1}^{n}\left(\Delta\mu_j^2 + \Delta\nu_j^2 + \Delta\gamma_j^2 + \Delta_{\delta_j}^{2}\right)\right]^{\frac{1}{2}}$$

Normalized Euclidean Distance:

$$M_{d_{16}}(F_1, F_2) = \left[\frac{1}{2n}\sum_{j=1}^{n}\left(\Delta\mu_j^2 + \Delta\nu_j^2 + \Delta\gamma_j^2 + \Delta_{\delta_j}^{2}\right)\right]^{\frac{1}{2}}$$

The Measure given by Dutta [22] as the extension of measure given by Wang and Xing [38]

$$M_{d_{17}}(F_1, F_2) = \frac{1}{n}\sum_{j=1}^{n}\left[\frac{\left(\Delta\mu_j + \Delta\nu_j + \Delta\gamma_j + \Delta\delta_j\right)}{4} + \frac{max\left(\Delta\mu_j, \Delta\nu_j, \Delta\gamma_j, \Delta_{\delta_j}\right)}{2}\right]$$

The measure given by Ganie [3] can be defined as

$$M_G(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\left[\left|\tan^{-1}\mu_{F_1}(a_j) - \tan^{-1}\mu_{F_2}(a_j)\right| + \left|\tan^{-1}\nu_{F_1}(a_j) - \tan^{-1}\nu_{F_2}(a_j)\right| + \left|\tan^{-1}\gamma_{F_1}(a_j) - \tan^{-1}\gamma_{F_2}(a_j)\right| + \left|\tan^{-1}\varphi_{F_1}(a_j) - \tan^{-1}\varphi_{F_2}(a_j)\right|\right]$$

## 4. Proposed distance measure

In this section, we proposed a novel distance measure for Picture Fuzzy Sets (PIFSs). Proposed distance measure and their properties are discussed in the section 4.1, 4.2 and 4.3. Our aim is to provide a novel measure for determining the distance between Picture Fuzzy Sets (PIFSs).

### 4.1 A novel distance measure for PFSs

Let $F_1$ and $F_2$ be any two IFSs $F_1 = \left\{\left(a_j, \mu_{F_1}(a_j), \nu_{F_1}(a_j), \gamma_{F_1}(a_j)\right) \middle| a_j \in U\right\}$ and $F_2 = \left\{\left(a_j, \mu_{F_2}(a_j), \nu_{F_2}(a_j), \gamma_{F_2}(a_j)\right) \middle| a_j \in U\right\}$ on U,where $\mu_{F_1}(a_j), \nu_{F_1}(a_j)$ and $\gamma_{F_1}(a_j)$ denotes the membership (positive), non-membership (negative) and neutrality respectively of $a_j \in U$ holds the following conditions $0 \le \mu_{F_1}(a_j) + \nu_{F_1}(a_j) + \gamma_{F_1}(a_j) \le 1$ and refusal degree $\varphi_{F_1} = 1 - \mu_{F_1}(a_j) - \nu_{F_1}(a_j) - \gamma_{F_1}(a_j)$ for all $a_j \in U$.then a PIFSs measure of distance is given by

$$M_{SB}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$$

**Theorem 4.1** The PIFSs measure of distance

$M_{SB}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_1}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_1}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$ is an effective distance measure for PIFSs.

**Proof:** To prove this measure, we will show that the proposed picture fuzzy distance measure $M_{SB}$ for any two sets $F_1$ and $F_2$ hold the conditions $M_1$ to $M_4$ in definition 2.6.

$\boldsymbol{M_1}$**.** As $0 \le \mu_{F_1}(a_j),\ \mu_{F_2}(a_j) \le 1\ \forall\ 1 \le j \le n$, so that we have $0 \le e^{-\mu_{F_1}(a_j)}, e^{-\mu_{F_2}(a_j)} \le 1$, and so $0 \le \left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| \le 1$. Similarly, we have $0 \le \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_1}(a_j)}\right| \le 1$, $0 \le \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_1}(a_j)}\right| \le 1$, and $0 \le \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right| \le 1$. Hence, we get $0 \le M_{SB}(F_1, F_2) \le 1\ \forall\ 1 \le j \le n$.

$\boldsymbol{M_2}$**.** To prove, $M_{SB}(F_1, F_2) = M_{SB}(F_2, F_1)$ we have,

$$\frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$$

$$= \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_2}(a_j)} - e^{-\mu_{F_1}(a_j)}\right| + \left|e^{-\nu_{F_2}(a_j)} - e^{-\nu_{F_1}(a_j)}\right| + \\ \left|e^{-\gamma_{F_2}(a_j)} - e^{-\gamma_{F_1}(a_j)}\right| + \left|e^{-\varphi_{F_2}(a_j)} - e^{-\varphi_{F_1}(a_j)}\right|\end{bmatrix}$$

$$= M_{SB}(F_2, F_1)$$

$\boldsymbol{M_3}$**.** Let $F_1 = F_2$ then $\mu_{F_1}(a_j) = \mu_{F_2}(a_j), \nu_{F_1}(a_j) = \nu_{F_2}(a_j), \gamma_{F_1}(a_j) = \gamma_{F_2}(a_j)\ and\ \varphi_{F_1}(a_j) = \varphi_{F_2}(a_j)\ \forall\ 1 \le j \le n$. Then we have

$$M_{SB}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_2}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_2}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_2}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_2}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix} = 0$$

$\boldsymbol{M_4}$ Let $F_1 \subseteq F_2 \subseteq F_3$, then $\mu_{F_1}(a_j) \le \mu_{F_2}(a_j) \le \mu_{F_3}(a_j), \nu_{F_1}(a_j) \ge \nu_{F_2}(a_j) \ge \nu_{F_3}(a_j)\ and\ \gamma_{F_1}(a_j) \le \gamma_{F_2}(a_j) \le \gamma_{F_3}(a_j) \forall\ 1 \le j \le n$. Thus, we have

$$\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| \le \left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_3}(a_j)}\right|,$$

$$\left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| \leq \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_3}(a_j)}\right|,$$

$$\left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| \leq \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_3}(a_j)}\right|$$

$$and$$

$$\left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right| \leq \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_3}(a_j)}\right|$$

Therefore,

$$\frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$$

$$\leq \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_3}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_3}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_3}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_3}(a_j)}\right|\end{bmatrix}.$$

So picture fuzzy distance measure $M_{SB}(F_1, F_2) \leq M_{SB}(F_1, F_3)$, also we can see that $M_{SB}(F_2, F_3) \leq M_{SB}(F_1, F_3)$ which shows picture fuzzy distance measure $M_{SB}$ is an effective PIFSs measure of distance.

**Theorem 4.3** The PIFSs measure of distance

$M_{SB}(F_1, F_2) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$ satisfies the following properties.

$a)$ $M_{SB}({F_1}^c,\ {F_2}^c) = M_{SB}(F_1, F_2)\ \forall\ F_1, F_2 \in$ PIFSs X.

$b)$ $M_{SB}(F_1,\ {F_2}^c) =\ M_{SB}(\ {F_1}^c,\ F_2)\ \forall\ F_1, F_2 \in$ PIFSs X.

$c)$ $M_{SB}(F_1,\ {F_1}^c) = 0$ iff $\mu_{F_1}(a_j) = \nu_{F_1}(a_j)\ \forall\ 1 \leq j \leq n.$

***Proof:*** We have

a) $$M_{SB}({F_1}^c,\ {F_2}^c) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$$

$$= \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\end{bmatrix}$$

$$= M_{SB}(F_1, F_2).$$

b) $$= \frac{1}{4n}\sum_{j=1}^{n}\left[\begin{array}{c} \left|e^{-\mu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right| \end{array}\right]$$

$$= \frac{1}{4n}\sum_{j=1}^{n}\left[\begin{array}{c} \left|e^{-\nu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\mu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \\ \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right| \end{array}\right]$$

$$= M_{SB}({F_1}^c, F_2)$$

c) $$M_{SB}(F_1, {F_1}^c) = 0 \Leftrightarrow \frac{1}{4n}\sum_{j=1}^{n}\left[\left|e^{-\mu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| + \left|e^{-\nu_{F_1}(a_j)} - e^{-\mu_{F_2}(a_j)}\right| + \left|e^{-\gamma_{F_1}(a_j)} - e^{-\gamma_{F_2}(a_j)}\right| + \left|e^{-\varphi_{F_1}(a_j)} - e^{-\varphi_{F_2}(a_j)}\right|\right] = 0$$

$$\Leftrightarrow \left|e^{-\mu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)}\right| = 0 \; \forall\, j$$

$$\Leftrightarrow e^{-\mu_{F_1}(a_j)} - e^{-\nu_{F_2}(a_j)} = 0 \; \forall\, j$$

$$\Leftrightarrow \mu_{F_1}(a_j) = \nu_{F_1}(a_j) \; \forall\, j.$$

**4.4 Experiments and Analysis**

We will compare the results of pre-existing measure defined in section 3 with proposed measure in section 4, in this section. We will use particular test sets to justify the rationality of proposed measure. Validating the picture fuzzy set measure with axiomatic condition is the key objective of any distance measure. If any picture fuzzy set is violating one or more axiomatic requirement of similarity measure, then we say that it produces counterintuitive situation. In addition, several well-known picture fuzzy sets are unable to accurately differentiate between various picture fuzzy set pairs. For example, in Problem 4.1, Problem 4.2 and Problem 4.3 we can see that picture fuzzy set produce equal distance measure for different sets. This situation is absurd and goes against the both counterintuitive and intuition.

**4.5 Superiority Analysis**

Here, we have compared the suggested and existed measures with numerical problems to show the distance between various Picture Fuzzy Sets (PIFSs).

**Problem 4.1** Consider six different types of PIFSs with every type containing two different PIFSs as given below:

Type-(a): $\{F_1 = \{(0.28, 0.55, 0.1)\},\ F_2 = \{(0.6, 0.27, 0.1)\}\}$

Type-(b): $\{F_1 = \{(0.28, 0.54, 0.1)\},\ F_2 = \{(0, 0.87, 0.1)\}\}$

Type-(c): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(0.6, 0.4, 0)\}\}$

Type-(d): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(0.5, 0.5, 0)\}\}$

Type-(e): $\{F_1 = \{(0.2, 0.5, 0.3)\},\ F_2 = \{(0.4, 0.4, 0.2)\}\}$

Type-(f): $\{F_1 = \{(0.1, 0.5, 0.1)\},\ F_2 = \{(0.2, 0.4, 0.2)\}\}$

**Table 1**: Distance between two picture fuzzy sets calculated by existing and proposed distance measure related to problem 1

| Measures | Type-(a) | Type-(b) | Type-(c) | Type-(d) | Type-(e) | Type-(f) |
|---|---|---|---|---|---|---|
| $M_{d_1}$ | **0.1600** | **0.1600** | **0.5000** | **0.5000** | **0.1000** | **0.1000** |
| $M_{d_2}$ | **0.2135** | **0.2135** | 0.6164 | 0.6124 | 0.1225 | 0.1000 |
| $M_{d_3}$ | **0.0800** | **0.0800** | **0.2500** | **0.2500** | 0.0500 | 0.0250 |
| $M_{d_4}$ | **0.1600** | **0.1600** | **0.5000** | **0.5000** | 0.1000 | 0.0500 |
| $M_{d_5}$ | **0.2055** | **0.2055** | 0.2373 | 0.2174 | 0.1429 | 0.0952 |
| $M_{d_6}$ | **0.1675** | **0.1675** | 0.2322 | 0.2110 | 0.1091 | 0.0677 |
| $M_{d_7}$ | **0.2055** | **0.2055** | 0.2373 | 0.2174 | 0.1429 | 0.0952 |
| $M_{d_8}$ | **0.1675** | **0.1675** | 0.2322 | 0.2110 | 0.1091 | 0.0677 |
| $M_{d_9}$ | **0.2000** | **0.2000** | **0.3333** | **0.3333** | 0.1333 | 0.1000 |
| $M_{d_{10}}$ | **0.4252** | **0.4252** | 0.7211 | 0.7071 | 0.2449 | 0.1732 |
| $M_{d_{11}}$ | **0.3200** | **0.3200** | 0.6000 | 0.5000 | 0.2000 | 0.1000 |
| $M_{d_{12}}$ | **0.3200** | **0.3200** | 0.6000 | 0.5000 | 0.2000 | 0.1000 |
| $M_{d_{13}}$ | **0.3200** | **0.3200** | **1.0000** | **1.0000** | **0.2000** | **0.2000** |
| $M_{d_{14}}$ | **0.3200** | **0.3200** | **1.0000** | **1.0000** | **0.2000** | **0.2000** |
| $M_{d_{15}}$ | **0.3020** | **0.3020** | 0.8718 | 0.8660 | 0.1732 | 0.1414 |
| $M_{d_{16}}$ | **0.3020** | **0.3020** | 0.8718 | 0.8660 | 0.1732 | 0.1414 |
| $M_{d_{17}}$ | **0.0640** | **0.0640** | **0.2000** | **0.2000** | 0.0400 | 0.0300 |
| $M_G$ | 0.1366 | 0.1315 | 0.4266 | 0.4282 | 0.0901 | 0.0932 |
| $\boldsymbol{M_{SB}}$ | 0.1079 | 0.1101 | 0.3532 | 0.3548 | 0.0725 | 0.0785 |

Bold numerical values display the unreasonable results

i) The distance measures for PIFSs derived from $M_{d_1}, M_{d_{13}}$ and $M_{d_{14}}$ display identical distances for distinct fuzzy sets in three pairs {Type-(a) and Type-(b)}, {Type-(c) and Type-(d)}, and {Type-(e) and Type-(f)}.

ii) The distance measure for PIFSs obtained from $M_{d_2}, M_{d_5}, M_{d_6}, M_{d_7}, M_{d_8}, M_{d_9}, M_{d_{10}}, M_{d_{11}}, M_{d_{12}}, M_{d_{15}}$ and $M_{d_{16}}$ display identical distances for two distinct types, namely {Type-(a) and Type-(b)}

iii) The distance measures for PIFSs derived from. $M_{d_2}, M_{d_5}, M_{d_6}, M_{d_7}, M_{d_8}, M_{d_9}, M_{d_{10}}, M_{d_{11}}, M_{d_{12}}, M_{d_{15}}$ and $M_{d_{16}}$ display identical distances for two different types in {Type-(a) and Type-(b)}, {Type-(c) and Type-(d)} and {Type-(e) and Type-(f)}.

iv) The PIFSs distance measures from $M_{d_{17}}$ display the same distance for two different PIFSs sets {Type -(a) and Type-(b)}, {Type-(c) and Type-(d)}.

v) The distance measures derived from $M_{d_{13}}$ and , $M_{d_{14}}$ for PIFSs display an identical distance of "1" in both Type-(c) and Type-(d). However, the distances between these PIFSs are not complement each other for distinct picture fuzzy sets.

vi) The proposed distance measure $M_{SB}$ computes the distance for all distinct types, avoiding any similarity and gives more accurate results compared to existing measure.

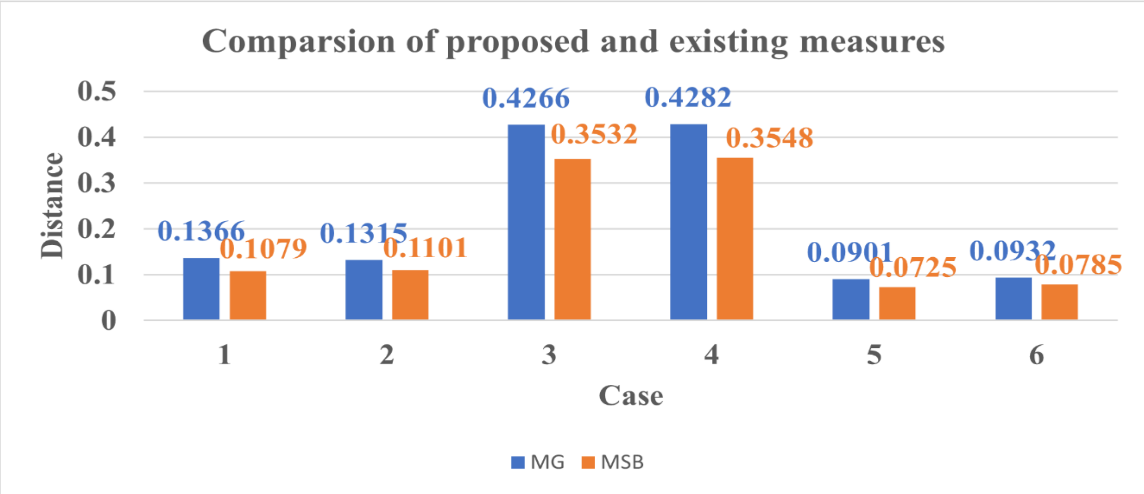



Figure1: Comparison of proposed and existing measures

The result of comparative analysis of proposed measure for this problem is discussed in Table 1 and Fig. 1, respectively.

**Problem 4.2** Consider the six different types of PIFSs with every type containing two different PIFSs as given below:

Type-(a): $\{F_1 = \{(0.2, 0.5, 0.3)\},\ F_2 = \{(0.4, 0.4, 0.2)\}\}$

Type-(b): $\{F_1 = \{(0.2, 0.5, 0.3)\},\ F_2 = \{(0.1, 0.4, 0.5)\}\}$

Type-(c): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(0.4, 0, 0.6)\}\}$

Type-(d): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(0.5, 0, 0.5)\}\}$

Type-(e): $\{F_1 = \{(0.2, 0, 0.8)\},\ F_2 = \{(0.3, 0, 0.7)\}\}$

Type-(f): $\{F_1 = \{(0.1, 0.6, 0.3)\},\ F_2 = \{(0.3, 0.3, 0.4)\}\}$

**Table 2**: Distance between two picture fuzzy sets calculated by existing and proposed distance measure related to problem 2

| Measures | Type-(a) | Type-(b) | Type-(c) | Type-(d) | Type-(e) | Type-(f) |
|---|---|---|---|---|---|---|
| $M_{d_1}$ | **0.1000** | **0.1000** | 0.4500 | 0.5000 | 0.0500 | 0.1500 |
| $M_{d_2}$ | **0.1225** | **0.1225** | 0.5523 | 0.6124 | 0.0707 | 0.1871 |
| $M_{d_3}$ | **0.0500** | **0.0500** | 0.2250 | 0.2500 | 0.0250 | 0.0750 |
| $M_{d_4}$ | **0.1000** | **0.1000** | 0.4500 | 0.5000 | 0.0500 | 0.1500 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $M_{d_5}$ | **0.1429** | **0.1429** | 0.2222 | 0.2174 | 0.0769 | 0.2000 |
| $M_{d_6}$ | **0.1091** | **0.1091** | 0.2098 | 0.2110 | 0.0606 | 0.1560 |
| $M_{d_7}$ | **0.1429** | **0.1429** | 0.2222 | 0.2174 | 0.0769 | 0.2000 |
| $M_{d_8}$ | **0.1091** | **0.1091** | 0.2098 | 0.2110 | 0.0606 | 0.1560 |
| $M_{d_9}$ | **0.1333** | **0.1333** | 0.3000 | 0.3333 | 0.0667 | 0.2000 |
| $M_{d_{10}}$ | **0.2449** | **0.2449** | 0.6403 | 0.7071 | 0.1414 | 0.3742 |
| $M_{d_{11}}$ | **0.2000** | **0.2000** | **0.5000** | **0.5000** | 0.1000 | 0.3000 |
| $M_{d_{12}}$ | **0.2000** | **0.2000** | **0.5000** | **0.5000** | 0.1000 | 0.3000 |
| $M_{d_{13}}$ | **0.2000** | **0.2000** | 0.9000 | **1.0000** | 0.1000 | 0.3000 |
| $M_{d_{14}}$ | **0.2000** | **0.2000** | 0.9000 | **1.0000** | 0.1000 | 0.3000 |
| $M_{d_{15}}$ | **0.1732** | **0.1732** | 0.7810 | 0.8660 | 0.1000 | 0.2646 |
| $M_{d_{16}}$ | **0.1732** | **0.1732** | 0.7810 | 0.8660 | 0.1000 | 0.2646 |
| $M_{d_{17}}$ | **0.0400** | **0.0400** | 0.1800 | 0.2000 | 0.0200 | 0.0600 |
| $M_G$ | 0.0901 | 0.0883 | 0.3825 | 0.4282 | 0.0395 | 0.1325 |
| $M_{SB}$ | 0.0725 | 0.0710 | 0.3150 | 0.3548 | 0.0313 | 0.1066 |

Bold numerical values show unreasonable results

i) The distance measures for PIFSs derived from $M_{d_{11}}$ and $M_{d_{12}}$ displays identical distances for distinct fuzzy sets in three pairs {Type-(a) and Type-(b)}, {Type-(c) and Type-(d)}, and {Type-(e) and Type-(f)}.

ii) $M_{d_1}, M_{d_2}, M_{d_3}, M_{d_4}, M_{d_5}, M_{d_6}, M_{d_7}, M_{d_8}, M_{d_9}, M_{d_{10}}, M_{d_{13}}, M_{d_{14}}, M_{d_{15}}, M_{d_{16}}, M_{d_{17}}$ continuously display an identical distance measurement in both Type-(a) and Type-(b)

iii) The distance measures derived from $M_{d_{13}}$ and $M_{d_{14}}$ for PIFSs display an identical distance of "1" in both Type-(c) and Type-(d). However, the distances between these PIFSs are not complement of each other for distinct picture fuzzy sets.

iv) The proposed distance measure $M_{SB}$ computes the distance for all distinct types, avoiding any similarity and gives more accurate results compared to existing measures.

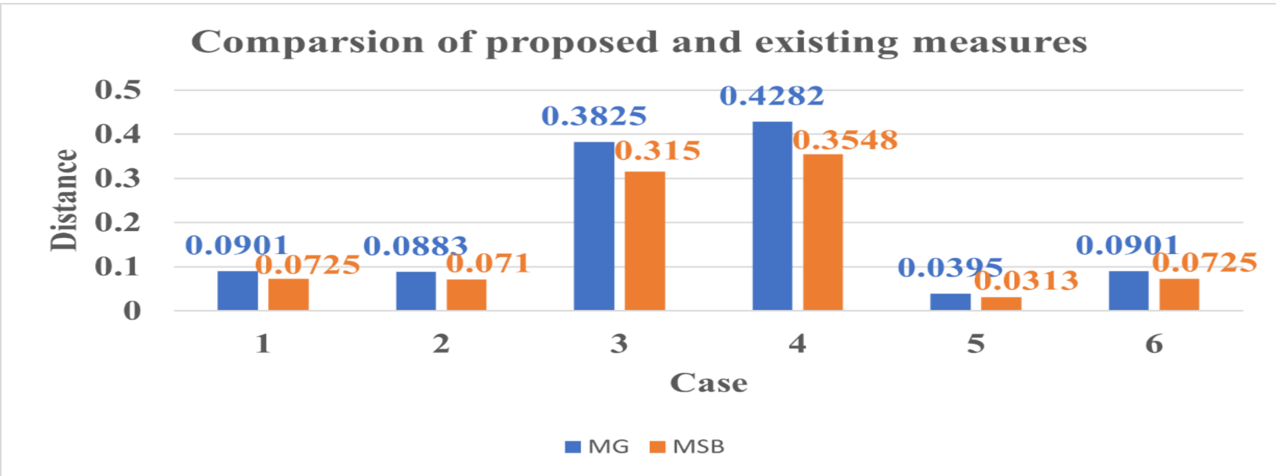



**Figure 2:** Comparison of proposed and existing measures.

The result of comparative analysis of proposed measure for this problem is discussed in Table 2 and Fig. 2, respectively.

**Problem 4.3** Consider five different types of PIFSs with every type containing two different PIFSs as given below:

Type-(a): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(1, 0, 0)\}\}$

Type-(b): $\{F_1 = \{(0.5, 0.1, 0.1)\},\ F_2 = \{(0.4, 0.2, 0.1)\}\}$

Type-(c): $\{F_1 = \{(0.4, 0.3, 0.1)\},\ F_2 = \{(0.3, 0.4, 0.1)\}\}$

Type-(d): $\{F_1 = \{(0.5, 0.2, 0.1)\},\ F_2 = \{(0.4, 0.2, 0.1)\}\}$

Type-(e): $\{F_1 = \{(0, 0, 0)\},\ F_2 = \{(0.5, 0.5, 0)\}\}$

Type-(f): $\{F_1 = \{(0.2, 0, 0.8)\},\ F_2 = \{(0.3, 0, 0.7)\}\}$

**Table 3**: Distance between two picture fuzzy sets calculated by existing and proposed distance measure related to problem 3

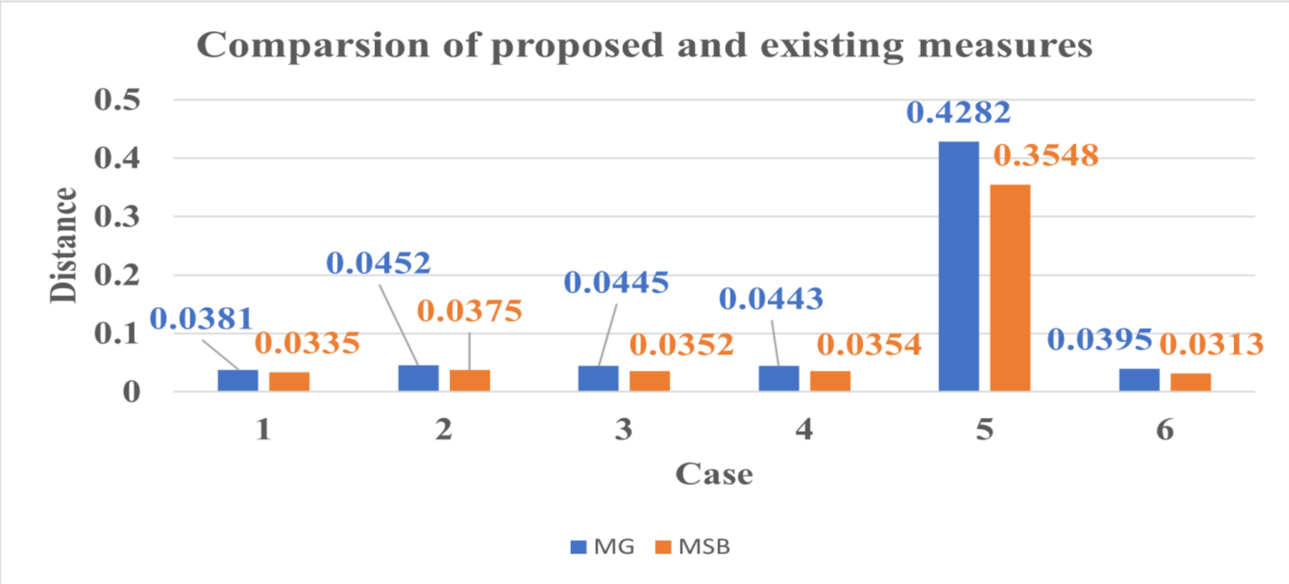
| Measures | Type-(a) | Type-(b) | Type-(c) | Type-(d) | Type-(e) | Type-(f) |
|---|---|---|---|---|---|---|
| $M_{d_1}$ | **0.0500** | **0.0500** | **0.0500** | **0.0500** | 0.5000 | **0.0500** |
| $M_{d_2}$ | **0.0707** | **0.0707** | **0.0707** | **0.0707** | 0.6124 | **0.0707** |
| $M_{d_3}$ | **0.0250** | **0.0250** | **0.0250** | **0.0250** | 0.2500 | **0.0250** |
| $M_{d_4}$ | **0.0500** | **0.0500** | **0.0500** | **0.0500** | 0.5000 | **0.0500** |
| $M_{d_5}$ | 0.1000 | 0.0893 | 0.0847 | 0.0656 | 0.2174 | 0.0769 |
| $M_{d_6}$ | 0.0739 | 0.0657 | 0.0638 | 0.0559 | 0.2110 | 0.0606 |
| $M_{d_7}$ | 0.1000 | 0.0893 | 0.0847 | 0.0656 | 0.2174 | 0.0769 |
| $M_{d_8}$ | 0.0739 | 0.0657 | 0.0638 | 0.0559 | 0.2110 | 0.0606 |
| $M_{d_9}$ | **0.0333** | **0.0667** | **0.0667** | **0.0333** | 0.3333 | **0.0667** |
| $M_{d_{10}}$ | **0.1000** | **0.1414** | **0.1414** | **0.1000** | 0.7071 | **0.1414** |
| $M_{d_{11}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 0.5000 | **0.1000** |
| $M_{d_{12}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 0.5000 | **0.1000** |
| $M_{d_{13}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 1.0000 | **0.1000** |
| $M_{d_{14}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 1.0000 | **0.1000** |
| $M_{d_{15}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 0.8660 | **0.1000** |
| $M_{d_{16}}$ | **0.1000** | **0.1000** | **0.1000** | **0.1000** | 0.8660 | **0.1000** |
| $M_{d_{17}}$ | **0.0200** | **0.0200** | **0.0200** | **0.0200** | 0.2000 | **0.0200** |
| $M_G$ | 0.0381 | 0.0452 | 0.0445 | 0.0443 | 0.4282 | 0.0395 |
| $M_{SB}$ | 0.0335 | 0.0375 | 0.0352 | 0.0354 | 0.3548 | 0.0313 |

i) The PIFSs distance measure in $M_{d_{10}}$ continuously display identical distances for two distinct fuzzy sets in various types, namely {Type -(a) and Type-(d)} and {Type-(c), Type-(c) and Type-(f)}.

ii) $M_{d_1}, M_{d_2}, M_{d_3}, M_{d_4}, M_{d_{11}}, M_{d_{12}}, M_{d_{13}}, M_{d_{14}}, M_{d_{15}}, M_{d_{16}}, M_{d_{17}}$ display an identical distance in all types {Type -(a) to Type-(d) and Type-(f)}.

iii) The proposed distance measure $M_{SB}$ computes the distance through various types without any similarity, and it avoids generating unreasonable results. It shows superior outcomes over existing measure.

**Figure 3**: Comparison of proposed and existing measures.




The result of comparative analysis of proposed measure for this problem is discussed in Table 3 and Fig. 3, respectively.

## 5. Medical Diagnosis (Decision)

Medical diagnosis [22] is the process to finding the disease or symptoms. The information needed for medical diagnosis is very difficult and gathered from a person's medical history and physical examination. Medical records contain uncertain information gathered, which can be useful for decision-making using fuzzy logic in everyday problems. In medical diagnosis (decision) many applications of FS theory and extension have been applied. It can be noted that in medical diagnosis some indications may be neutral impact on disease. We can see by an example that degree of neutrality for headache and temperature to the disease chest and stomach. Accordingly, indications chest and stomach pain have neutral impact on the disease to typhoid, malaria and viral fever. So, it becomes very important to consider degree of neutrality. Regarding neutrality PFSs in medical diagnosis is essential for its practical application in healthcare sector. How we can use PFSs in medical problems can be seen in below example.

### 5.1 Significance of distance measures in medical diagnosis

Sometimes diagnosis is very challenging, as there are lots of symptoms and sign are nonspecific. For example, we can see that tiredness of body, is a sign of many disorders and not clearly seen what is wrong. It is very difficult to identifying the relationship between disease and patients. Measures has an important role in medical diagnosis, it helps us how to relate the relationship between disease and patients and disease and symptoms. The small distance between the relationship of disease and patients shows the better relationship while large distance shows weak relation. So, it can be seen that if the distance between disease and patient is small then it can be concluded that patient has the disease.

### 5.2 Methodology

In this section, we will discuss the algorithmic steps detailing how to apply the medical diagnosis into problem-solving. We are defining disease within the set $\tilde{\mathcal{D}}$, providing patient information in set $\tilde{\mathcal{P}}$, and symptoms of both patients and disease in set $\tilde{\mathcal{S}}$. The algorithm steps for applying the Picture Fuzzy Set Relationship (PFSR) to a medical problem are as follows:

Step 1: Using medical knowledge to construct a set Containing both patient symptoms and disease symptoms.

Step 2: Defining the positive, negative, and neutral degrees within the dataset.

Step 3: Employing the proposed measure to compute the distance between the disease and patients.

Step 4: Identifying the shortest distance between the patient and disease, indicating the patient's most probably suffering from the disease.

Step 5: If the medical expert finds the result unsatisfactory, then reconstruct the Picture Fuzzy Set Relationship (PFSR) and reapply Steps 3 and 4 for further results.

**Problem 5.1** Picture fuzzy relation between patient and symptoms are given in Table 4, and relation between disease and symptoms are given in Table 5. Here, $\tilde{\mathcal{D}}_i$, $\tilde{\mathcal{S}}_i$ and $\tilde{\mathcal{P}}_i$ for $i = 1$ to 5 represents disease, symptoms and patient respectively. Here, $(\mu, \nu, \gamma)$ represents the positive, negative and neutral membership degree of the PFSs.

**Table 4**: Relation between symptoms of the patients

| PFSR | $\tilde{\mathcal{S}}_1$ | $\tilde{\mathcal{S}}_2$ | $\tilde{\mathcal{S}}_3$ | $\tilde{\mathcal{S}}_4$ | $\tilde{\mathcal{S}}_5$ |
|---|---|---|---|---|---|
| $\tilde{\mathcal{P}}_1$ | $(0.1,0.2,0.4)$ | $(0.2,0.3,0.5)$ | $(0.8,0.1,0)$ | $(0.6,0.2,0.1)$ | $(0.8,0.1,0)$ |
| $\tilde{\mathcal{P}}_2$ | $(0.9,0.1,0)$ | $(0.3,0.2,0.4)$ | $(0.1,0.6,0.2)$ | $(0.2,0.5,0.2)$ | $(0.1,0.6,0.2)$ |
| $\tilde{\mathcal{P}}_3$ | $(0.1,0.5,0.2)$ | $(0.1,0.4,0.3)$ | $(0.7,0.2,0.1)$ | $(0.3,0.2,0.4)$ | $(0.7,0.2,0.1)$ |
| $\tilde{\mathcal{P}}_4$ | $(0.3,0.5,0.1)$ | $(0.1,0.7,0.2)$ | $(0.114,0.3,0.2)$ | $(0.3,0.4,0.2)$ | $(0.4,0.3,0.2)$ |

**Table 5**: Relation between symptoms of the disease

| PFSR | $\tilde{\mathcal{S}}_1$ | $\tilde{\mathcal{S}}_2$ | $\tilde{\mathcal{S}}_3$ | $\tilde{\mathcal{S}}_4$ | $\tilde{\mathcal{S}}_5$ |
|---|---|---|---|---|---|
| $\tilde{\mathcal{D}}_1$ | $(0.2,0.5,0.3)$ | $(0.8,0.1,0)$ | $(0.1,0.5,0.3)$ | $(0.2,0.4,0.35)$ | $(0,0.35,0.5)$ |

| | | | | | |
|---|---|---|---|---|---|
| $\widetilde{\mathcal{D}}_2$ | $(0.8, 0, 0)$ | $(0.2, 0.3, 0.35)$ | $(0.1, 0.5, 0.3)$ | $(0.2, 0.3, 0.4)$ | $(0.2, 0.3, 0.4)$ |
| $\widetilde{\mathcal{D}}_3$ | $(0.2, 0.4, 0.3)$ | $(0.1, 0.6, 0.2)$ | $(0.3, 0.3, 0.4)$ | $(0.2, 0.3, 0.35)$ | $(0.6, 0.1, 0.2)$ |
| $\widetilde{\mathcal{D}}_4$ | $(0, 0.5, 0.4)$ | $(0.1, 0.5, 0.3)$ | $(0.7, 0, 0)$ | $(0.7, 0, 0.1)$ | $(0.2, 0.35, 0.4)$ |
| $\widetilde{\mathcal{D}}_5$ | $(0.1, 0.5, 0.35)$ | $(0.1, 0.5, 0.25)$ | $(0.4, 0, 0)$ | $(0.4, 0.2, 0.3)$ | $(0.3, 0.4, 0.2)$ |

**Table 6**: Calculated distances between disease and patients using existing and proposed measure

| PFSR | $\widetilde{\mathcal{D}}_1$ | | $\widetilde{\mathcal{D}}_2$ | | $\widetilde{\mathcal{D}}_3$ | | $\widetilde{\mathcal{D}}_4$ | | $\widetilde{\mathcal{D}}_5$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ |
| $\tilde{\mathcal{P}}_1$ | 0.2722 | 0.2258 | 0.2260 | 0.1870 | 0.1692 | 0.1404 | 0.1482 | **0.1246** | 0.1624 | 0.1339 |
| $\tilde{\mathcal{P}}_2$ | 0.1557 | 0.1272 | 0.0846 | **0.0699** | 0.1946 | 0.1594 | 0.2586 | 0.2146 | 0.2285 | 0.1891 |
| $\tilde{\mathcal{P}}_3$ | 0.2168 | 0.1811 | 0.1831 | 0.1515 | 0.1033 | **0.0855** | 0.1464 | 0.1240 | 0.1111 | 0.0920 |
| $\tilde{\mathcal{P}}_4$ | 0.1670 | 0.1392 | 0.1515 | 0.1245 | 0.0817 | **0.0680** | 0.1590 | 0.1340 | 0.1153 | 0.0979 |

Bold numerical values show the disease by which patient is suffering.

Table 6, displays the correlation between the patients and their respective disease with bold values, using relationship between symptoms of patient and relationship between symptoms of disease given in Table 4 and Table 5 respectively. Particularly, the Table 6 indicates that patient $\tilde{\mathcal{P}}_1$ is affected with disease $\widetilde{\mathcal{D}}_4$, patient $\tilde{\mathcal{P}}_2$ has disease $\widetilde{\mathcal{D}}_2$ patient $\tilde{\mathcal{P}}_3$ is suffering from disease $\widetilde{\mathcal{D}}_3$ and patient $\tilde{\mathcal{P}}_4$ also has disease $\widetilde{\mathcal{D}}_3$. Also, proposed measure $M_{SB}$ shows the superior value compare to $M_G$.

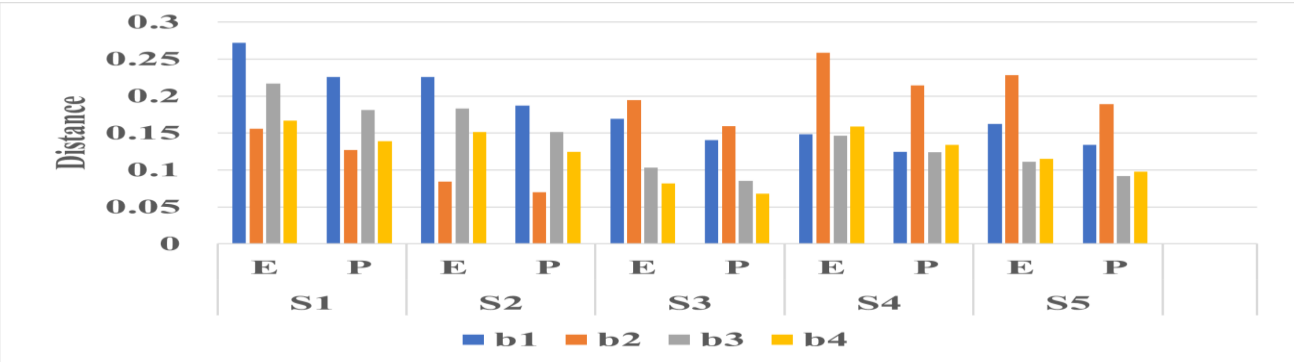



**Figure 4**: Comparisons of distance of the disease with the patients using existing measure $M_G$ and proposed measure $M_{SB}$.

The result of comparative analysis of proposed measure for this problem is discussed in Table 6 and Fig. 4, respectively.

**Problem 5.2** Picture fuzzy relation between patient and symptoms are given in Table 7, and relation between disease and symptoms are given in Table 8.

**Table 7**: Relation between symptoms of the patients

| PFSR | $\tilde{\mathcal{S}}_1$ | $\tilde{\mathcal{S}}_2$ | $\tilde{\mathcal{S}}_3$ | $\tilde{\mathcal{S}}_4$ | $\tilde{\mathcal{S}}_5$ |
|---|---|---|---|---|---|
| $\tilde{\mathcal{P}}_1$ | $(0.1, 0.5, 0.2)$ | $(0.1, 0.5, 0.3)$ | $(0.1, 0.6, 0.2)$ | $(0.3, 0.3, 0.2)$ | $(0.8, 0.1, 0)$ |

| | | | | | |
|---|---|---|---|---|---|
| $\tilde{\mathcal{P}}_2$ | $(0.2, 0.4, 0.3)$ | $(0.2, 0.3, 0.4)$ | $(0.3, 0.2, 0.3)$ | $(0.7, 0, 0.1)$ | $(0.4, 0.2, 0.3)$ |
| $\tilde{\mathcal{P}}_3$ | $(0.3, 0.4, 0.2)$ | $(0.2, 0.3, 0.4)$ | $(0.6, 0.2, 0.1)$ | $(0.2, 0.3, 0.4)$ | $(0.5, 0.2, 0.1)$ |
| $\tilde{\mathcal{P}}_4$ | $(0.2, 0.3, 0.3)$ | $(0.2, 0.6, 0.2)$ | $(0.4, 0.2, 0.3)$ | $(0.2, 0.3, 0.3)$ | $(0.2, 0.6, 0.1)$ |

**Table 8**: Relation between symptoms of the disease

| PFSR | $\tilde{\mathcal{S}}_1$ | $\tilde{\mathcal{S}}_2$ | $\tilde{\mathcal{S}}_3$ | $\tilde{\mathcal{S}}_4$ | $\tilde{\mathcal{S}}_5$ |
|---|---|---|---|---|---|
| $\widetilde{\mathcal{D}}_1$ | $(0.1, 0.2, 0.3)$ | $(0.5, 0.2, 0.3)$ | $(0.4, 0.2, 0.4)$ | $(0.1, 0.2, 0.6)$ | $(0.5, 0.5, 0)$ |
| $\widetilde{\mathcal{D}}_2$ | $(0.5, 0.1, 0.3)$ | $(0.1, 0.5, 0.3)$ | $(0.1, 0.2, 0.5)$ | $(0.3, 0.2, 0.4)$ | $(0.8, 0.1, 0)$ |
| $\widetilde{\mathcal{D}}_3$ | $(0.3, 0.2, 0.4)$ | $(0.1, 0.6, 0.2)$ | $(0.2, 0.2, 0.3)$ | $(0.1, 0, 0.9)$ | $(0.2, 0.5, 0.2)$ |
| $\widetilde{\mathcal{D}}_4$ | $(0.4, 0.2, 0.3)$ | $(0.7, 0.1, 0.2)$ | $(0.5, 0.1, 0.2)$ | $(0.4, 0.3, 0.1)$ | $(0.1, 0.5, 0.2)$ |
| $\widetilde{\mathcal{D}}_5$ | $(0.1, 0.2, 0.5)$ | $(0.2, 0.4, 0.3)$ | $(0.3, 0.5, 0.1)$ | $(0.2, 0.7, 0.1)$ | $(0.5, 0.2, 0.1)$ |

**Table 9:** Calculated distances between disease and patients using existing and proposed measure

| PFSR | $\widetilde{\mathcal{D}}_1$ | | $\widetilde{\mathcal{D}}_2$ | | $\widetilde{\mathcal{D}}_3$ | | $\widetilde{\mathcal{D}}_4$ | | $\widetilde{\mathcal{D}}_5$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ | $M_G$ | $M_{SB}$ |
| $\tilde{\mathcal{P}}_1$ | 0.1791 | 0.1470 | 0.1001 | **0.0821** | 0.1946 | 0.1605 | 0.2066 | 0.1704 | 0.1147 | 0.0948 |
| $\tilde{\mathcal{P}}_2$ | 0.1737 | 0.1455 | 0.1424 | **0.1174** | 0.1485 | 0.1203 | 0.1439 | 0.1184 | 0.1439 | 0.1189 |
| $\tilde{\mathcal{P}}_3$ | 0.1377 | 0.1153 | 0.1251 | 0.1029 | 0.1596 | 0.1311 | 0.1457 | 0.1204 | 0.1066 | **0.0865** |
| $\tilde{\mathcal{P}}_4$ | 0.1174 | 0.0970 | 0.1434 | 0.1184 | 0.1075 | **0.0905** | 0.1173 | 0.0966 | 0.1340 | 0.1099 |

Bold numerical values show the disease by which patient is suffering.

Table 9, displays the correlation between the patients and their respective disease with bold values, using relationship between symptoms of patient and relationship between symptoms of disease given in Table 7 and Table 8 respectively. Particularly, the Table 9 indicates that patient $\tilde{\mathcal{P}}_1$ is affected with disease $\widetilde{\mathcal{D}}_2$, patient $\tilde{\mathcal{P}}_2$ has disease $\widetilde{\mathcal{D}}_2$ patient $\tilde{\mathcal{P}}_3$ is suffering from disease $\widetilde{\mathcal{D}}_5$ and patient $\tilde{\mathcal{P}}_4$ also has disease $\widetilde{\mathcal{D}}_3$. Also, proposed measure $M_{SB}$ shows the superior value compare to $M_G$.

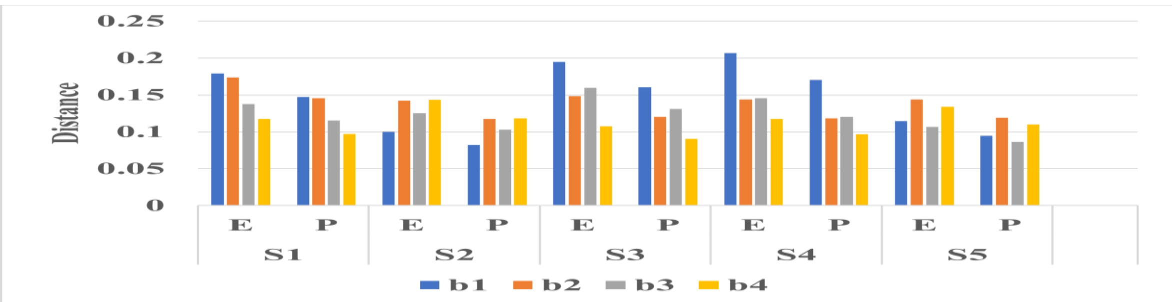



**Figure 5:** Comparisons of distance of the disease with the patients using existing measure $M_G$ and proposed measure $M_{SB}$.

The result of comparative analysis of proposed measure for this problem is discussed in Table 9 and Fig. 5, respectively.

### 5.3 Pattern analysis

The classical problem was expected to solve using an artificial neuronal network which is a case of pattern classification, i.e., handwritten characters[39], [40], [41]. Pixels represented the object of reality; frequency patterns represent a linguistic signal and a sound also came under the concept of pattern. The first machine for pattern classification was developed by Frank Rosenblatt at Cornell University, New York in 1957 and 1958. Here we apply the proposed measure and compare the performance with the existing measures with the help of examples which shows the superiority of proposed measure.

### 5.4 Algorithm based on stated measure

Consider $A = \{a_1, a_2, a_3 \ldots\ldots, a_n\}$ be finite universal set and assume the n patterns $B = \{F_1, F_2, F_3 \ldots, F_n\}$ which can be expressed by (PIFSs) as $F_j = \left\{\left(a_i, \mu_{F_j}(a_i), \nu_{F_j}(a_i), \gamma_{F_j}(a_i) \middle| a_i \in A\right)\right\}$ and $k$ test sample $C = \{C_1, C_2, C_3 \ldots C_k\}$ which can be expressed by (PIFSs) as

$C_p = \left\{\left(a_i, \mu_{F_j}(a_i), \nu_{F_j}(a_i), \gamma_{F_j}(a_i) \middle| a_i \in A\right)\right\}$ the steps on pattern can be follows as

**Step 1** Find the measure between the given pattern $F_j$ and test sample $C_k$, by using the new (PIFSs) method

$$M_{SB}(F_j, C_k) = \frac{1}{4n}\sum_{j=1}^{n}\begin{bmatrix}\left|e^{-\mu_{F_j}(a_j)} - e^{-\mu_{C_k}(a_j)}\right| + \left|e^{-\nu_{F_j}(a_j)} - e^{-\nu_{C_k}(a_j)}\right| + \\ \left|e^{-\gamma_{F_j}(a_j)} - e^{-\gamma_{C_k}(a_j)}\right| + \left|e^{-\varphi_{F_j}(a_j)} - e^{-\varphi_{C_k}(a_j)}\right|\end{bmatrix}$$

**Step 2** Pick the minimum value between the PIFSs $F_j$ and $C_k$ calculating in the step-1

Symbolically $M_{SB}(F_\alpha, C_k) = \min\limits_{1 \le j \le n} M_{SB}(F_j, C_k)$

**Step 3** Now, text sample $C_k$ is assigned for the pattern $F_\alpha$, where

$$\alpha = arg \min_{1 \le j \le n} M_{SB}(F_j, C_k)$$

**Problem 5.3** Consider $F_1, F_2, F_3$ and $F$ be the four-pattern taken in the form of PIFSs as:

$F_1 = \{(0.4, 0.3, 0), (0.6, 0.1, 0.1), (0.4, 0.3, 0.1), (0.7, 0, 0.2), (0.5, 0.3, 0.2)\}$

$F_2 = \{(0.2, 0.1, 0.5), (0.3, 0.3, 0.3), (0.7, 0.1, 0.1), (0.1, 0.5, 0.2), (0.2, 0.3, 0.4)\}$

$F_3 = \{(0.3, 0.4, 0.2), (0.5, 0.3, 0.1), (0.1, 0.3, 0.4), (0.2, 0.5, 0.3), (0.4, 0.3, 0.1)\}$

$F = \{(0.4, 0.3, 0.2), (0.4, 0.2, 0.2), (0.6, 0.2, 0.1), (0.7, 0.1, 0), (0.3, 0.4, 0.2)\}$

We need to check the measure between $F_j$ and $F$ where $j$ takes values 1, 2 and 3. The pattern $F$ is categorized under $F_j$ where $1 \le j \le 3$, if the measure between $F$ and $F_j$ is smallest. The calculated measure values between $F$ and $F_j$ where $1 \le j \le 3$, using the proposed and existing measure, displayed in table 10 and graphically represented in Figure 6.

**Table 10**: Computed values of distance measures with existing and proposed measure $M_{SB}$

| Measures | $(F_1, F)$ | $(F_2, F)$ | $(F_3, F)$ | Results |
|---|---|---|---|---|
| $M_{d_1}$ | 0.1000 | 0.1500 | 0.1700 | $F_1$ |
| $M_{d_2}$ | 0.1265 | 0.2074 | 0.2236 | $F_1$ |
| $M_{d_3}$ | 0.0500 | **0.0650** | **0.0650** | Unable to classify |
| $M_{d_4}$ | 0.1000 | 0.1597 | 0.1628 | $F_1$ |
| $M_{d_5}$ | 0.3750 | 0.5154 | 0.4755 | $F_1$ |
| $M_{d_6}$ | 0.3491 | 0.4011 | 0.3880 | $F_1$ |
| $M_{d_7}$ | 0.1250 | 0.1872 | 0.1775 | $F_1$ |
| $M_{d_8}$ | 0.1054 | 0.1725 | 0.1686 | $F_1$ |
| $M_{d_9}$ | 0.1000 | 0.1867 | 0.1933 | $F_1$ |
| $M_{d_{10}}$ | 0.1000 | 0.1833 | 0.1929 | $F_1$ |
| $M_{d_{11}}$ | 0.2000 | **0.2600** | **0.2600** | Unable to classify |
| $M_{d_{12}}$ | 0.0894 | 0.1428 | 0.1456 | $F_1$ |
| $M_{d_{13}}$ | 1.0000 | 1.5000 | 1.7000 | $F_1$ |
| $M_{d_{14}}$ | 0.2000 | 0.3000 | 0.3400 | $F_1$ |
| $M_{d_{15}}$ | 0.4000 | 0.6557 | 0.7071 | $F_1$ |
| $M_{d_{16}}$ | 0.8944 | 1.4663 | 1.5811 | $F_1$ |
| $M_{d_{17}}$ | **0.0400** | **0.0400** | 0.0200 | Unable to classify |
| $M_G$ | 0.0926 | 0.1360 | 0.1552 | $F_1$ |
| $M_{SB}$ | 0.0794 | 0.1123 | 0.1292 | $F_1$ |

Table 10 illustrates that F is closest to $F_1$ as proved by all distance measures and $M_{d_3}$ $M_{d_{11}}$ and $M_{d_{17}}$ suggests an unreasonable result. Our proposed distance measure $M_{SB}$ continuously provides better results and closely with $F_1$. It can be noticed that F is close to $F_1$ according to all distance measures, and the proposed measure verify this result. Consequently, F is grouped with $F_1$.

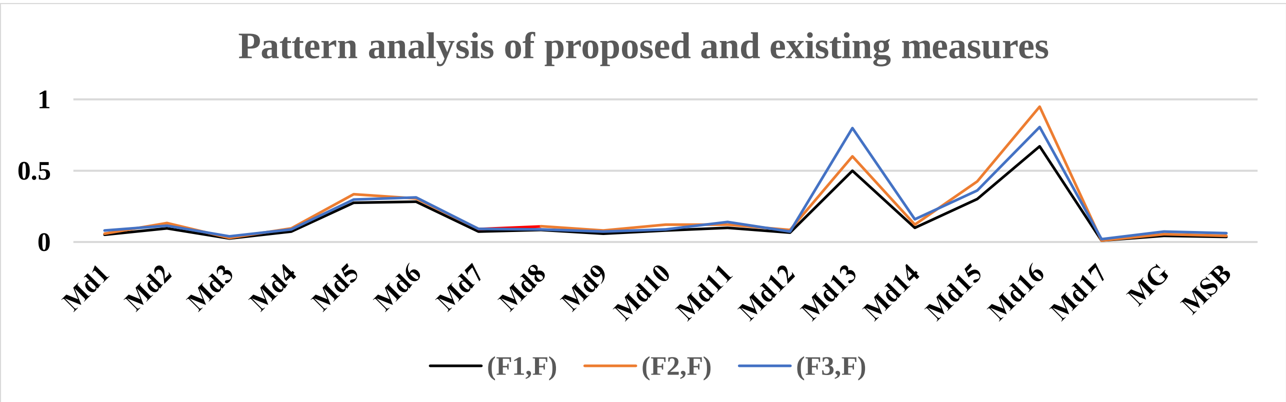



**Figure 6** Pattern analysis of proposed and existing measures.

The result of comparative analysis of proposed measure for this problem is discussed in Table 10 and Fig. 6, respectively.

**Problem 5.4** Consider $F_1, F_2, F_3$ and $F$ be the four-pattern taken in the form of PFSs as:

$F_1 = \{(0.4, 0, 0.5), (0.7, 0, 0.3), (0.5, 0, 0.3), (0.7, 0, 0.3), (0.2, 0.2, 0.6)\}$

$F_2 = \{(0.2, 0, 0.7), (0.7, 0, 0.3), (0.5, 0, 0.2), (0.7, 0, 0.3), (0.1, 0.5, 0.4)\}$

$F_3 = \{(0.4, 0, 0.6), (0.7, 0, 0.2), (0.5,0, 0.4), (0.7, 0, 0.2\ ), (0.3, 0.4, 0.3)\}$

$F\ = \{(0.3, 0, 0.6), (0.7, 0, 0.3), (0.4, 0, 0.3), (0.7, 0, 0.3), (0.5, 0.1, 0.4)\}$

We need to find the measure between $F_j$ and $F$ where $j$ takes values 1, 2 and 3. The pattern $F$ is classified with $F_j$ where $1 \leq j \leq 3$, if the measure between $F$ and $F_j$ is smallest. The calculated measure values between $F$ and $F_j$ where $1 \leq j \leq 3$, using the proposed and existing measure, are given in the table 11 and graphically represented in Figure 7.

**Table 11:** Computed values of distance with existing measure and proposed measure $M_{SB}$

| Measure | $(F_1, F)$ | $(F_2, F)$ | $(F_3, F)$ | Results |
|---|---|---|---|---|
| $M_{d_1}$ | 0.0500 | 0.0600 | 0.0800 | $F_1$ |
| $M_{d_2}$ | 0.0949 | 0.1342 | 0.1140 | $F_1$ |
| $M_{d_3}$ | 0.0250 | 0.0300 | 0.0400 | $F_1$ |
| $M_{d_4}$ | 0.0742 | 0.0949 | 0.0894 | $F_1$ |
| $M_{d_5}$ | 0.2759 | 0.3333 | 0.2991 | $F_1$ |
| $M_{d_6}$ | 0.2810 | 0.3056 | 0.3112 | $F_1$ |
| $M_{d_7}$ | 0.0734 | 0.0909 | 0.0922 | $F_1$ |
| $M_{d_8}$ | 0.0836 | 0.1091 | 0.0887 | $F_1$ |
| $M_{d_9}$ | 0.0600 | 0.0800 | 0.0733 | $F_1$ |
| $M_{d_{10}}$ | 0.0825 | 0.1200 | 0.0872 | $F_1$ |
| $M_{d_{11}}$ | 0.1000 | 0.1200 | 0.1400 | $F_1$ |
| $M_{d_{12}}$ | 0.0663 | 0.0849 | 0.0721 | $F_1$ |
| $M_{d_{13}}$ | 0.5000 | 0.6000 | 0.8000 | $F_1$ |
| $M_{d_{14}}$ | 0.1000 | 0.1200 | 0.1600 | $F_1$ |
| $M_{d_{15}}$ | 0.3000 | 0.4243 | 0.3606 | $F_1$ |
| $M_{d_{16}}$ | 0.6708 | 0.9487 | 0.8062 | $F_1$ |
| $M_{d_{17}}$ | 0.0100 | 0.0100 | 0.0200 | Unable to classify |
| $M_G$ | 0.0433 | 0.0535 | 0.0741 | $F_1$ |
| $M_{SB}$ | 0.0345 | 0.0434 | 0.0625 | $F_1$ |

Table 11 illustrates that F is closest to $F_1$ as proved by all distance measures and $M_{d_{17}}$ shows an unreasonable result. Our proposed distance measure $M_{SB}$ continuously provides better results and very closely to $F_1$. It is observed that F is close to $F_1$ calculating by all distance measures, and the proposed measure verify this result. Consequently, F is classified with $F_1$.

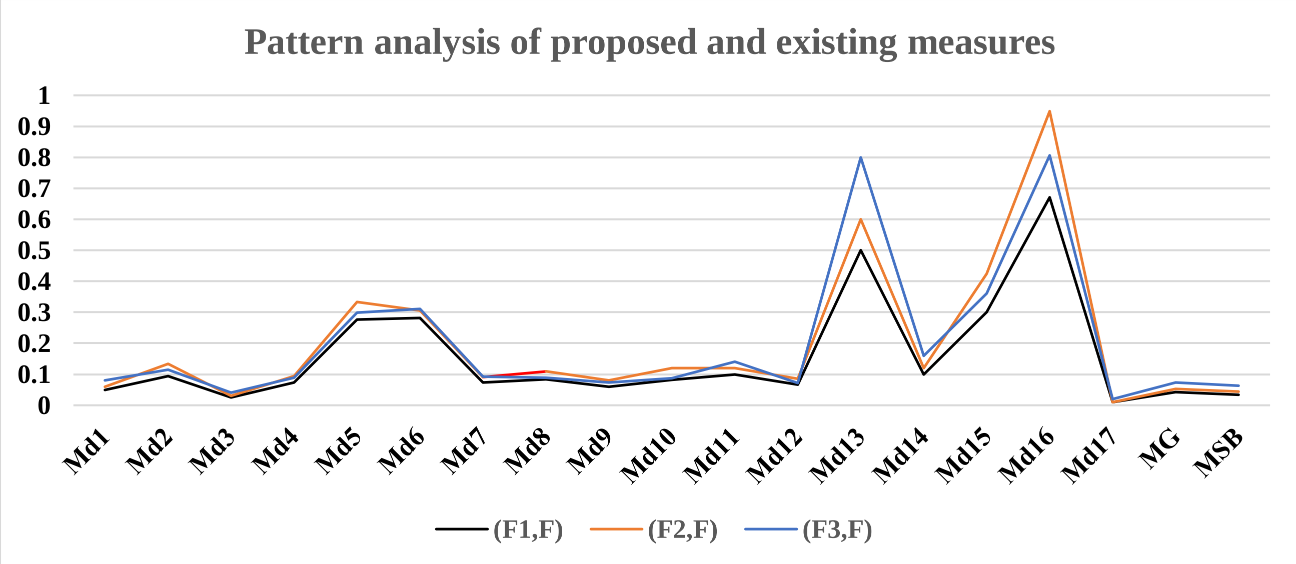



Figure 7: Pattern analysis of proposed and existing measures.

The result of comparative analysis of proposed measure for this problem is discussed in Table 11 and Fig. 7, respectively.

## 6. Conclusion

This paper introduces a new method to measure distances for picture fuzzy sets and demonstrates its efficiency in identifying the relationships and differences between fuzzy elements. Through various comparative analysis with existing measures, it consistently shows superior performance, and provide more accurate and meaningful results. Future efforts can be focused on refining the algorithm for advanced computational efficiency and exploring its applicability in various field such as pattern analysis, machine learning, artificial intelligence etc. Additionally, advanced techniques such as machine learning or optimization methods may contribute to improvement of the proposed distance measure. Further studies can also explore real-world applications and validate the measure's effectiveness in solving complex problems in areas like pattern recognition, decision-making, and image processing.

### Conflict of interest

The authors confirm that they have no conflict of interest.

### Data availability statement

All the data supporting this study's findings are provided in the article.

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**Refrences**

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