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Exploring Stolarsky-3 Mean Cordial Labeling Properties in Graph Classes with a Python Module

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Abstract:

A graph G= (V,E), where V is the set of vertices and E is the set of edges, with p vertices and q edges. A graph G is referred to as a Stolarsky-3 Mean cordial graph if we can assign distinct labels to each vertex $x \in V$ from the set {0,1,2}, denoted by f(x), and distinct labels to each edge $e = uv \in E$ based on the values assigned to the endpoints u and v. The label for the edge e = uv is calculated using one of the following Stolarsky-3 Mean cordial formula:

$$f(e = uv) = \sqrt{\frac{f(u)^2 + \mathrm{f(u)F(v)} + f(v)^2}{3}}$$

The function f is referred to as a Stolarsky-3 mean cordial labeling if the conditions $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$. Hold for i, j $\varepsilon\{0,1,2\}$, where $v_f(x)$ and $e_f(x)$ represent the number of vertices and edges respectively, labeled with x (x= 0,1,2). A graph that admits such a labeling is called a mean cordial graph. Such that the edge labels are derived from the flooring function of the Stolarsky-3 mean of the labels of the two end vertices of each edge. In this paper, we have created python module to analysis the Stolarsky-3 mean cordial labeling properties of various graph classes, including Path (P_n), Cycle (C_n), Wheel (W_n), Star Graph ($K_{1,n}$ or S_n) & Wheel and Path Graph (W_nP_m).

Keywords: Stolarsky-3 Mean, Mean Cordial, Path, Cycle, Wheel, Star Graph

1. Introduction

One important topic of research in graph theory is graph labeling, which is the process of assigning labels typically numbers to a graph's vertices, edges, or both in accordance with predetermined guidelines [1]. Over the years, various types of labeling techniques have been developed, each with distinct characteristics and applications. Among these, cordial labeling has garnered particular interest due to its relatively relaxed conditions compared to more restrictive forms such as graceful or harmonious labeling [2]. A function assigns labels to the vertices and edges of a graph such that the number of vertices (or edges) labeled with 0 or 1 differs by at most one [3]. This type of labeling has been widely studied for its simplicity and applicability to different types of graphs.

The researcher explored how assigning labels based on the arithmetic mean of vertices can lead to distinct edge labels [4, 5]. This work laid the groundwork for further investigations into mean-based graph labeling. Harmonic mean of vertex labels to assign edge labels, providing a different perspective on mean-based labeling techniques [6, 7]. This research expanded the application of mean-based labeling in various types of graphs, contributing to the development of new labeling methodologies.

Stolarsky Mean Labeling technique [8-11] utilizes a mean calculation based on the Stolarsky mean, which generalizes several other mean functions. Although specific literature on Stolarsky Mean Labeling is limited, it builds upon the earlier works on mean and harmonic mean labeling, offering a novel approach to graph labeling. In-depth explorations of Stolarsky mean in graph labeling and its potential applications, particularly in cryptography, and Network analysis Applications. Stolarsky-3 mean labeling remains a crucial area of study within graph theory, focusing on assigning labels to vertices and edges in a way that satisfies specific conditions [12]. Till now no one is analyzing Stolarsky-3 mean cordial labeling techniques.

Stolarsky-3 mean cordial labeling is a relatively recent addition to this field, expanding on concepts introduced by earlier labeling techniques. This research introduces a new approach by pinpointing vertex combinations and formulating mathematical models that describe Stolarsky-3 mean cordial labeling. Research on Stolarsky-3 Mean Labeling is likely to have implications in areas where graph labeling is crucial, such as network design and communication theory.

2. **Methods:**

We have developed the python module to find the graph labeling properties.

2.1. Python Module

```
import itertools

import math

def stolarsky_3_mean(u, v):

   return math.floor(math.sqrt((u**2 + u*v + v**2) / 3))

# Function to check the conditions for valid Stolarsky-3 mean cordial labeling

def is_valid_labeling(s1, s2, s3, s4, s5, s6, m1, m2, m3, m4):

   return (

      abs(s1 - s2) <= 1 and abs(s2 - s3) <= 1 and abs(s1 - s3) <= 1 and

      abs(s4 - s5) <= 1 and abs(s5 - s6) <= 1 and abs(s4 - s6) <= 1 and

      s1 + s2 + s3 == m1 and s4 + s5 + s6 == m4

   )

# Main function to generate and validate Stolarsky-3 mean cordial labeling

def stolarsky_3_mean_cordial_labeling(n, m1, m2, m3, m4):

   combinations = itertools.product([0, 1, 2], repeat=n)
```

```
    valid_combinations = []  # To store valid labelings
    for combination in combinations:
        s4 = s5 = s6 = 0
        for i in range(len(combination) - 1):  # Assuming the graph is a path (Pn)
            u, v = combination[i], combination[i + 1]
            edge_label = stolarsky_3_mean(u, v)
            if edge_label == 0:
                s4 += 1
            elif edge_label == 1:
                s5 += 1
            elif edge_label == 2:
                s6 += 1
        if is_valid_labeling(s1, s2, s3, s4, s5, s6, m1, m2, m3, m4):
            valid_combinations.append((combination, (s1, s2, s3, s4, s5, s6)))
    print("Valid Stolarsky-3 Mean Cordial Labelings:")
    for combo, counts in valid_combinations:
        print(f"Vertex Labels: {combo}, Counts: {counts}")
```

stolarsky_3_mean_cordial_labeling(n, m1, m2, m3, m4)he process, printing only those combinations that satisfy the specified conditions.

This Python module will analyze and determine the Stolarsky-3 mean cordial labeling characteristics in various types of graphs. Let me know if you'd like further assistance with this paper

3.Results and Analysis

Table 1. Comparative analyses of 3 different mean labeling with respect to various graphs.

The table provided compares different types of graph labeling with respect to mean cordial labeling, Stolarsky-3 mean labeling, and Stolarsky-3 mean cordial labeling. Let's break down and analyze these findings across different graph types:

Path Graph (P_n)

Theorem 3.1: Every Path P_n is Stolarsky-3 Mean Cordial.

Proof:

Consider a Path P_n with vertices $u_1, u_2, \ldots, u_n$.

Define $f: V(P_n)$____ $\{1, 2, \ldots, q+1\}$.

Therefore $f(u_i) = i$, $1 \leq i \leq n$.

Then the edge labels are distinct.

Case 1:n≡0(mod3)

Let n=3t.f(u_i) =0,1≤i≤t, f(u_{t+i}) =1, 1≤i≤t, f(u_{2t+i})=2, 1≤i≤t.

Then v_f(0) =v_f(1) =v_f(2) =t and e_f(0) =t-1, e_f(1)=e_f(2) =t.

Case 2:n≡1(mod3)

Let n=3t+1.f(u_i) =0, 1≤i≤t, f(u_{t+i})=1,1≤i≤t+1,f(u_{2t+1+i})=2, 1≤i≤t+1.

Then v_f(0) =t+1, v_f(1) =v_f(2) =t and e_f(0) =e_f(1) =e_f(2) =t.

Case 3:n≡2(mod3)

Let n=3t+2, f(u_i)=0, 1≤i≤t, f(u_{t+i})=1, 1≤i≤t+1, f(u_{2t+1+i})=2, 1≤i≤t+1.

Then v_f(0) =t+1, v_f(1)=v_f(2)=t and e_f(0) =t+1, e_f(1)=e_f(2)=t.

Hence the above three cases satisfies the condition |v_f(i)-v_f(j)|≤1 and |e_f(i)-e_f(j) |≤1 for all i, j ∈{0, 1,2}.

Hence f is a Stolarsky-3 Mean Cordial labeling.

The Stolarsky-3 Mean Cordial Labeling of the Path Graph (P_n) is illustrated in Figure 1.

The Mean Cordial Labeling method is applicable to graphs with any number of vertices. Similarly, Stolarsky-3 Mean Labeling is also effective for graphs with any number of vertices n. Furthermore, Stolarsky-3 Mean Cordial Labeling remains valid and applicable for all values of n, ensuring its versatility across different graph sizes (Table 1). All three labeling methods work well for path graphs, no matter how many vertices they have. This is because path graphs are simple and linear, making them easy to label.

Cycle Graph (C_n)

Theorem 3.2:The Cycle C_n is Stolarsky-3 Mean Cordial iff n≡1, 2(mod3).

Proof:

Consider a Cycle C_n with vertices u_1, u_2,...., $u_n u_1$.

Define f:V(Cn) ____ {1, 2,...,q+1}

Therefore f(u_i) =i,1≤i≤n.

Since the edge labels are distinct.

Case 1: n≡0(mod3)

Let n=3t.If C_n admits Stolarsky-3Mean Cordial labeling f.

Then v_f(0) =v_f(1) =v_f(2)=t and e_f(0) =e_f(1) =e_f(2) =t.

Assign 0's to t number of vertices C_n.

Then we get e_f(0) >t.

Hence f is not a Stolarsky-3Mean Cordial labeling.

Case 2:n≡1(mod3)

Let n=3t+1, Assign the label 1 to t+1 vertices and the labels 0 and 2 to the remaining each of t vertices.

Then $v_f(0)=v_f(1)=t$, $v_f(2)=t+1$ and $e_f(0)=e_f(2)=t$, $e_f(1)=t+1$.

Hence f is Stolarsky-3 Mean Cordial labeling.

Case 3:n≡2(mod3)

Let n=3t+2, Assign the label 1 to t vertices and the labels 1 and 2 to the remaining each of t vertices.

Then $v_f(0)=v_f(1)=t+1$, $v_f(2)=t$ and $e_f(0)=e_f(2)=t+1$, $e_f(1)=t$.

Hence f is Stolarsky-3Mean Cordial labeling.

The Stolarsky-3 Mean Cordial Labeling of the Cycle Graph (C_n) is illustrated in Figure 2.

Mean Cordial Labeling is Works for most, but not when (n = 3, 6, 9,…) etc. (multiples of 3). This happens because of an imbalance in how vertices and edges are connected in these cases. Stolarsky-3 Mean Labeling is Works for any n, even multiples of 3. Stolarsky-3 Mean Cordial Labeling is Same as Mean Cordial Labeling, it does not work when (n = 3, 6, 9,…) etc (Table 1). Stolarsky-3 Mean Labeling is more flexible for cycle graphs, while the other two methods have trouble with graphs where is a multiple of 3.

Wheel Graph (W_n)

Theorem 3.3:The Wheel graph W_n is not a Stolarsky-3 Mean Cordial for any n.

Proof:

Consider the Wheel graph W_n, where C_n is a cycle with vertices u_1, u_2,....., $u_n u_1$ and there is a central vertex u.

Case 1: n≡0(mod3)

Let n=3t, a contradiction.

Case 2:n≡1(mod3)

Let n=3t+1, again a contradiction.

Case 3:n≡2(mod3)

Let n=3t+2,we get a contradiction.

Therefore Contradiction arise from attempting to label the edges while maintaining distinct labels.

Hence the Wheel graph W_n is not a Stolarsky-3Mean Cordial for any n.

Mean Cordial Labeling is not applicable to wheel graphs. Similarly, Stolarsky-3 Mean Labeling is also ineffective for these graphs(Table 1). Additionally, Stolarsky-3 Mean Cordial Labeling does not apply to wheel graphs either. Wheel graphs are tricky because the central hub vertex makes it hard to balance the labels. None of the three methods work for this type of graph.

Star Graph $K_{1,n}$ or S_n

Theorem 3.4:The Star graph($K_{1,n}$) (S_n) is a Stolarsky-3 Mean Cordial if $n\leq 15$.

Proof:

Consider the Star graph ($K_{1,n}$) with vertices v_1, v_2,..., v_n respectively.

Case 1:for $2\leq n\leq 8$

Assign u=1, v_1=2,v_i =2i-1 for $2\leq i\leq 8$.

Case 2: for $9\leq n\leq 15$

Assign u=1, v_1=2 and for v_i increase systematically to ensure unique values.

Case 3:n>15

If n>15, label u=1 and v_i=2i-3 for $4\leq i\leq n$.

Therefore $K_{1,n}$ is not a Stolarsky-3 Mean Cordial for all n>15.

From case 1,2,3,we conclude that K1, n is Stolarsky-3 Mean Cordial if $n\leq 15$

The Stolarsky-3 Mean Cordial Labeling of the Star Graph (Sn) is illustrated in Figure 3.

Mean Cordial Labeling: Only works when n < 3. Stolarsky-3 Mean Labeling: Works when n < 16, but not for larger . Stolarsky-3 Mean Cordial Labeling: Works for any n. The Stolarsky-3 Mean Cordial Labeling is the best option for star graphs, as it works no matter how many vertices the graph has. The other two methods only work for smaller graphs.

Wheel and Path Graph W_nP_m

Theorem 3.5:The Wheel Graph W_n connected with a Path Graph P_m(denoted as W_nP_m) is Stolarsky - 3 Mean Cordial Labeling for any n.

Proof:

Consider the graph W_nP_m,which consists of the Wheel graph W_n connected to a Path graph Pm.

The Wheel graph W_n has a cycle C_n with the vertices $u_1,u_2,\dots,u_n$ and a central vertex u connected to all other vertices in the cycle.The Path graph Pm consists of vertices $v_1,v_2,\dots,v_n$.

Case 1:n+m≡0(mod3)

Let n+m=3t,where t is an integer.

$v_f(0)=v_f(1)=v_f(2)=t,e_f(0)=e_f(1)=e_f(2)=t$.

Hence it is Stolarsky-3 Mean Cordial.

Case 2:n+m≡1(mod3)

Let n+m=3t+1,where t is an integer.

$v_f(0)+v_f(2)=t,v_f(1)=t+1,e_f(0)=e_f(1)=e_f(2)=t$.

Hence it is Stolarsky-3Mean Cordial.

Case3:n+m≡2(mod3)

Let n+m=3t+2,where t is an integer.

$v_f(0)=v_f(2)=t+1,v_f(1)=t,e_f(0)=e_f(2)=t+1,e_f(1)=t$.

Hence it is Stolarsky-3 Mean Cordial.

In all cases,the graph satisfies the Stolarsky-3 Mean Cordail labeling.

Hence the graph W_nP_m is Stolarsky-3 Mean Cordial for any n.

The Stolarsky-3 Mean Cordial Labeling of the Wheel and Path Graph (W_nP_m) is illustrated in Figure 4.

Mean Cordial Labeling works for any n.Stolarsky-3 Mean Labeling does not work. Stolarsky-3 Mean Cordial Labeling works for any n. Combining a wheel and a path makes Stolarsky-3 Mean Labeling fail, but the other two methods can handle this graph type(table 1).

This comparative analysis highlights how different graph structures interact with various labeling schemes, providing insights into their mathematical properties and practical applications in graph theory.

4. Conclusion

Different patterns emerge when different graph architectures are analyzed for compatibility with Mean Cordial Labeling, Stolarsky-3 Mean Labeling, and Stolarsky-3 Mean Cordial Labeling. The versatility of path graphs (P_n) is demonstrated by their support for all three labeling techniques. Cycle graphs only face issues for multiples of 3 with Mean Cordial and Stolarsky-3 Mean Cordial Labeling. Wheel graphs cannot be labeled by any method due to their complex structure. Star graphs are best labeled with Stolarsky-3 Mean Cordial Labeling, which works for all. Combined wheel and path graphs are compatible with Mean Cordial and Stolarsky-3 Mean Cordial Labeling. In summary, Stolarsky-3 Mean Cordial Labeling is the most versatile method, but wheel graphs remain a challenge for all. These findings support specific applications by highlighting the significance of network topology in assessing labeling feasibility.

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Table 1. Comparative analyses of 3 different mean labeling with respect to various graphs.

Graph Type	Mean Cordial labeling (n is number of vertices)	Stolarsky-3 Mean Labeling (n is number of vertices)	Stolarsky-3 Mean Cordial Labeling(n is number of vertices)
Path P_n	for any n	for any n	for any n
Cycle C_n	for any n, Except n=3,6,9..	for any n	for any n, Except n=3,6,9..
Wheel W_n	No	No	No
Star Graph $K_{1,n}$ or S_n	n<3	n<16	for any n
Wheel and Path Graph W_nP_m	for any n	No	for any n

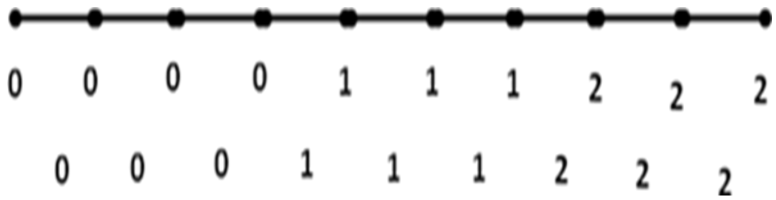

Fig. 1. Stolarsky-3 Mean cordial labeling of Path (P_{10} ➔0000111222) graph

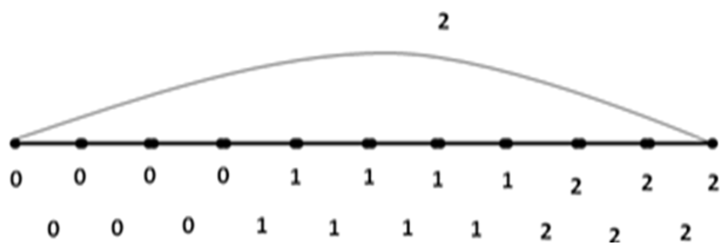

Fig. 2. Stolarsky-3 Mean Cordial labeling of Cycle (C_{11} ➔00001111222) graph.

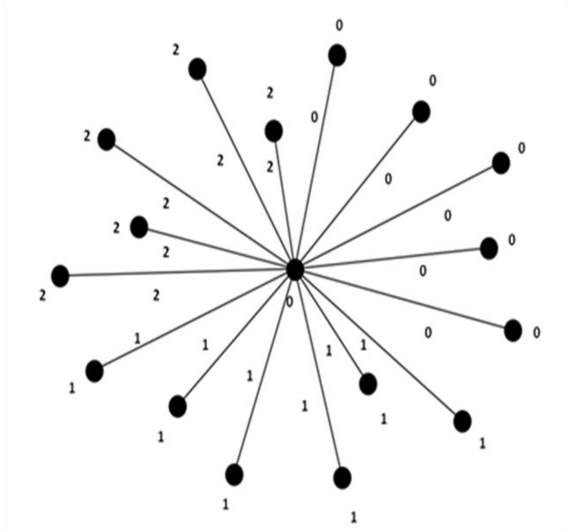

Fig. 3. Stolarsky-3 Mean Cordial labeling of Star (S_{16} ➔00000011111122222) graph

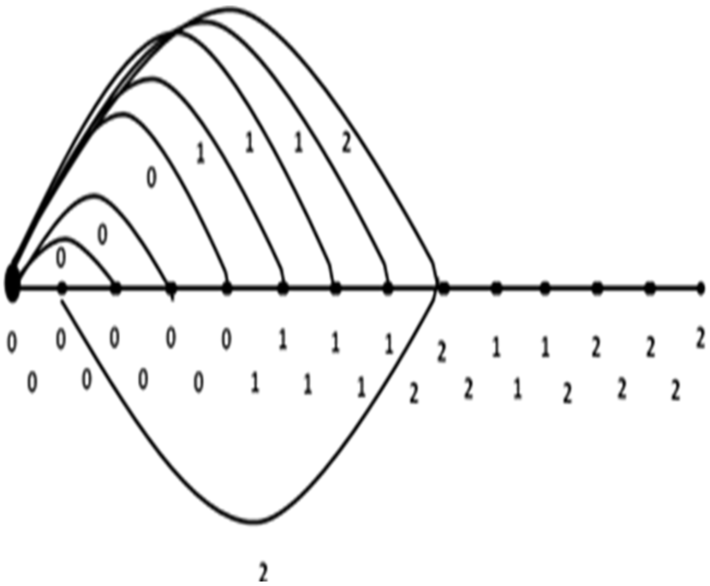

Fig.4. Stolarsky-3 Mean Cordial labeling of Wheel and Path Graph (W_9P_6 ➔00000111211222) graph.