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Odd Hamming Distance Labeling of Some Path Related Graphs

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Abstract

Binary data strings of equal length are compared using the metric called hamming distance. It is the number of bit positions in which the two binary strings differ. The hamming distance between two binary strings m and n of equal length is denoted by hd(m,n). We introduced the concept of Hamming distance labeling and odd Hamming distance labeling. In this paper, it is shown that Path graph, Star graph, One point union of Path graphs, Coconut tree, are Odd hamming distance labeled graphs and obtained their Odd hamming distance number. Both Hamming and Odd Hamming distance labeling are used to send secret messages in Cryptography.

Keywords: Hamming Distance, Odd Hamming Distance Labeling, Path graph, Star graph, One-point union for Path of graphs, Coconut tree graph.

1. Introduction

Let G = (V, E) be a graph with vertex set V and edge set E. A Path graph $P_m,\ m \geq 1$ is an alternating sequence of vertices and edges, beginning and ending with vertices in which each edge is incident with two vertices immediately preceding and following it. Edges and vertices appear only once in a path. A Path graph of length m has m+1 vertices and m edges[2].The complete bipartite graph of the form $K_{1,n}$ is a Star graph with n+1 vertices and it is denoted by $S_n, n \geq 1$[3].The One point union of Path graph $P_m^n\ n, m \geq 2,$ is obtain by replacing each edge of a Star graph $K_{1,n}$ by Path graph P_m, where n is the number of pendant edges in Star graph and m is the length of the Path graph[6].A Coconut tree CT(n,m), $n \geq 2, m \geq 1$ is the graph obtained from the Path P_m by appending n new pendant edges at an end vertex of P_m[7].

Graph labeling is a function defined on the vertex set or edge set subject to certain conditions enforced on the number of vertices p or on the number of edges q or on both p and q[1]. The concept of graph labeling was introduced in the year 1967 by Rosa and it was further developed by Graham and Sloane in 1980[4]. We introduced the concept of Hamming distance labeling and proved that some Path related graphs are Hamming distance graphs [8]. In this paper, prove the existence of Odd hamming distance labeling of some path related Graphs. Here the notation $[x]_2$ denotes the binary conversion of the number x.

2. Odd Hamming Distance Labelling of some Graphs

2.1. Definition

Let G = (V, E) be a graph. A function $f: V \rightarrow N \cup \{0\}$ is said to be an Odd hamming distance labeling if there exist an induced function $f^* : E \rightarrow \{1,3,5, \ldots, \mathrm{n}\}$ such that for every $uv \in E, f^*(uv) = hd([f(u)]_2, [f(v)]_2)$ satisfying the following conditions:

(i) For every vertex $v \epsilon V$, the set of all edges incident with v receive distinct odd labels.

(ii) For every edge $e = uv$, the adjacent vertices u and v receive distinct labels.

A graph which admits Odd hamming distance labeling is called Odd hamming distance graph.The Odd hamming distance number of a graph G is the least positive integer n such that $2^n - 1 \geq k$, where k = max $\{f(v)/v \in V\}$ and it is denoted by $\eta'_{hd}(\mathrm{G})$.

2.2.1. Algorithm: Odd hamming distance labeling of P_m graph

Procedure: Vertex labeling of P_m graph, m ≥ 1

Input: Path graph P_m

$V \leftarrow \{v_i \;/ 0 \leq i \leq m\}$

$v_0 \leftarrow 0;$

for $i = 1$ to m do

$$v_i \leftarrow \begin{cases} 1 & \text{if } i \equiv 1 (\text{mod} 4) \\ 6 & \text{if } i \equiv 2 (\text{mod} 4) \\ 2 & \text{if } i \equiv 3 (\text{mod} 4) \\ 5 & \text{if } i \equiv 0 (\text{mod} 4) \end{cases};$$

end for

end procedure

Output: The labeled vertices of Path graph P_m.

2.3.2.Theorem The Path graph $P_m, m \geq 1$ is an Odd hamming distance graph and the Odd hamming distance number is $\eta'_{hd}(P_m) = \begin{cases} 1 & if\ m = 1 \\ 3 & if\ m > 1 \end{cases}$.

Proof: Let us consider the Path graph P_m with vertex set $V = \{v_i/0 \leq i \leq m\}$ and edge set $E = \{v_i v_{i+1} \;/\; 0 \leq i \leq m-1\}$. Define a function $f: V \rightarrow N \cup \{0\}$ such that $f(v_i) \neq f(v_{i+1}), 0 \leq i \leq m - 1$ as given in the above algorithm 3.3.1. hence the adjacent vertices receive distinct labels.The edge labels are obtained as follows:

$f^*(v_0 v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = hd([0]_2, [1]_2) = hd(00000, 00001) = 1$

For $1 \leq i \leq m - 1$

Case (i): When $i \equiv 1(\text{mod } 4)$;$f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([1]_2, [6]_2) = 3$

Case (ii): When $i \equiv 2(\text{mod} 4)$;$f^*(v_i v_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([6]_2, [2]_2) = 1$

Case (iii): When $i \equiv 3(\mathrm{mod}\ 4)$;$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([2]_2, [5]_2) = 3$

Case (iv): When $i \equiv 0(\mathrm{mod} 4)$;$f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = hd([5]_2, [1]_2) = 1$

From all the above cases, all the adjacent edges receive distinct odd labels. Hence the Path graph P_m admits Odd hamming distance labeling and the Odd hamming distance number is $\eta'_{hd}(P_m) = \begin{cases} 1 & if\ m = 1 \\ 3 & if\ m > 1 \end{cases}$.

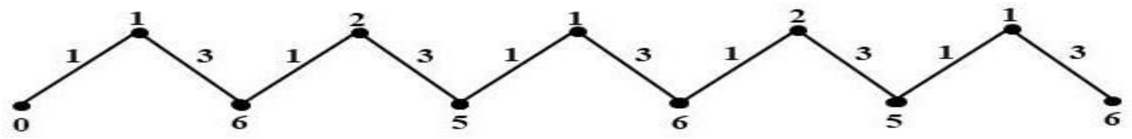

Figure 3.Odd Hamming Distance P_{10} graph.

3.3.3. Algorithm: Odd hamming distance labeling of S_n graph.

Procedure: Vertex labeling of S_n graph $n \geq 1$.

Input: Star graph S_n

$V \leftarrow \{v_i\ /0 \leq i \leq n\}$

$v_0\ \leftarrow 0$;

for $i = 1\ to\ n$ do

$v_i\ \leftarrow 2^{2i-1} - 1$;

end for

end procedure

Output: The labeled vertices of star graph. S_n.

3.3.4.Theorem The Star graph $S_n, n \geq 1$ is an Odd hamming distance graph and the Odd hamming distance number is $\eta'_{hd}(S_n) = 2n - 1$,where n is the number of pendant edges.

Proof: Let us consider the Star graph S_n with vertex set $V = \{v_i\ /0 \leq i \leq n\}$ and edge set $E = \{v_0v_i\ /\ 1 \leq i \leq n\}$. Define a function $f: V \leftarrow N \cup \{0\}$ such that $f(v_0) \neq f(v_i), 1 \leq i \leq n$ as given in the above algorithm 3.3.3. Here all the adjacent vertices receive distinct labels.The edge labels are obtained as follows:

For $1 \leq i \leq n$;

$f^*(v_0v_i) = hd([f(v_0)]_2, [f(v_i)]_2) = hd([0]_2, [2^{2i-1} - 1]_2) = 2i - 1$.

For each i, the corresponding vertex label and edge label are given in the following table:

i	1	2	3	4	5	6	7	…… …	N

$f(v_i)$	1	7	31	127	511	2047	8191	………	$2^{2n-1} - 1$
e = $hd([f(v_0)]_2, [f(v_i)]_2)$	1	3	5	7	9	11	13	………	$2n - 1$

where $f(v_0)$ = 0.From the above table it is clear that all the adjacent edges receive distinct odd labels. Hence the star graph S_n admits Odd hamming distance labeling and the Odd hamming distance number is $\eta'_{hd}(S_n) = 2n - 1$ for any $n \geq 1$.

3.3.5. Algorithm: Odd hamming distance labeling of P_m^n graph

Procedure: Vertex labeling of P_m^n, graph $m, n \geq 2$.

Input: One point union of Path graph P_m^n.

$V \leftarrow \{v_{ij} /1 \leq i \leq n, 0 \leq j \leq m; v_{10} = v_{20} = v_{30} = \cdots = v_{n0}\}$

$v_0 \leftarrow 0; v_0 = v_{10} = v_{20} = v_{30} = \cdots = v_{n0}$

for $j = 1$ *to* m do

$$v_{1j} \leftarrow \begin{cases} 1 \text{ if } j \equiv 1(mod4) \\ 6 \text{ if } j \equiv 2(mod4) \\ 2 \text{ if } j \equiv 3(mod4) \\ 5 \text{ if } j \equiv 0(mod4) \end{cases}; \quad v_{2j} \leftarrow \begin{cases} 7 \text{ if } j \equiv 1(mod4) \\ 3 \text{ if } j \equiv 2(mod4) \\ 4 \text{ if } j \equiv 3(mod4) \\ 0 \text{ if } j \equiv 0(mod4) \end{cases};$$

end for

for $i = 3$ *to* n do

$v_{i1} \leftarrow 2^{2i-1} - 1;\ v_{i2} \leftarrow 2^{2i-2} - 1;$

$v_{i3} \leftarrow 2^{2i-5} - 1;\ v_{i4} \leftarrow 2^{2i-6} - 1;$

for $j = 5$ *to* m do

if $i = 3$

$v_{ij} \leftarrow (v_{(i-1)(j-4)});$

else

$v_{ij} \leftarrow (v_{(i-2)(j-2)});$

end if

end for

end for

end procedure

Output: The labeled vertices of one point union of path graph P_m^n.

3.3.6.Theorem The one point union of path graphs $P_m^n, m, n \geq 2$ is an Odd hamming distance graph and the Odd hamming distance number $\eta'_{hd}(P_m^n) = 2n - 1$.

Proof: Let us consider the one point union of path graphs P_m^n with vertex set $V = \{v_{ij} /1 \leq i \leq n, 0 \leq j \leq m; v_{10} = v_{20} = v_{30} = \cdots = v_{n0}\}$, let $v_0 = v_{i0}, 1 \leq i \leq n$ and edge set $E = \{ v_{ij}v_{i(j+1)} / 1 \leq i \leq n, 0 \leq j \leq m - 1\}$. This graph has mn+1 vertices and mn edges. Define a function $f: V \to N \cup \{0\}$ such that $f(v_{ij}) \neq f(v_{i(j+1)}), 1 \leq i \leq n , 0 \leq j \leq m - 1$ as given in the above algorithm 3.3.6. hence the adjacent vertices receive distinct labels.The edge labels are obtained as follows:

For $1 \leq i \leq n$,

$hd([f(v_0)]_2, [f(v_{i1})]_2)$ = $2i - 1$. Which is given in the following table, here $f(v_0) = 0$.

i	1	2	3	4	5	6	7	………	N
$f(v_{i1})$	1	7	31	127	511	2047	8191	………	$2^{2n-1} - 1$
e = $hd([f(v_0)]_2, [f(v_{i1})]_2)$	1	3	5	7	9	11	13	………	$2n - 1$

For $1 \leq i \leq n$

Case (i): when $j \equiv 1(mod4)$

$f^*(v_{11}v_{12})$ = $hd([f(v_{11})]_2, [f(v_{12}))]_2)$ = hd$([1]_2, [6]_2)$ = 3.

$f^*(v_{21}v_{22})$ = $hd([f(v_{21})]_2, [f(v_{22}))]_2)$ = hd$([7]_2, [3]_2)$ = 1

i	3	4	5	6	7	………	n
$f(v_{i1})$	31	127	511	2047	8191	……….	$2^{2n-1} - 1$
$f(v_{i2})$	15	63	255	1023	4095	……….	$2^{2n-2} - 1$
e = $hd([f(v_{i1})]_2, [f(v_{i2})]_2)$	1	1	1	1	1	1	1

For $1 \leq i \leq n, 5 \leq j \leq m$

$f^*(v_{ij}v_{i(j+1)})$ = $hd([f(v_{ij})]_2, [f(v_{i(j+1)}))]_2)$ = 1.

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i	1	2	3	4	5	…	n
$f(v_{\mathrm{ij}})$	1	7	7=$v_{(3-1)(\mathrm{j}-4)}$	4= $v_{(4-2)(\mathrm{j}-2)}$	1=$v_{(5-2)(\mathrm{j}-2)}$	…	$v_{(\mathrm{n}-2)(\mathrm{j}-2)}$
$f(v_{i(j+1)})$	6	3	3=$v_{(3-1)(\mathrm{j}-3)}$	0= $v_{(4-2)(\mathrm{j}-1)}$	0=$v_{(5-2)(\mathrm{j}-1)}$	…	$v_{(\mathrm{n}-2)(\mathrm{j}-1)}$
e $= hd([f(v_{\mathrm{ij}})]_2, [f(v_{i(j+1)})]_2$)	3	1	1	1	1		1

Case (ii): when $j \equiv 2(mod4)$

$f^*(v_{12}v_{13}) = hd([f(v_{12})]_2, [f(v_{13}))]_2) =$ hd($[6]_2, [2]_2$) = 1.

$f^*(v_{22}v_{23}) = hd([f(v_{22})]_2, [f(v_{23}))]_2) =$ hd($[3]_2, [4]_2$) = 3.

i	3	4	5	6	7	………	n
$f(v_{\mathrm{i}2})$	15	63	255	1023	4095	……….	$2^{2n-2}-1$
$f(v_{i3})$	1	7	31	127	511	……….	$2^{2n-5}-1$
e $= hd([f(v_{\mathrm{i}2})]_2, [f(v_{i3})]_2)$	3	3	3	3	3	3	3

For $1 \le i \le n, 6 \le j \le m$

$f^*(v_{ij}v_{i(j+1)}) = hd([f(v_{ij})]_2, [f(v_{i(j+1)}))]_2)$ = 3.

i	1	2	3	4	5	…	n
$f(v_{\mathrm{ij}})$	6	3	3=$v_{(3-1)(\mathrm{j}-4)}$	0= $v_{(4-2)(\mathrm{j}-2)}$	0= $v_{(5-2)(\mathrm{j}-2)}$	…	$v_{(\mathrm{n}-2)(\mathrm{j}-2)}$
$f(v_{i(j+1)})$	2	4	4=$v_{(3-1)(\mathrm{j}-3)}$	7= $v_{(4-2)(\mathrm{j}-1)}$	7= $v_{(5-2)(\mathrm{j}-1)}$	…	$v_{(\mathrm{n}-2)(\mathrm{j}-1)}$
e= $hd([f(v_{\mathrm{ij}})]_2, [f(v_{i(j+1)})]_2)$	1	3	3	3	3	3	3

Case (iii): when $j \equiv 3(mod4)$

For $1 \leq i \leq n$

i	1	2	3	4	5	…	N
$f(v_{i3})$	2	4	1	$7 = v_{(4-2)(j-2)}$	$31 = v_{(5-2)(j-2)}$	…	$v_{(n-2)(j-2)}$
$f(v_{i4})$	5	0	0	$3 = v_{(4-2)(j-1)}$	$15 = v_{(5-2)(j-1)}$	…	$v_{(n-2)(j-1)}$
e $= hd([f(v_{i1})]_2, [f(v_{i2})]_2)$	3	1	1	1	1	1	1

For $1 \leq i \leq n, 7 \leq j \leq m$

$f^*(v_{ij}v_{i(j+1)}) = hd([f(v_{ij})]_2, [f(v_{i(j+1)})]_2) = 1.$

i	1	2	3	4	5	…	n
$f(v_{ij})$	2	4	$4 = v_{(3-1)(j-4)}$	$7 = v_{(4-2)(j-2)}$	$7 = v_{(5-2)(j-2)}$	…	$v_{(n-2)(j-2)}$
$f(v_{i(j+1)})$	5	0	$0 = v_{(3-1)(j-3)}$	$3 = v_{(4-2)(j-1)}$	$3 = v_{(5-2)(j-1)}$	…	$v_{(n-2)(j-1)}$
e $= hd([f(v_{ij})]_2, [f(v_{i(j+1)})]_2)$	3	1	1	1	1	1	1

Case (iv): when $j \equiv 0 (mod 4)$

i	1	2	3	4	5	…	n
$f(v_{i4})$	5	0	0	$3 = v_{(4-2)(j-2)}$	$15 = v_{(5-2)(j-2)}$	…	$v_{(n-2)(j-2)}$
$f(v_{i5})$	3	7	$7 = v_{(3-1)(j-3)}$	$4 = v_{(4-2)(j-1)}$	$1 = v_{(5-2)(j-1)}$	…	$v_{(n-2)(j-1)}$
$e = hd([f(v_{i2})]_2, [f(v_{i3})]_2)$	3	1	1	1	1	1	1

For $1 \leq i \leq n, 8 \leq j \leq m$

$f^*(v_{ij}v_{i(j+1)}) = hd([f(v_{ij})]_2, [f(v_{i(j+1)})]_2) = 3.$

i	1	2	3	4	5	…	n
$f(v_{\text{ij}})$	5	0	0=$v_{(3-1)(\text{j}-4)}$	3=$v_{(4-2)(\text{j}-2)}$	3=$v_{(5-2)(\text{j}-2)}$	…	$v_{(\text{n}-2)(\text{j}-2)}$
$f(v_{i(j+1)})$	1	7	7=$v_{(3-1)(\text{j}-3)}$	4=$v_{(4-2)(\text{j}-1)}$	4=$v_{(5-2)(\text{j}-1)}$	…	$v_{(\text{n}-2)(\text{j}-1)}$
e= $hd([f(v_{\text{ij}})]_2, [f(v_{i(j+1)})]_2)$	3	1	1	1	1	1	1

From all the above cases, all the adjacent edges receive distinct odd labels. Hence the one point union of Path graphs P_m^n, admits Odd hamming distance labeling and the Odd hamming distance number $\eta'_{hd}(P_m^n)$ is $2n-1$.

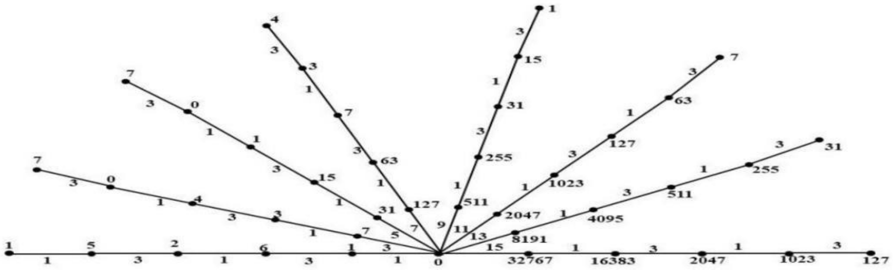

Figure 5 Odd Hamming Distance P_5^8 graph

3.3.7. Algorithm: Odd hamming distance labeling of CT(n,m) graph

Procedure: Vertex labeling of Coconut tree CT(n,m)

Input: Coconut tree graph CT(n,m).

$\text{V} \leftarrow \left\{\{\text{u}_\text{i} / 0 \le \text{i} \le \text{m}\} \cup \{\text{v}_\text{j} / 1 \le \text{j} \le \text{n}\}\right\}$

$\text{u}_0 \leftarrow 0;$

for i = 1 to m do

$$\text{u}_\text{i} \leftarrow \begin{cases} 1 & \text{if i} \equiv 1(\text{mod}4) \\ 6 & \text{if i} \equiv 2(\text{mod}4) \\ 2 & \text{if i} \equiv 3(\text{mod}4) \\ 5 & \text{if i} \equiv 0(\text{mod}4) \end{cases}$$

end fo

for j = 1 to n do

$$\text{v}_\text{i} \leftarrow \begin{cases} 6 & \text{if i} \equiv 1(\text{mod}4) \\ 2 & \text{if i} \equiv 2(\text{mod}4) \\ 5 & \text{if i} \equiv 3(\text{mod}4) \\ 1 & \text{if i} \equiv 0(\text{mod}4) \end{cases}$$

end for

for j = 2 to n do

$v_j \leftarrow 2^{2j+1} - u_m + 1)$

end for

end procedure

Output: The labeled vertices of Coconut tree graph CT(n,m).

3.3.8.Theorem The Coconut tree graph CT(n,m) is an Odd hamming distance graph and the Odd hamming distance number is $\eta'_{hd}(\mathrm{CT(n,m)}) = 2n + 1$.

Proof: Let us consider the coconut tree graph CT(n,m) with vertex set $V = \left\{\{u_i / 0 \le i \le m\} \cup \{v_j / 1 \le j \le n\}\right\}$ and edge set $E = \left\{\{u_i u_{i+1} / 0 \le i \le m-1\} \cup \{u_m v_j / 1 \le j \le n\}\right\}$.This graph has m+n+1 vertices and m+n edges. Define a function $f: V \to N \cup \{0\}$ such that $f(u) \ne f(v)$ for any two adjacent vertices u and v as given in the above algorithm 3.3.7.Hence all the adjacent vertices receive distinct labels.The edge labels are obtained as follows:

$f^*(u_0 u_1) = hd([f(u_0)]_2, [f(u_1)]_2) = hd([0]_2, [1]_2) = 1$.

For $1 \le i \le m$, where $u_{m+1} = v_1$

Case (i): if $i \equiv 1(mod\ 4)$; $f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = hd([1]_2, [6]_2) = 3$.

Case (ii): if $i \equiv 2(mod\ 4)$; $f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = hd([6]_2, [2]_2) = 1$.

Case (iii): if $i \equiv 3(mod\ 4)$; $f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = hd([2]_2, [5]_2) = 3$.

Case (iv): if $i \equiv 0(mod\ 4)$; $f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = hd([5]_2, [1]_2) = 1$.

For $2 \le j \le n$

Case (i): if $m \equiv 1(mod\ 4)$

j	2	3	4	5	6	………	n
$f(u_m)$	1	1	1	1	1	……….	1
$f(v_j)$	30	126	510	2046	8190	……….	$2^{2j+1} - 2$
e = $hd([f(u_m)]_2, [f(v_j)]_2)$	5	7	9	11	13	19	2n+1

Case (ii): if $m \equiv 2(mod\ 4)$

j	2	3	4	5	6	………	n

$f(u_{\mathrm{m}})$	6	6	6	6	6	……….	6
$f(v_j)$	25	121	505	2041	8185	……….	$2^{2j+1} - 7$
e $= hd([f(u_{\mathrm{m}})]_2, [f(v_j)]_2)$	5	7	9	11	13	19	2n+1

Case (iii): if $m \equiv 3(mod\ 4)$

j	2	3	4	5	6	………	n
$f(u_{\mathrm{m}})$	2	2	2	2	2	……….	2
$f(v_j)$	29	125	509	2045	8189	……….	$2^{2j+1} - 3$
e $= hd([f(u_{\mathrm{m}})]_2, [f(v_j)]_2)$	5	7	9	11	13	19	2n+1

Case (iv): if $m \equiv 0(mod\ 4)$

j	2	3	4	5	6	………	n
$f(u_{\mathrm{m}})$	5	5	5	5	5	……….	5
$f(v_j)$	26	122	506	2042	8186	……….	$2^{2j+1} - 6$
e $= hd([f(u_{\mathrm{m}})]_2, [f(v_j)]_2)$	5	7	9	11	13	19	2n+1

From all the above cases, all the adjacent edges receive distinct odd labels. Hence the coconut tree graph CT(n,m) admits Odd hamming distance labeling and the Odd hamming distance number $\eta'_{hd}(\mathrm{CT(n, m)})$ is $2n + 1$.

3. Conclusion

In this paper, the Odd hamming distance number of some Path related graphs were obtained.

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