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Stability of Malicious Object in SIEQAR Model

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Abstract:

The proposed model is SIEQAR (Susceptible-Infected-Exposed-Quarantine-Antidotal-Recovered) which is extension of SAIR model. In this model we discussed Basic Reproduction number for MFE (Malware Free Equilibrium) point. We discussed about Local stability at that point, also Endemic equilibrium point ids discussed.

Keywords: Quarantine, Antidotal, Malware Free Equilibrium ¸ Endemic Equilibrium, reproduction number..

1. Introduction: Computer virus is nothing but it is a code. Malware is computer program which destroy the important files from computer. Computer virus is similar like biological virus[1]. More than 1000 papers are discussed for many type of model [2-3]. Common models are Susceptible-Infectious- Susceptible (SIS) model[4-5], Susceptible- Infected- Recovered (SIR) model[6-7], Kaveri et.al.[8,9] discussed the different type of model.

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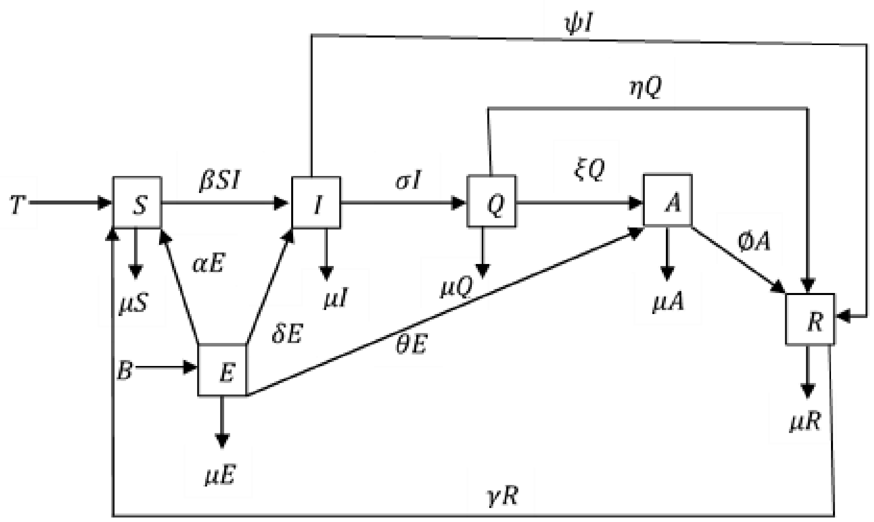
2. Formulation of model:



Schematic diagram for viruses in network

$$\left.\begin{aligned}\frac{dS}{dt} &= T - \beta SI + \alpha E + \gamma R - \mu S\\ \frac{dI}{dt} &= \beta SI + \delta E - (\mu + \psi + \sigma)I\\ \frac{dE}{dt} &= B - (\alpha + \mu + \delta + \theta)E\\ \frac{dQ}{dt} &= \sigma I - (\xi + \mu + \eta)Q\\ \frac{dA}{dt} &= \xi Q + \theta E - (\mu + \phi)A\\ \frac{dR}{dt} &= \eta Q + \psi I + \phi A - (\mu + \gamma)R\end{aligned}\right\} \quad \ldots \text{(i)}$$

All parameters are positive.

i.e., $S \geq 0, I \geq 0,\ E \geq 0,\ Q \geq 0,\ A \geq 0,\ R \geq 0.$

∴ total population $N = S + I + E + Q + A + R$.

Then, $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dE}{dt} + \frac{dQ}{dt} + \frac{dA}{dt} + \frac{dR}{dt}$.

$\Rightarrow \frac{dN}{dt} = T + B - \mu N$

Solving $\lim_{t\to\infty} \sup(S + I + E + Q + A + R) \leq \frac{T + B}{\mu}$

Hence the feasible region for system (1) is,

$$\Omega = \left\{(S, I, E, Q, A, R): S, I, E, Q, A, R \geq 0;\ S + I + E + Q + A + R \leq \frac{T + B}{\mu}\right\}$$

The Malware Free Equilibrium (MFE) of system (1) is denoted by E^o

$i.e.,\ \ E^o = (S, I, E, Q, A, R) = \left(\frac{T + B}{\mu}, 0, 0, 0, 0, 0\right)$

Description of parameters used in above ODE system.

S is Number of susceptible nodes.

I is Number of infected nodes.

E is Number of exposed nodes.

Q is Number of quarantined nodes

A is Number of antidotal nodes

R is Number of recovered nodes

β is Coefficient of transmission for susceptible individuals

σ is rate of quarantine for infectious individuals

ψ is rate of recovery for infectious individuals

ξ is rate of antidotal for quarantine individuals

ϕ is rate of recovery for antidotal individuals

η is rate of recovery for quarantine individuals

θ is rate of antidotal for exposed individuals

δ is rate of infective for exposed individuals

γ is rate of recovered individuals to susceptible

α is rate of exposed individuals to susceptible

μ is death rate due to other than the malicious objects

Basic Reproduction Number:

For convenient we take four classes for calculation of basic reproduction number, from ODE system (1).

$$\frac{dI}{dt} = \beta SI + \delta E - (\mu + \psi + \sigma)I$$

$$\frac{dE}{dt} = B - (\alpha + \mu + \delta + \theta)E$$

$$\frac{dQ}{dt} = \sigma I - (\xi + \mu + \eta)Q$$

$$\frac{dA}{dt} = \xi Q + \theta E - (\mu + \phi)A$$

By using next generation matrix $\frac{dx}{dt} = f - v$.

Where,

$$f = \begin{bmatrix} \beta SI \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} (\mu + \psi + \sigma)I - \delta E \\ (\alpha + \mu + \delta + \theta)E - B \\ (\xi + \mu + \eta)Q - \sigma I \\ (\mu + \phi)A - \xi Q - \theta E \end{bmatrix}$$

F is Jacobian of f at MFE is $\begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

V is Jacobian of v at MFE is $\begin{bmatrix} \mu + \psi + \sigma & -\delta & 0 & 0 \\ 0 & \alpha + \mu + \delta + \theta & 0 & 0 \\ -\sigma & 0 & \xi + \mu + \eta & 0 \\ 0 & -\theta & -\xi & \mu + \phi \end{bmatrix}$

Then,

$$V^{-1} = \begin{bmatrix} \frac{1}{\mu+\psi+\sigma} & \frac{\delta}{(\mu+\psi+\sigma)(\alpha+\mu+\delta+\theta)} & 0 \\ 0 & \frac{1}{(\alpha+\mu+\delta+\theta)} & 0 \\ \frac{\sigma}{(\mu+\psi+\sigma)(\xi+\mu+\eta)} & \frac{\sigma\delta}{(\mu+\psi+\sigma)(\alpha+\mu+\delta+\theta)(\xi+\mu+\eta)} & \frac{1}{(\xi+\mu+\eta)} \\ \frac{\sigma\xi}{(\mu+\psi+\sigma)(\xi+\mu+\eta)(\mu+\phi)} & \frac{(\mu+\psi+\sigma)\theta(\xi+\mu+\eta)+\delta\sigma\xi}{(\mu+\psi+\sigma)(\alpha+\mu+\delta+\theta)(\xi+\mu+\eta)(\mu+\phi)} & \frac{\xi}{(\xi+\mu+\eta)(\mu+\phi)} \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} \frac{\beta}{\mu+\psi+\sigma} & \frac{\beta\delta}{(\mu+\psi+\sigma)(\alpha+\mu+\delta+\theta)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The dominant eigen values $R_0 = \frac{\beta}{(\mu+\psi+\sigma)}$

3. Local Stability at MFE (Malware Free Equilibrium) Point:

For equilibrium points in the steady state of ODE system (i).

$$\left.\begin{aligned} &T - \beta SI + \alpha E + \gamma R - \mu S = 0 \\ &\beta SI + \delta E - (\mu+\psi+\sigma)I = 0 \\ &B - (\alpha+\mu+\delta+\theta)E = 0 \\ &\sigma I - (\xi+\mu+\eta)Q = 0 \\ &\xi Q + \theta E - (\mu+\phi)A = 0 \\ &\eta Q + \psi I + \phi A - (\mu+\gamma)R = 0 \end{aligned}\right\} \quad \text{... (ii)}$$

Theorem 1: The $E^*$of system (i) is locally asymptotically stable (LAS) if $R_0 < 1$.

Proof: The Jacobian of system (i)

$$J_1 = \begin{bmatrix} -\mu & 0 & 0 & 0 & 0 & \gamma \\ 0 & \beta S^* - (\mu+\psi+\sigma) & \delta & 0 & 0 & 0 \\ 0 & 0 & -(\alpha+\mu+\delta+\theta) & 0 & 0 & 0 \\ 0 & \sigma & 0 & -(\xi+\mu+\eta) & 0 & 0 \\ 0 & 0 & \theta & \xi & -(\mu+\phi) & 0 \\ 0 & \psi & 0 & \eta & \phi & -(\mu+\gamma) \end{bmatrix}$$

The characteristic equation of J_1 is $|J_1 - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -(\mu+\lambda) & 0 & 0 & 0 & 0 & \gamma \\ 0 & \beta S^* - (\mu+\psi+\sigma+\lambda) & \delta & 0 & 0 & 0 \\ 0 & 0 & -(\alpha+\mu+\delta+\theta+\lambda) & 0 & 0 & 0 \\ 0 & \sigma & 0 & -(\xi+\mu+\eta+\lambda) & 0 & 0 \\ 0 & 0 & \theta & \xi & -(\mu+\phi+\lambda) & 0 \\ 0 & \psi & 0 & \eta & \phi & -(\mu+\gamma+\lambda) \end{vmatrix} = 0$$

The eigen vlues of$|J_1 - \lambda I| = 0$ are

$\lambda_1 = -\mu < 0$

$\lambda_2 = \beta S^* - (\mu + \psi + \sigma)$

$\lambda_3 = -(\alpha + \mu + \delta + \theta) < 0$

$\lambda_4 = -(\xi + \mu + \eta) < 0$

$\lambda_5 = -(\mu + \phi) < 0$

$\lambda_6 = -(\mu + \gamma) < 0$

Clearly it has five negative real roots and the root $\lambda_2 = \beta S^* - (\mu + \psi + \sigma) < 0$

If $\beta S^* < (\mu + \psi + \sigma)$.

Clearly all roots are negative real roots so system (i) is Malware free equilibrium and is localy asymptotically stable.

4. Endemic Equilibrium point :

The endemic Equilibrium (EE) Point $P^*(S^*, I^*, E^*, Q^*, A^*, R^*)$, which can be calculated by using system (ii)

Thus,

$$S^* = \frac{\{T(\alpha + \mu + \delta + \theta) + \alpha\beta\}(\xi + \mu + \eta)(\mu + \phi)(\mu + \gamma) + \gamma[I^*(\alpha + \mu + \delta + \theta)[\{\eta\sigma + \psi(\xi + \mu + \eta)\}(\mu + \phi) + \phi\xi\sigma] + \theta B(\xi + \mu + \eta)]}{(\alpha + \mu + \delta + \theta)(\xi + \mu + \eta)(\mu + \phi)(\mu + \gamma)(BI^* + \mu)},$$

$$I^* = \frac{\delta E}{(\mu + \psi + \sigma - \beta S^*)}, \quad E^* = \frac{B}{\alpha + \mu + \delta + \theta}, \quad Q^* = \frac{\sigma I^*}{(\xi + \mu + \eta)},$$

$$A^* = \frac{\xi\sigma I^* + \theta B(\xi + \mu + \eta)}{(\alpha + \mu + \delta + \theta)(\xi + \mu + \eta)(\mu + \phi)},$$

$$R^* = \frac{I^*(\alpha + \mu + \delta + \theta)[\{\eta\sigma + \psi(\xi + \mu + \eta)\}(\mu + \phi) + \phi\xi\sigma] + \theta B(\xi + \mu + \eta)}{(\alpha + \mu + \delta + \theta)(\xi + \mu + \eta)(\mu + \phi)(\mu + \gamma)}$$

From above calculation, the EE state exist.

5. Conclusion: In above paper, the proposed model is SIEQAR (Susceptible-Infected-Exposed-Quarantine-Antidotal-Recovered) which is extension of SAIR model.This model is useful for getting Malware Free Equilibrium (MFE) point. We discussed about Local stability of MFE point and Endemic Equilibrium (EE) point. locally asymptotically stable (LAS) if $R_0 < 1$ MFE point.

In 2-dimenssion and 3- dimension several graphs of parameters are discussed. The above paper is useful for controlling the virus in all computer network.

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