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Rainbow Dynamic Coloring in Few Brick Product Graphs

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Abstract: Consider a connected graph that is nontrivial, defined as a coloring $c : V(G) \to \{1, 2, \ldots, k\},\ k \in N$ of the vertices of G. A rainbow dynamic coloring of a graph is a dynamic coloring, and a minimum number of colors required, such that every pair of vertices is connected by at least one path whose within vertices have different colors. The minimum k for which k-vertex coloring exists is called the rainbow dynamic coloring of G, denoted by rdyc(G). In this paper, we determine the rdyc of some graphs of brick products $C(2n, m, r)$ associated with odd cycles for $m = 1$.

Objectives: To find the rdyc of few brick product graphs.

Conclusions: In this paper, we obtain the rainbow dynamic coloring of fewbrick product graphs $C(2n, m, r)$ for $m = 1$ and $r = 3, 5, 7$.

Keywords: rainbow vertex connection number, dynamiccoloring, brick product, rainbow dynamic coloring.

1. Introduction

All graphs considered in this paper are simple, finite, and undirected. Let G be non-trivial with a vertex coloring $c : V(G) \to \{1, 2, \ldots, k\}, k \in N$. In a proper vertex-colored graph G, a path P is in a rainbow path if no two vertices P are of the same color, except possibly the end vertices of P. If a rainbow path connects every two vertices of G, then G is a rainbow connected to the vertex. A proper vertex coloring of a connected graph G that results in a connected graph with vertex rainbow is a rainbow vertex coloring of G. The minimum number of colors needed for the vertex rainbow coloring of G is the vertex rainbow connection number rvc(G). Bruce Montgomery introduced a relatively new concept in vertex coloring, called dynamic coloring, in 2001 [2]. A dynamic graph coloring is a proper coloring of the set of vertex such that each vertex of degree at least two of its neighbors receives at least two different colors. Krivelevich and Yuster introduced a rainbow vertex coloring concept in 2010 [3]. A rainbow vertex connection number, rvc(G) of a connected graph, is the minimum number of colors required to color its vertices. Every pair of vertices is connected by at least one path, which has different colors within the vertices. A rainbow dynamic coloring of a graph is not just a theoretical concept but a practical one. It is a dynamic coloring, and a minimum number of colors is required such that every pair of vertices is connected by at least one path whose within vertices have different colors. The minimum k for which k-vertex coloring exists is called the dynamic rainbow coloring of G, denoted by rdyc(G). We define a brick product graph associated with an odd cycle.

2. Results

Definition

Let m, n and r be positive integers. A cycle of order 2n is indicated by the notation $C_{2n} = v_1, v_2, \ldots. v_{2n-1}, v_{2n} = v_1$. The brick product (m, r) of C_{2n} represented by *C*(2*n*, *m*, *r*) is defined in the following way in two cases.

1. We need *r* to be odd and greater than 1 for *m* = 1. Next, fromC_{2n}, *C*(2*n*, *m*, *r*) can be determined by adding chords $v_{2k}(v_{2k+r})$, is obtained by adding chords$v_{2k}(v_{2k+r})$, *k* = 1, 2, ..., *n*, where the computation is performed modulo 2*n*.

2. We need *m* + *r* to be even for *m* > 1. Then *C*(2*n*, *m*, *r*) is obtained by first taking the disjoint union of m copies of C_{2n}, that is, $C_{2n}(1)$, $C_{2n}(2)$, ..., $C_{2n}(m)$ where for every *i* = 1, 2, ..., *m*, $C_{2n}(i) = v_{i1,}$ $v_{i1,}, v_{i1} \ldots\ldots v_{i2n,}$. Next, an edge (also known as a brick edge) is drawn to join $(v_{i,}, v_{k,})$ to $(v_{i+1,}, v_{k,})$ for each odd *i* = 1, 2, ..., *m* − 1 and each even *k* = 0, 1, 2, ..., 2*n* − 2,. Similarly, for each even *i* = 1, 2, ...*m* − 1 and each odd *k* = 1, 2, ..., 2*n* − 1, an edge (also known as a brick edge) is drawn to join $(v_{i+1,}, v_{k,})$. Lastly, an edge (referred to as a hooking edge) is created to join $(v_{1,}, v_{k,})$ to $(v_{m,}, v_{k+r,})$ for each odd *k* = 1, 2, ..., 2*n* − 1. A flat edge is an edge in *C*(2*n*, *m*, *r*) that is either a brick or a hooked edge.

Theorem 1. *Consider G* = (2*n*, *m*, *r*). *Then, for r* = 3 *and m* = 1, *and n* ≥ 3,

$$rdyc(G) = \begin{cases} 4 & for\ 3 \le n \le 7 \\ 5 & for\ 8 \le n \le 10 \\ \left\lfloor \frac{2n}{3} \right\rfloor - 1, 2n \cong 1(mod3) \\ \left\lfloor \frac{2n}{3} \right\rfloor - 2, 2n \cong 0(mod3) \\ \left\lfloor \frac{2n}{3} \right\rfloor - 1, 2n \cong 2(mod3 \end{cases}$$

The vertex set of G is defined as $V(G) = \{v_1, v_2, \ldots\ldots, v_{2n-1}, v_{2n} = v_1\}$ and the edge set of G is defined as $E(G) = \{e_i : 1 \le i \le 2n\} \cup \{e_i' : 1 \le i \le n\}$ where e_i represents the cycle edge (v_{i-1}, v_i) and e_i' represents the brick edge (v_{2k}, v_{2k+r}), $k = 0,1,2, \ldots.., n$. In this case, 2*k* + *r* is computed modulo 2*n*. Let's define coloring to G's vertices in the following manner.

Case 1. 3 ≤ *n* ≤ 7

Coloring is defined by $v_{i+4k} = i, 1 \le i \le 4, 0 \le k \le \left\lceil \frac{n}{2} \right\rceil - 1$

Case 2. 8 ≤ *n* ≤ 10

for n=8 Coloring is defined by $v_{i+5k} = i$, 1 ≤ *i* ≤ 5, 0 ≤ *k* ≤ 2, $v_{2n} = 2$

for *n* = 9, 10 Coloring is defined by $v_{i+5k} = i$, 1 ≤ *i* ≤ 5, 0 ≤ *k* ≤ 3

Case 3. 2*n* ≡ 1 (mod 3)

Coloring is defined by $v_i + \left\lceil \frac{n}{2} \right\rceil k = i\ for\ 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, 0 \le k \le 3.$

It is clear from this color pattern that *rdyc*(*G*) =rdyc(G) $= \left\lfloor \frac{2n}{3} \right\rfloor - 1.$

Case 4. 2*n* ≡ 0 (mod 3)

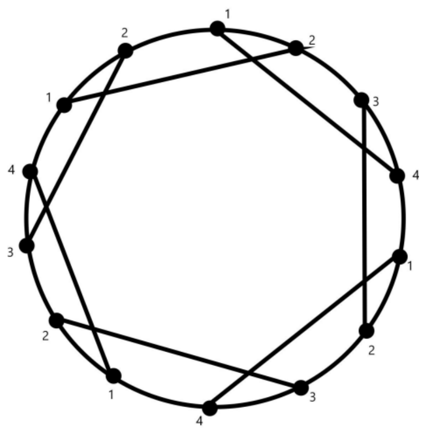
Coloring is defined by $v_{i+\left(\left\lceil \frac{2n}{3} \right\rceil - 2\right)k=i}$ for $1 \le \mathrm{i} \le \left\lceil \frac{2\mathrm{n}}{3} \right\rceil - 2, 0 \le \mathrm{k} \le 3.$ □

It is clear from this color pattern that $\mathrm{rdyc(G)} = \left\lfloor \frac{2\mathrm{n}}{3} \right\rfloor - 2$

Case 5. 2*n* ≡ 2 (mod 3)

Coloring is defined by $v_{i+\left(\left\lceil \frac{2n}{3} \right\rceil - 1\right)k=i}$ for $1 \le \mathrm{i} \le \left\lceil \frac{2\mathrm{n}}{3} \right\rceil - 1, 0 \le \mathrm{k} \le 3.$ □

It is clear from this color pattern that $\mathrm{rdyc(G)} = \left\lfloor \frac{2\mathrm{n}}{3} \right\rfloor - 1.$



□

□ □ **Figure 1: A graph illustrating the way colors are assigned to brick products in C(14,1,3)**

Theorem 2. *Consider G* = (2*n*, *m*, *r*). *Then, for r* = 7 *and m* = 1, *and n* ≥ 5,

$$rdyc(G) = \begin{cases} 4 \; for \; n = \; 5, 7 \le n \le 12 \\ \left\lfloor \frac{n}{2} \right\rfloor \quad for \;\; n = 6, 13 \\ \left\lfloor \frac{n}{2} \right\rfloor - 1 \;\; for \;\; n \ge 14 \end{cases}$$

Let us define the vertex set and edge set as in theorem1. Let's define coloring to G's vertices in the following manner.

Case 1.

for *n* = 5, 7, 9, 11

Coloring is defined by $v_{i+4k} = i$, 1 ≤ *i* ≤ 4, $0 \le \mathrm{k} \le \left\lfloor \frac{\mathrm{n}}{2} \right\rfloor - 1$; *vi*+4*k* = *i* + 1, 1 ≤ *i* ≤ 2, $\mathrm{k} = \left\lfloor \frac{\mathrm{n}}{2} \right\rfloor$

for *n* = 8, 10, 12

Coloring is defined by $v_{i+4k} = i$, 1 ≤ *i* ≤ 4, $0 \le \mathrm{k} \le \frac{\mathrm{n}}{2} - 1.$

Case 2. n=6,13

Coloring is defined by $v_{i+\left(\frac{n}{2}\right)k=i}$ for $1 \leq \mathrm{i} \leq \left\lfloor\frac{\mathrm{n}}{2}\right\rfloor, 0 \leq \mathrm{k} \leq \left\lfloor\frac{\mathrm{n}}{3}\right\rfloor + 1$. □

It is clear from this color pattern that $\mathrm{rdyc(G)} = \left\lfloor\frac{\mathrm{n}}{2}\right\rfloor$

Case 3. $\mathrm{n} \geq 14$

Coloring is defined by $v_{\left(\left\lfloor\frac{n}{2}\right\rfloor-1\right)k} = i$ for $1 \leq \mathrm{i} \leq \left\lfloor\frac{\mathrm{n}}{2}\right\rfloor - 1, 0 \leq \mathrm{k} \leq 4$. □

It is clear from this color pattern that $\mathrm{rdyc(G)} = \left\lfloor\frac{\mathrm{n}}{2}\right\rfloor - 1$.

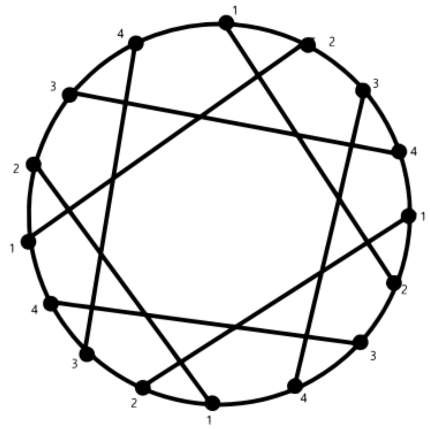


Figure 2: A graph illustrating the way colors are assigned to brick products in C(16,1,5)

Theorem 3. *Consider G* = (2*n*, *m*, *r*)*. Then, for r* = 7 *and m* = 1*, and n* ≥ 7*,*

$$rdyc(G) = \begin{cases} 3 & for\ \ n = 7 \\ 4 \text{ for n} = 7,8,10 \leq \text{n} \leq 14 & \\ 5 & \text{for n} = 15,17,16 \\ 6 & \text{for n} = 16,18 \\ \left\lfloor\frac{n}{3}\right\rfloor + 2, & 2n \cong 2(mod3) \\ \left\lfloor\frac{n}{3}\right\rfloor + 1, & 2n \cong 1(mod3) \\ \frac{n}{3} + 1, & 2n \cong 0(mod3 \end{cases}$$

Let us define the vertex set and edge set as in theorem1. Let's define coloring to G's vertices in the following

manner.

Case 1. *n* = 9

Coloring is defined by $v_{i+3k} = i, 1 \leq i \leq 3, 0 \leq k \leq 5$.

Case 2. *n* = 7, 8, 10 ≤ *n* ≤ 14

Coloring is defined by $v_{i+4k} = i, 1 \leq i \leq 4, 0 \leq k \leq \left\lfloor\frac{n}{2}\right\rfloor - 1$.

Case 3. n = 15, 17

Coloring is defined by $v_{i+4k} = i, 1 \leq i \leq 4, 0 \leq k \leq \left\lceil \frac{n}{2} \right\rceil - 1$.

Case 4. n = 16, 18

for n = 16 Coloring is defined by $v_{i+6k} = i$, $1 \leq i \leq 6$, $0 \leq k \leq 4$; $v_{i+6k} = i + 1$, $1 \leq i \leq 2$, $k = \left\lfloor \frac{n}{3} \right\rfloor$.

for n = 18, $v_{i+6k} = i$, $1 \leq i \leq 6$, $0 \leq k \leq 5$

Case 5. $2n \equiv 2 \pmod 3$

Coloring is defined by $v_{i+\left(\left\lfloor \frac{n}{3} \right\rfloor+2\right)k}$ =i, $1 \leq i \leq \left\lfloor \frac{n}{3} + 2 \right\rfloor$, $0 \leq k \leq 4$ for $2n \leq 50$, $0 \leq k \leq 5$ for $2n > 50$

It is clear from this color pattern that $rdyc(G) = \left\lfloor \frac{n}{3} \right\rfloor + 2$.

Case 6. $2n \equiv 1 \pmod 3$

Coloring is defined by $v_{i+\left(\left\lfloor \frac{n}{3} \right\rfloor+1\right)k}$ =i, $1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor + 1$, $0 \leq k \leq 5$

It is clear from this color pattern that $rdyc(G) = \left\lfloor \frac{n}{3} \right\rfloor + 1$

Case 7. $2n \equiv 0 \pmod 3$

Coloring is defined by $v_{i+\left(\frac{n}{3}+1\right)k} = i$, for $1 \leq i \leq \frac{n}{3} + 1, 0 \leq k \leq 5$.

It is clear from this color pattern that $rdyc(G) = \left\lfloor \frac{n}{3} \right\rfloor + 1$.

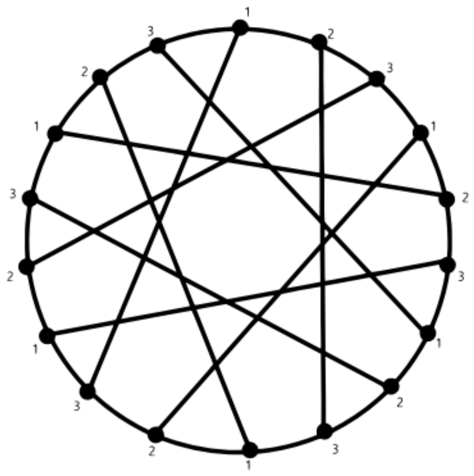


Figure 3: A graph illustrating the way colors are assigned to brick products in C(18,1,7)

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