

An Analytical Approach to Solve Nipah Virus Infection Model

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Abstract: An epidemic is the rapid spread of disease to a large number of hosts in a given population within a short period of time. The SIR model is that it assumes homogeneous mixture of populations, however this is an over simplification of the reality of epidemic process. The Nipah virus, known for its severe outbreaks and high mortality rates, necessitates effective modeling techniques for understanding its transmission and potential control measure. In this paper, initially we discuss the stability analysis and the q – Homotopy Analysis Transform Method (q – HATM) is compared with CFDTM graphically. The q – Homotopy Analysis Transform Method offers a series solution that converges rapidly, while the CFDTM provides robust numerical approximation. Comparative analysis highlights the strengths and limitations of each method in terms of accuracy, computational efficiency, and applicability to real world scenarios. The q – Homotopy Analysis Transform Method used to solve the system of non-linear differential equations which arises due to Nipah virus infection

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1. Introduction

Clinical manifestations of the Nipah virus infection in humans range from asymptomatic illness to severe respiratory infection and deadly encephalitis. An estimate of the case fatality rate ranges from 40 percent to 75 percent. Depending on regional capacity for epidemiological monitoring and clinical care, this rate may vary per epidemic. Humans can become infected with the Nipah virus through close contact with infected individuals or animals, particularly bats and pigs. The first major outbreak occurred in Malaysia in 1999, where the virus was linked to pigs. Later outbreaks in Bangladesh and India showed that it could also spread directly between people through bodily fluids and secretions.

In 2001, it was noted that the virus might spread within a hospital environment in Siliguri, India, where medical employees or patients accounted for 75 percent of recorded cases. About half of the reported cases in Bangladesh between 2001 and 2008 were caused by transmission from person to person while treating infected patients. Individuals who are infected initially exhibit symptoms such as headaches, fever, vomiting, sore throat and myalgia (muscle pain). These symptoms can then progress to dizziness, sleepiness, altered awareness, and neurological indications that suggest severe encephalitis. It is thought that the incubation time (how long it takes for symptoms to appear after an infection) lasts

between 4 and 14 days [6, 8, 14, 24]

The Nipah Virus, a novel type of virus in the Parayxoviridae family and a member of the genus Henipavirus, has gained attention as an emerging zoonotic virus in the south-east and south Asian area. Because of this, epidemiology defines health and illness in terms of the frequencies and distributions of factors and situations in a population or in a particular subset of a population. Even though there have only been a few Nipah virus outbreaks, it is a matter of public health since it affects a broad variety of animals and leads to severe illness and even death in humans. As the effectiveness of antiviral medications is unsatisfactory, the majority of treatments are symptomatic and supportive. In light of the extremely high case fatality rate, effective and stringent control and preventative measures are required. This study applies stability analysis and the q – HATM method to Nipah virus (NiV) infections and explores potential q – HATM preventative and control measures [1, 2, 21, 25].

Finding exact solutions to differential equations with fractional orders appears to be substantially more challenging than carrying out the same tasks for equations with corresponding integer orders. Thus, much emphasis has been paid to developing effective numerical and analytical methodologies for estimating answers to this kind of issue. The Homotopy perturbation method, Laplace decomposition method, Homotopy analysis method (HAM) [21], and Adomian decomposition method are a few methods used at present. The q – HATM procedure is yet another really potent technique. This q – HATM technique merges the LTM and HAM when $q \in [0, 1/m]$

Definition 1.1 [26]. The fractional R-L derivative of a function $f(t)$ is determined as

$$J^\varphi(f(t)) = \frac{1}{\Gamma(\varphi)} \int_0^t (t - \varrho)^{\varphi-1} (f(\varrho)) d\varrho \tag{1}$$

Definition 1.2 [26] The Caputo fractional order derivative of $f \in C$ is presented as follows :

$$D_t^\varphi(f(t)) = \begin{cases} \frac{d^m f(t)}{dt^m} & \text{if } \varphi = m \in N \\ \frac{1}{\Gamma(m - \varphi)} \int_0^t (t - \varrho)^{m-\varphi-1} (f^m(\varrho)) d\varrho & \text{if } m - 1 < \varphi < m, m \in N \end{cases} \tag{2}$$

Definition 1.3. [26] The LT of $f(t)$ with respect to fractional Caputo derivative is

$$\mathcal{L}[D_t^\varphi(f(t))] = s^\varphi F(s) - \sum_{r=0}^{m-1} s^{\varphi-r-1} f^{(r)}(0+), (m - 1 < \varphi \leq m), \tag{3}$$

where $F(s)$ is LT of $f(t)$.

Here we solve fractional differential equations to illustrate how the suggested method can be applied.

$$D_t^\varphi \varrho(\zeta, t) + R \varrho(\zeta, t) + N \varrho(\zeta, t) = g(\zeta, t), m - 1 < \varphi \leq m, \tag{4}$$

$$\varrho(\zeta, 0) = g(\zeta), \tag{5}$$

where $D_t^{\wp} \varrho(\zeta, \iota)$ symbolise the Caputo derivative of $\varrho(\zeta, \iota)$,

Utilizing the LT of equation (4), we obtain the following equation

$$\mathcal{L}[\varrho(\zeta, \iota)] - \frac{\varrho(\zeta, 0)}{s} + \frac{(1-\wp + \frac{\wp}{s^\wp})}{\mathcal{B}(\wp)} \{ \mathcal{L}[\mathcal{R}\varrho(\zeta, \iota)] + \mathcal{L}[\mathcal{N}\varrho(\zeta, \iota)] - \mathcal{L}[g(\zeta, \iota)] \} = 0 \tag{6}$$

Following are the steps involved in creating a homotopy for a primary function that is non zero:

$$\begin{aligned} \mathcal{N}[\vartheta(\zeta, \iota; q)] = & \mathcal{L}[\vartheta(\zeta, \iota; q)] - \frac{\varrho(\zeta, 0)}{s} + \frac{(1-\wp + \frac{\wp}{s^\wp})}{\mathcal{B}(\wp)} \{ \mathcal{L}[\mathcal{R}\vartheta(\zeta, \iota; q)] + \mathcal{L}[\mathcal{N}\vartheta(\zeta, \iota; q)] \} \\ & - \frac{(1-\wp + \frac{\wp}{s^\wp})}{\mathcal{B}(\wp)} \mathcal{L}[g(\zeta, \iota; q)] \end{aligned} \tag{7}$$

where $q \in [0, \frac{1}{m}]$ and a real function of ζ, ι and q is $\vartheta(\zeta, \iota; q)$.

For primary functions that are non zero, we establish the following homotopy:

$$(1 - \eta q)\mathcal{L}\{\vartheta(\zeta, \iota; q) - \varrho(\zeta, 0)\} = \eta q \mathcal{N}\{\vartheta(\zeta, \iota; q)\}, \tag{8}$$

where \mathcal{L} is denotes the LT, the constant parameter appears to be $q \in [0, \frac{1}{m}]$, ($m \geq 1$) and $\eta \neq 0$ is a supporting variable, $\vartheta(\zeta, \iota; q)$ is an undetermined function, $\varrho_0(\zeta, \iota)$ is an initial estimation of $\varrho(\zeta, \iota; q)$.

For $q = 0$ and $q = \frac{1}{m}$ respectively, the outcomes listed below are acceptable:

$$\vartheta(\zeta, \iota; 0) = \varrho_0(\zeta, \iota), \vartheta(\zeta, \iota; \frac{1}{m}) = \varrho(\zeta, \iota). \tag{9}$$

By expanding q from 0 to $\frac{1}{m}$, the result $\vartheta(\zeta, \iota; q)$ converges from $\varrho_0(\zeta, \iota)$ to $\varrho(\zeta, \iota)$. Taylor's theorem is used close to q to enlarge the component of the series structure $(\zeta, \iota; q)$:

$$\vartheta(\zeta, \iota; q) = \varrho_0(\zeta, \iota) + \sum_{r=1}^{\infty} \varrho_r(\zeta, \iota) q^r, \tag{10}$$

where

$$\varrho_r(\zeta, \iota) = \frac{1}{r!} \left. \frac{\partial^r \vartheta(\zeta, \iota; q)}{\partial q^r} \right|_{q=0}. \tag{11}$$

By using the basic linear operators $\varrho_0(\zeta, \iota)$, m and η , we could find one of the solutions for equation (4). At $q = \frac{1}{m}$, the series $\vartheta(\zeta, \iota; q)$ converges:

$$\varrho(\zeta, \iota) = \varrho_0(\zeta, \iota) + \sum_{r=1}^{\infty} \varrho_r(\zeta, \iota) \left(\frac{1}{m}\right)^r. \tag{12}$$

By dividing $r!$, computing for $q = 0$ after differentiating the zeroth order deformation equation r times with respect to q

$$\mathcal{L}[\varrho_r(\zeta, \iota) - k_r \varrho_{r-1}(\zeta, \iota)] = \eta \mathfrak{R}_r(\vec{\varrho}_{r-1}), \tag{13}$$

where

$$\varrho_r = [\varrho_0(\zeta, \iota), \varrho_1(\zeta, \iota), \dots, \varrho_r(\zeta, \iota)]. \tag{14}$$

Eq. (15), is reduced by using inverse LT,

$$\varrho_r(\zeta, \iota) = k_r \varrho_{r-1}(\zeta, \iota) + \hbar \mathcal{L}^{-1} \left[\mathfrak{R}_r \left(\underline{\varrho}_{r-1} \right) \right], \tag{15}$$

where the vectors are determined as

$$\begin{aligned} \mathfrak{R}_r(\underline{\varrho}_{r-1}) = & \mathcal{L}[\varrho_{r-1}(\zeta, \iota)] - \left(1 - \frac{k_r}{m} \right) \left(\frac{\varrho(\zeta, 0)}{s} + \frac{\left(1 - \wp + \frac{\wp}{s^\wp} \right)}{\mathcal{K}(\wp)} \mathcal{L}[g(\zeta, \iota)] \right) \\ & + \frac{\left(1 - \wp + \frac{\wp}{s^\wp} \right)}{\mathcal{K}(\wp)} \mathcal{L}[\mathcal{R}\varrho_{r-1}(\zeta, \iota) + \mathcal{H}_{r-1}], \end{aligned} \tag{16}$$

and

$$k_r = \begin{cases} 0 & \text{if } r \leq 1 \\ m & \text{if } r > 1 \end{cases}, \quad \mathcal{H}_r = \frac{1}{r!} \left[\frac{\partial^r \vartheta(\zeta, \iota; q)}{\partial q^r} \right]_{q=0}, \quad \text{and } \vartheta(\zeta, \iota; q) = \vartheta_0 + q\vartheta_1 + q^2\vartheta_2 + \dots,$$

we have,

$$\begin{aligned} \varrho_r(\zeta, \iota) = & (k_r + \hbar) \varrho_{r-1}(\zeta, \iota) - \left(1 - \frac{k_r}{m} \right) \mathcal{L}^{-1} \left(\sum_{k=0}^{n-1} s^{\rho-k-1} \varrho^{(k)}(\zeta, 0) + \frac{1}{s^\wp} \mathcal{L}[g(\zeta, \iota)] \right) \\ & + \hbar \mathcal{L}^{-1} \frac{1}{s^\rho} (\mathcal{L}[\mathcal{R}\varrho_{r-1}(\zeta, \iota)] + \mathcal{H}_{r-1}). \end{aligned} \tag{17}$$

Therefore, by resolving the above, we may derive the iterative term $\varrho_r(\zeta, t)$. The series solution of the q - HATM method is denoted by

$$\varrho(\zeta, \iota) = \varrho_0(\zeta, \iota) + \sum_{r=1}^{\infty} \varrho_r(\zeta, \iota) \left(\frac{1}{m} \right)^r. \tag{18}$$

2. MATHEMATICAL FORMULATION

The main tenet of this investigation is the notion of fractional calculus generalization. We believe that mathematical modelling is a very important component in understanding the illness and its transmission. To sustain the amounts of data produced on host pathogen communication, several types of models must be integrated. There are several models that may be used to study the transmission of a disease, however some techniques provide more precise and consistent results than others. To capture the ideal treatment for the disease, the system of nonlinear equations in this work is using the stability analysis and q - HATM technique. In the current work, we used the epidemiology model created by Sultana and N. Podder to observe the spread of a virus and to gather data for the epidemic model under consideration. The total population $N(t)$ is divided into three mutually disjoint compartments, namely susceptible $S(t)$, infected $I(t)$ and recovered $R(t)$ such that $N(t) = S(t) + I(t) + R(t)$. To explain the fluctuation of NiV infections in the community, we take into account the framework of non-linear differential equations given below, which is a form of typical SIR disease model [24].

$$\begin{aligned}
 \frac{dS}{dt} &= \theta N(t) - \beta S(t)J(t) - \tau S(t), \\
 \frac{dJ}{dt} &= \beta S(t)J(t) - (\psi + \tau + \delta)J(t), \\
 \frac{dR}{dt} &= \psi J(t) - \tau R(t), \\
 \frac{dN}{dt} &= \theta N(t) - \delta J(t) - \tau N(t),
 \end{aligned} \tag{19}$$

where the parameter β is the effective contact rate, θ is the natural birth rate and τ is the natural mortality rate, ψ is the recovery rate. δ is represent the disease induced death rate. The associated values of the parametric quantity are $\beta = 0.75$, $\delta = 0.01$, $\tau = 0.002$, $\psi = 0.005$, $\theta = 0.03$ and the initial values are $S(0) = 0.90$, $I(0) = 0.05$, $R(0) = 0.05$. Using the above assumptions, the model of the fractional-order dynamical system represented mathematically (19) as follows [24].

3. STABILITY ANALYSIS

In this section, we discuss the local stability of endemic equilibrium and disease-free equilibrium of system (1) by using Hurwitz criterion. Since the first three equations of system (1) are independent of $N(t)$, without loss of generality, we omit this one and then the system (1) is reduced to the following :

$$\begin{aligned}
 \frac{dS}{dt} &= \theta N(t) - \beta S(t)J(t) - \tau S(t), \\
 \frac{dJ}{dt} &= \beta S(t)J(t) - (\psi + \tau + \delta)J(t), \\
 \frac{dR}{dt} &= \psi J(t) - \tau R(t).
 \end{aligned} \tag{20}$$

3.1 Equilibrium Points.

For the disease-free equilibrium points point E_0

$$E_0 = \left(\frac{\theta}{\tau}, 0, 0\right).$$

The Jacobian of the system (2) is given by,

$$J = \begin{bmatrix} -\beta J - \tau & \beta S & 0 \\ \beta J & \beta S - \delta - \psi - \tau & 0 \\ 0 & \psi & -\tau \end{bmatrix}$$

while the Jacobian of free equilibrium point E_0 is

$$J(E_0) = \begin{bmatrix} -\tau & \frac{-\beta\theta}{\tau} & 0 \\ 0 & \frac{\beta\theta - \delta - \psi\tau - \tau^2}{\tau} & 0 \\ 0 & \psi & -\tau \end{bmatrix}.$$

For the reproduction number to be largest eigenvalues or spectral radius of characteristic equation $|F V^{-1} - \lambda I| = 0$, for the model in (3), the associated matrices F and V for the new infectious terms

and the remaining transition terms evaluated at the disease-free equilibrium are respectively given by

$$F = \begin{bmatrix} \frac{\beta\theta}{\tau} & 0 \\ 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \psi + \delta + \tau & 0 \\ -\psi & \tau \end{bmatrix}.$$

From which the next generation matrix can be calculated as

$$F V^{-1} = \begin{bmatrix} \frac{\beta\theta}{\tau(\psi + \delta + \tau)} & 0 \\ 0 & 0 \end{bmatrix},$$

where $R_0 = \frac{\beta\theta}{\tau(\psi + \delta + \tau)}$ is the required reproduction number.

Theorem 3.1. For $R_0 > 1$, there exist a positive disease equilibrium point E^* .

Proof. We set the left side of the system (20) is equal to zero. Then we have E^*

$$\theta - \beta S J - \tau S = 0 \tag{21}$$

$$\beta S J - (\psi + \delta + \tau) J = 0 \tag{22}$$

$$\psi J - \tau R = 0. \tag{23}$$

From (22) we have

$$\beta S J = (\psi + \delta + \tau) J$$

$$\beta S = (\psi + \delta + \tau)$$

$$S = \frac{(\psi + \delta + \tau)}{\beta}.$$

It follows from (21)

$$\begin{aligned} \theta - \beta S J - \tau S &= 0 \\ \theta &= (\beta J + \tau) S \end{aligned}$$

$$\beta I + \tau = \frac{\theta}{S}$$

$$\beta I = \frac{\theta}{S} - \tau$$

$$\beta I = \frac{\beta\theta}{\psi + \delta + \tau} - \tau$$

$$\beta I = \frac{\beta\theta - \tau\psi - \tau\delta - \tau^2}{\psi + \delta + \tau}$$

$$I = \frac{\beta\theta - \tau\psi - \tau\delta - \tau^2}{\beta(\psi + \delta + \tau)}$$

$$I = \frac{-\tau(\psi + \delta + \tau) + \frac{s\theta + \tau}{\beta(\psi + \delta + \tau)}}{I}$$

$$I = \frac{\tau(R_0 - \delta + \tau)\tau(R_0 - 1)}{\beta(\psi + \delta + \tau)}$$

From (23) we have

$$\begin{aligned}\psi J - \tau \mathcal{R} &= 0 \\ \psi J &= \tau \mathcal{R} \\ \tau \mathcal{R} &= \frac{\tau(\beta\theta - \tau\psi - \tau\delta - \tau^2)}{\beta(\psi + \delta + \tau)} \\ \mathcal{R} &= \frac{\tau(\beta\theta - \tau(\psi + \delta + \tau))}{\beta\tau(\psi + \delta + \tau)} \\ \mathcal{R} &= \frac{\psi\tau(\psi + \delta + \tau) - \left(\frac{\beta\theta}{\psi + \delta + \tau} - 1\right)}{\beta\tau(\psi + \delta + \tau)} \\ \mathcal{R} &= \frac{\psi}{\beta}(R_0 - 1)\end{aligned}$$

$$\mathcal{S} = \frac{\psi + \delta + \tau}{\beta}$$

$$I = \frac{\tau}{\beta}(R_0 - 1)$$

$$R = \frac{\Psi}{\beta}(R_0 - 1)$$

from which it can clearly be seen that, the equilibrium exists only if $R_0 > 1$.

4. q - HATM SOLUTION FOR THE PROJECTED MODEL

We will discuss the model presented in this section, by mean of q - HATM solution approach. The system of equations of fractional order (19) are now defined as follows

$$\begin{aligned}D_t^\rho \mathcal{S}(t) &= \theta \mathcal{N}(t) - \beta \mathcal{S}(t) \mathcal{J}(t) - \tau \mathcal{S}(t), \\ D_t^\rho \mathcal{J}(t) &= \beta \mathcal{S}(t) \mathcal{J}(t) - (\psi + \tau + \delta) \mathcal{J}(t), \\ D_t^\rho \mathcal{R}(t) &= \psi \mathcal{J}(t) - \tau \mathcal{R}(t), \\ D_t^\rho \mathcal{N}(t) &= \theta \mathcal{N}(t) - \delta \mathcal{J}(t) - \tau \mathcal{N}(t)\end{aligned}\tag{24}$$

with initial conditions

$$\mathcal{S}(0) = \mathcal{S}_0, \mathcal{J}(0) = \mathcal{J}_0, \mathcal{R}(0) = \mathcal{R}_0, \mathcal{N}(0) = \mathcal{N}_0.\tag{25}$$

Applying the LT to the system (19) we obtain equation.

$$\begin{aligned}\mathcal{L}\{\mathcal{S}(t)\} - \frac{1}{s}\mathcal{S}_0 - \frac{1}{B(\varphi)}\left(1 - \varphi + \frac{\varphi}{s^\varphi}\right)\mathcal{L}\{\theta \mathcal{N}(t) - \beta \mathcal{S}(t) \mathcal{J}(t) - \tau \mathcal{S}(t)\} &= 0, \\ \mathcal{L}\{\mathcal{J}(t)\} - \frac{1}{s}\mathcal{J}_0 - \frac{1}{B(\varphi)}\left(1 - \varphi + \frac{\varphi}{s^\varphi}\right)\mathcal{L}\{\beta \mathcal{S}(t) \mathcal{J}(t) - (\psi + \tau + \delta) \mathcal{J}(t)\} &= 0, \\ \mathcal{L}\{\mathcal{R}(t)\} - \frac{1}{s}\mathcal{R}_0 - \frac{1}{B(\varphi)}\left(1 - \varphi + \frac{\varphi}{s^\varphi}\right)\mathcal{L}\{\psi \mathcal{J}(t) - \tau \mathcal{R}(t)\} &= 0, \\ \mathcal{L}\{\mathcal{N}(t)\} - \frac{1}{s}\mathcal{N}_0 - \frac{1}{B(\varphi)}\left(1 - \varphi + \frac{\varphi}{s^\varphi}\right)\mathcal{L}\{\theta \mathcal{N}(t) - \delta \mathcal{J}(t) - \tau \mathcal{N}(t)\} &= 0.\end{aligned}\tag{26}$$

Next taking non-linear operator to (26) we get ,

$$\begin{aligned}
 \mathcal{N}^1[\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4] &= \mathcal{L}\{\vartheta_1(t; q)\} - \frac{1}{s} \mathcal{S}_0 - \frac{1}{\mathcal{B}(\varphi)} \left(1 - \varphi + \frac{\varphi}{s^\varphi}\right) \\
 &\quad \times \mathcal{L}\{\theta \vartheta_4(t; q) - \beta \vartheta_1(t; q) \vartheta_2(t; q) - \tau \vartheta_1(t; q)\}, \\
 \mathcal{N}^2[\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4] &= \mathcal{L}\{\vartheta_2(t; q)\} - \frac{1}{s} \mathcal{J}_0 - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\{\beta \vartheta_1(t; q) \vartheta_2(t; q) - (\psi + \tau + \delta) \vartheta_2(t; q)\}, \\
 \mathcal{N}^3[\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4] &= \mathcal{L}\{\vartheta_3(t; q)\} - \frac{1}{s} \mathcal{R}_0 - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\{\psi \vartheta_2(t; q) - \tau \vartheta_3(t; q)\} \\
 \mathcal{N}^4[\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4] &= \mathcal{L}\{\vartheta_4(t; q)\} - \frac{1}{s} \mathcal{N}_0 - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\{\theta \vartheta_4(t; q) - \delta \vartheta_2(t; q) - \tau \vartheta_4(t; q)\}. \tag{27}
 \end{aligned}$$

For $\mathcal{H}(x, t) = 1$ the r^{th} order deformation equation is presented as

$$\begin{aligned}
 \mathcal{L}[\mathcal{S}_r(t) - K_r \mathcal{S}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{1,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}], \\
 \mathcal{L}[\mathcal{J}_r(t) - K_r \mathcal{J}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{3,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}], \\
 \mathcal{L}[\mathcal{R}_r(t) - K_r \mathcal{R}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{1,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}], \\
 \mathcal{L}[\mathcal{N}_r(t) - K_r \mathcal{N}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{3,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}],
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 \mathcal{L}[\mathcal{R}_r(t) - K_r \mathcal{R}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{4,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] \\
 \mathcal{L}[\mathcal{N}_r(t) - K_r \mathcal{N}_{r-1}(t)] &= \mathfrak{h} \mathfrak{R}_{5,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}]
 \end{aligned}$$

where,

$$\begin{aligned}
 \mathfrak{R}_{1,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] &= \mathcal{L}\{\mathcal{S}_{r-1}(t)\} - \left(1 - \frac{K_r}{n}\right) \frac{\mathcal{S}_0}{s} - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\left\{\theta \mathcal{N}_{r-1}(t) - \beta \sum_{i=0}^{r-1} \mathcal{S}_i(t) \mathcal{J}_{r-1-i}(t) - \tau \mathcal{S}_{r-1}(t)\right\}, \\
 \mathfrak{R}_{2,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] &= \mathcal{L}\{\mathcal{J}_{r-1}(t)\} - \left(1 - \frac{K_r}{n}\right) \frac{\mathcal{J}_0}{s} - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\left\{\beta \sum_{i=0}^{r-1} \mathcal{S}_i(t) \mathcal{J}_{r-1-i}(t) - (\psi + \tau + \delta) \mathcal{J}_{r-1}(t)\right\}, \\
 \mathfrak{R}_{3,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] &= \mathcal{L}\{\mathcal{R}_{r-1}(t)\} - \left(1 - \frac{K_r}{n}\right) \frac{\mathcal{R}_0}{s} - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\{\psi \mathcal{J}_{r-1}(t) - \tau \mathcal{R}_{r-1}(t)\}, \\
 \mathfrak{R}_{4,r}[\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] &= \mathcal{L}\{\mathcal{N}_{r-1}(t)\} - \left(1 - \frac{K_r}{n}\right) \frac{\mathcal{N}_0}{s} - \frac{1}{\mathcal{B}(\wp)} \left(1 - \wp + \frac{\wp}{s^\wp}\right) \\
 &\quad \times \mathcal{L}\{\theta \mathcal{N}_{r-1}(t) - \delta \mathcal{J}_{r-1}(t) - \tau \mathcal{N}_{r-1}(t)\}.
 \end{aligned}$$

Taking inverse LT on both side of (28) we have

$$\begin{aligned}
 \mathcal{S}_r(l) &= K_r \mathcal{S}_{r-1}(l) + \mathfrak{h} \mathcal{L}^{-1} \{ \mathfrak{R}_{1,r} [\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] \}, \\
 \mathcal{J}_r(l) &= K_r \mathcal{J}_{r-1}(l) + \mathfrak{h} \mathcal{L}^{-1} \{ \mathfrak{R}_{3,r} [\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] \}, \\
 \mathcal{R}_r(l) &= K_r \mathcal{R}_{r-1}(l) + \mathfrak{h} \mathcal{L}^{-1} \{ \mathfrak{R}_{4,r} [\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] \}, \\
 \mathcal{N}_r(l) &= K_r \mathcal{N}_{r-1}(l) + \mathfrak{h} \mathcal{L}^{-1} \{ \mathfrak{R}_{5,r} [\vec{\mathcal{S}}_{r-1}, \vec{\mathcal{J}}_{r-1}, \vec{\mathcal{R}}_{r-1}, \vec{\mathcal{N}}_{r-1}] \}.
 \end{aligned} \tag{29}$$

On solving the system (29), we get

$$\begin{aligned}
 \mathcal{S}_0(l) &= 0.90 \\
 \mathcal{J}_0(l) &= 0.05, \\
 \mathcal{R}_0(l) &= 0.05, \\
 \mathcal{N}_0(l) &= 1.00, \\
 \mathcal{S}_1(l) &= \frac{0.005550 \mathfrak{h}}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\}, \\
 \mathcal{J}_1(l) &= \frac{-0.032900 \mathfrak{h}}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\}, \\
 \mathcal{R}_1(l) &= \frac{-0.00015 \mathfrak{h}}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\}, \\
 \mathcal{N}_1(l) &= \frac{-0.02750 \mathfrak{h}}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\}, \\
 \mathcal{S}_2(l) &= \frac{0.005550 \mathfrak{h}(\mathfrak{n} + \mathfrak{h})}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\} \\
 &\quad - \frac{0.0211632750 \mathfrak{h}^2}{[\mathcal{B}(\wp)]^2} \left(1 - 2\wp + \wp^2 + \frac{2\wp(1 - \wp)t^\wp}{\Gamma(\wp + 1)} + \frac{\wp^2 t^{2\wp}}{\Gamma(2\wp + 1)} \right), \\
 \mathcal{J}_2(l) &= \frac{-0.032900 \mathfrak{h}(\mathfrak{n} + \mathfrak{h})}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\} \\
 &\quad + \frac{0.0214400750 \mathfrak{h}^2}{[\mathcal{B}(\wp)]^2} \left(1 - 2\wp + \wp^2 + \frac{2\wp(1 - \wp)t^\wp}{\Gamma(\wp + 1)} + \frac{\wp^2 t^{2\wp}}{\Gamma(2\wp + 1)} \right), \\
 \mathcal{R}_2(l) &= \frac{-0.00015 \mathfrak{h}(\mathfrak{n} + \mathfrak{h})}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\} \\
 &\quad + \frac{0.000164200 \mathfrak{h}^2}{[\mathcal{B}(\wp)]^2} \left(1 - 2\wp + \wp^2 + \frac{2\wp(1 - \wp)t^\wp}{\Gamma(\wp + 1)} + \frac{\wp^2 t^{2\wp}}{\Gamma(2\wp + 1)} \right), \\
 \mathcal{N}_2(l) &= \frac{-0.02750 \mathfrak{h}(\mathfrak{n} + \mathfrak{h})}{\mathcal{B}(\wp)} \left\{ 1 - \wp + \frac{\wp t^\wp}{\Gamma(\wp + 1)} \right\} \\
 &\quad + \frac{0.00044100 \mathfrak{h}^2}{[\mathcal{B}(\wp)]^2} \left(1 - 2\wp + \wp^2 + \frac{2\wp(1 - \wp)t^\wp}{\Gamma(\wp + 1)} + \frac{\wp^2 t^{2\wp}}{\Gamma(2\wp + 1)} \right).
 \end{aligned} \tag{30}$$

and so on simplifying the above system of equations and the values given can be acquired. As described by the solutions of the q – HATM, we obtain the series as follows:

$$\begin{aligned} \mathcal{S}(t) &= \mathcal{S}_0(t) + \sum_{r=1}^{\infty} \mathcal{S}_m(t) \left(\frac{1}{m}\right)^r, \\ \mathcal{J}(t) &= \mathcal{J}_0(t) + \sum_{r=1}^{\infty} \mathcal{J}_m(t) \left(\frac{1}{m}\right)^r, \\ \mathcal{R}(t) &= \mathcal{R}_0(t) + \sum_{r=1}^{\infty} \mathcal{R}_m(t) \left(\frac{1}{m}\right)^r, \end{aligned} \tag{31}$$

5. RESULT AND DISCUSSION

This study establishes the local stability of both the endemic and disease-free equilibriums of system (19) using the Hurwitz criterion. we made use of q – HATM technique (q – HATM) to provide a suitable answer to a set of non-linear equations and solve an epidemic Nipah virus illness model. The initial requirements regarding the model that is offered in this paper are $S(0) = S_0 = 0.90$, $I(0) = I_0 = 0.05$, $R(0) = R_0 = 0.05$. A series of solutions have been examined in order to understand the behaviour of the model. We establish and identify a technique for $S(t)$, $I(t)$ and $R(t)$ using variations in fractional orders (\wp) with regard to time (t). It is evident from the diagram that the planned version matches the order and provides more comfort. As a consequence, success is predicted when the fractional operator is applied to the future version. The diagram shows how the sequence has a substantial impact on the anticipated model and provides additional flexibility. Here, we evaluated the tabular data and diagrams produced by q – HATM. Table 1, 2 and 3 show the answer to $S(t)$, $I(t)$ and $R(t)$ correspondingly for variations in fractional orders (\wp) with regard to time (t) acquired by q – HATM. From the charts, it is clear that the system’s time and order significantly influence the values of $S(t)$, $I(t)$ and $R(t)$. Additionally, we record embedding behavior using the value of $\wp = 0.6, 0.7, 0.8, 0.9$ and for classical order $\wp = 1$ and the order of the fractional differential coefficient at $h = -1$ and $m = 1$. As can be seen, the proposed system must imitate behaviors’ with regard to parameters and fractional order. The proposed fractional operator also gives other exciting outcomes to explore and project the future of the proposed model. The recent study may aid in understanding the lethal virus because epidemic models heavily rely on genetic characteristics.

TABLE 1. Table of the susceptible class of $S(t)$ for various \wp values

t	$\wp = 0.6$	$\wp = 0.7$	$\wp = 0.8$	$\wp = 0.9$	$\wp = 1$
0	0.8943938760	0.8964303052	0.8980434690	0.8992333672	0.9000000000
1	0.8723831960	0.8740240779	0.8765310374	0.8798538867	0.8838683625
2	0.8556267369	0.8515618372	0.8483633345	0.8465448548	0.8465734500
3	0.8393687934	0.8272321253	0.8141067684	0.800756083	0.7881152625
4	0.8232160227	0.8011902661	0.7744866197	0.7433693061	0.7084938000
5	0.8070412178	0.7735978359	0.7299972459	0.6749925801	0.6077090625
6	0.7907912215	0.7445877208	0.6810028025	0.5960768778	0.48576105500

TABLE 2. Table of the infective class of $I(t)$ for various \wp values

I	$\wp = 0.6$	$\wp = 0.7$	$\wp = 0.8$	$\wp = 0.9$	$\wp = 1$
0	0.06659041200	0.06179960675	0.05743760300	0.05350440075	0.05000000000
1	0.1072058919	0.1055129490	0.1026609786	0.09866297055	0.09362003750
2	0.1336278505	0.1413923592	0.1485607804	0.1545145311	0.1586801500
3	0.1577451162	0.1772442353	0.1988958201	0.2218808745	0.2451803375
4	0.1807804428	0.2137417469	0.2536356816	0.3002924200	0.3531206000
5	0.2031954688	0.2510705611	0.3125881524	0.3893329883	0.4825009375
6	0.2252195365	0.2892854813	0.3755563451	0.4886561530	0.6333213500

TABLE 3. Table of the recovered class of $R(t)$ for various \wp values

t	$\wp = 0.6$	$\wp = 0.7$	$\wp = 0.8$	$\wp = 0.9$	$\wp = 1$
0	0.05008627200	0.05005977800	0.05003656800	0.05001664200	0.05000000000
1	0.05046117025	0.05049310148	0.05052099088	0.05054365211	0.05056050000
2	0.05069644883	0.05082942615	0.05097484619	0.05112829150	0.051285200
3	0.05090780422	0.05115653905	0.05145245440	0.05179309994	0.05217410000
4	0.05110747387	0.05148349032	0.05195773351	0.05253751992	0.053227200
5	0.05130015308	0.05181334016	0.05249103017	0.05335975052	0.05444450000
6	0.05148820996	0.05214739658	0.05305185035	0.05425793308	0.055826000

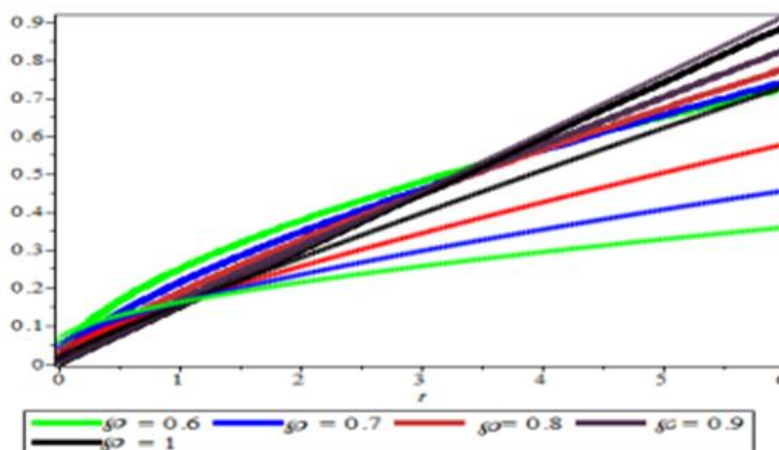


Figure 1: Plot of q – HATM and CFDTM solution for $S(t)$ with respect to t at $h = -1, ; m = 1,$ for varying \wp .

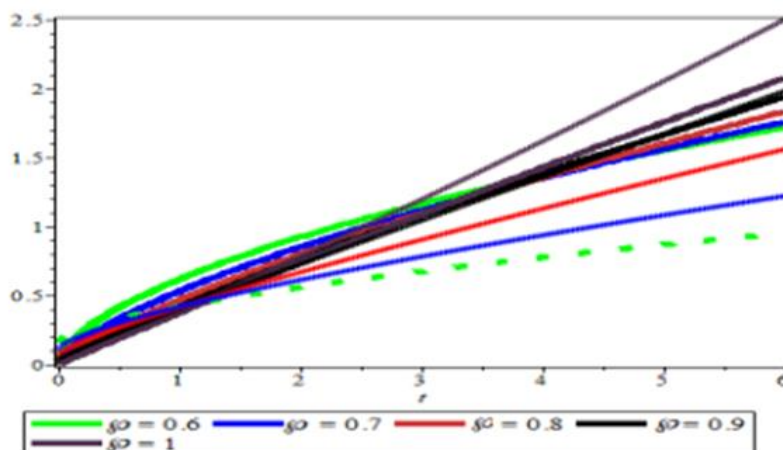


Figure 2: Plot of q – HATM and CFDTM solution for $I(t)$ with respect to t at $h = -1, ; m = 1,$ for varying ϕ .

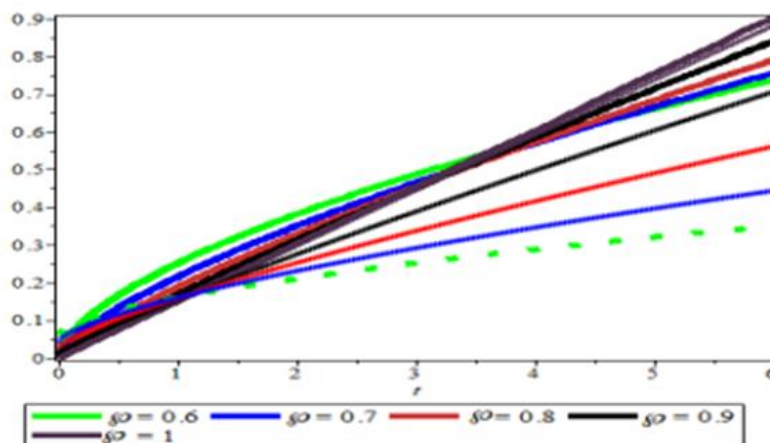


Figure 3: Plot of q – HATM and CFDTM solution for $R(t)$ with respect to t at $h = -1, ; m = 1,$ for varying ϕ .

6 CONCLUSION

The complex structure of fractional calculus is studied using Hurwitz criterion, the local stability of the disease-free equilibrium and endemic equilibrium of system is proved and utilizing the q – Homotopy Analysis Transform Method in the current inquiry of Nipah viral sickness. The results are provided in a series form that converges quickly. We have stabilized a more accurate and realistic model by taking into account the Caputo fractional derivatives. The present technique also saves time and does not call for any deviation while solving nonlinear models. It is concluded that findings for real, imagined, and other phenomenal variables may be derive using the proposed nonlinear model.

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