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Stability Analysis by Routh Hurwitz on Dengue Epidemic Model

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Abstract:

Dengue is a serious disease, according to the WHO. Humans contract this illness when female mosquitoes, particularly the Aedes aegypti/yellow fever mosquito, bite them. Humans can contract this disease when bitten by an infected mosquito carrying any of the four virus strains designated by DENV-1 to DENV-4. As an extension of the SIR model, we suggested the SIHRV (Susceptible Infectious Hospital Recovery Vaccine) model for this purpose. The threshold number $R_0$is determined by applying the Routh-Hurwity and Jacobian criteria. The model is unstable if $R_0 > 1$ and stable if $R_0 < 1$. In order to improve recovery, we also determine the endemic and disease-free equilibrium points.

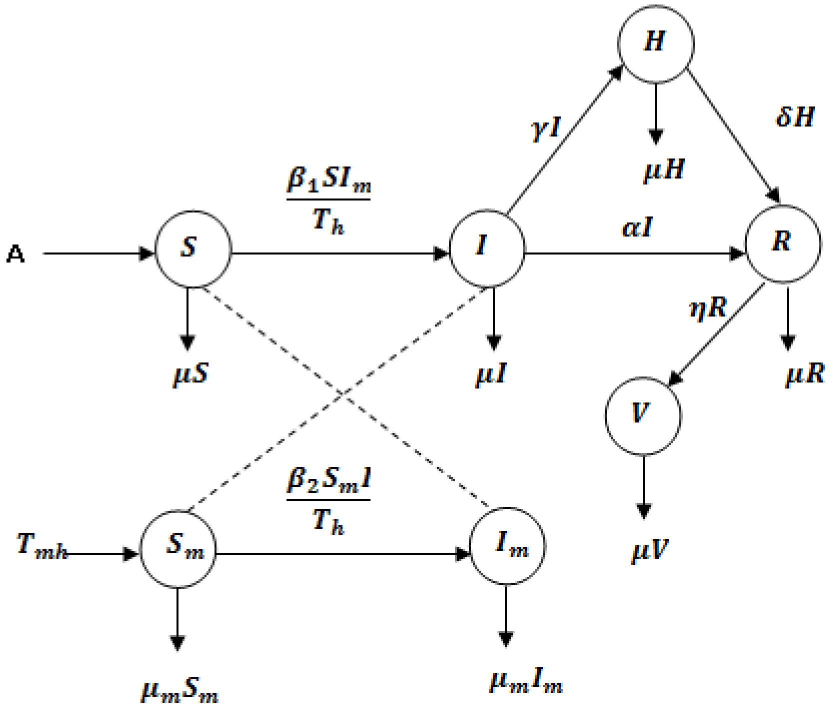
Keywords: Mathematical model, vaccination, Endemic equilibrium, Routh-Hurwitz criteria.

AMS Classification 2010 : 93A30, 92B05, 92D30

Introduction Dengue fever is a major problem in many industrialized and developing countries' tropical and subtropical regions. Every year, millions of cases from all around the world are reported to the World Health Organization (WHO). The female Yellow Fever (Aedes aegypti) The main vector of this disease is the mosquito. Any one of the four virus strains—designated DENV-1 through DENV4—carried by mosquito bites can infect humans. Humans only get a temporary cross-immunity to the other strains of dengue after being bitten by one, but they are permanently immune to that strain.

Usually appearing 4–10 days after the virus enters the bitten person's bloodstream, dengue symptoms can last for up to a week or two. Dengue fever can now be prevented by receiving the New Serotype Dengue Fever Vaccine. Preventive measures are the alternative, essentially costly or impractical, way to stay unaffected. This work uses generalized fractional order derivatives to generate the suggested Dengue Epidemic Model. It is demonstrated that fractional order equations are more suitable than integer order equations for simulating biological, economic, and social systems when memory effects are important. Humans are therefore frequently chosen by mosquitoes based on their prior encounters with them, especially on their location and degree of defense. Thus, a perfect model to enable a nation to be disease free can be built based on the examination of the transmission history and fractional order calculus. Numerous authors [1,2] have written about the various kinds of models that are employed in computation. **Sulami** [3] , **Hamdam** [4], **Defterli** [5] Discuss the SIR model by fractional order within the Caputo fractional operator.

SIHRV Model for transmission of viruses



Schematic figure for SIHRV Model

$$\left.\begin{aligned} \dot{S} &= A - \frac{\beta_1 S I_m}{T_h} - \mu S \\ \dot{I} &= \frac{\beta_1 S I_m}{T_h} - \gamma I - \alpha I - \mu I \\ \dot{R} &= \alpha I + \delta H - \eta R - \mu R \\ \dot{H} &= \gamma I - \delta H - \mu H \\ \dot{V} &= \eta R - \mu V \\ \dot{S_m} &= T_m - \frac{\beta_2 S_m I}{T_h} - \mu_m S_m \\ \dot{I}_m &= \frac{\beta_2 S_m I}{T_h} - \mu_m I_m \end{aligned}\right\} \quad \dots(1)$$

Condition : $S + I + R + H + V = T_h$

Description of parameters used in above ODE:

A is influx rate.

T_h is total population of human.

T_m is total population of mosquito.

S is total number of susceptible human population.

I is total number of infective human population.

H is total number of hospitalized human population.

R is total number of recovered human population.

V is total number of vaccinated human population.

μ death rate in human population.

S_m is total number of susceptible mosquito population.

I_m is total number of infective mosquito population

μ_m death rate in mosquito population.

β_1 Transfer coefficient from human to mosquito.

β_2 Transfer coefficient from mosquito to human.

α is recovery rate by infective human population.

γ is the hospitalized rate by infective human population.

δ is recovery rate by hospitalized human population.

η is vaccinated rate by recovered human population.

Some definitions of fractional derivatives [6,7] are implimented. The Riemann –Liouville derivatives of ith order is defined as

$${}_aI_x^i\, f(x) = \frac{1}{\Gamma i}\int_a^x (x-t)^{i-1} f(t)dt$$

$${}_aD_x^i f(x) = \frac{d^n}{dx^n}\ {}_aI_x^{n-i}\, f(x)$$

$$= \frac{d^n}{dx^n}\frac{1}{\Gamma(n-i)}\int_a^x (x-t)^{n-i-1} f(t)dt \quad where, i\ \epsilon\ \mathbb{R}^+\ \ and\ \ n-1 < i < n$$

An alternative definition by Caputo is

$${}_aD_x^i\, f(x) = \ {}_aI_x^{n-i}\left\{\frac{d^n}{dx^n} f(x)\right\}$$

In this paper we will use Caputo fractional derivative for the system (1)

$$\left.\begin{aligned} D_x^i S &= A - \frac{\beta_1 S I_m}{T_h} - \mu S \\ D_x^i I &= \frac{\beta_1 S_m I}{T_h} - \gamma I - \alpha I - \mu I \\ D_x^i R &= \alpha I + \delta H - \eta R - \mu R \\ D_x^i H &= \gamma I - \delta H - \mu H \\ D_x^i V &= \eta R - \mu V \\ D_x^i S_m &= T_m - \frac{\beta_2 S_m I}{T_h} - \mu_m S_m \\ D_x^i I_m &= \frac{\beta_2 S_m I}{T_h} - \mu_m I_m \end{aligned}\right\} \quad \dots (2)$$

All parameters are assumed to be non-negative from system (1)

$$D_x^i T_h = A - \mu T_h \quad \dots \quad (3)$$

Let $\Omega = \{(S, I, R, H, V, S_m, I_m): S, I, R, H, V, S_m, I_M \leq 0,\ S + I + R + H + V \leq A/\mu\}$

3. Equilibrium Points :

$$D_x^i S = 0,\ \ D_x^i I = 0,\ \ D_x^i R = 0,\ \ D_x^i H = 0,\ \ D_x^i V = 0,\ \ D_x^i S_m = 0,\ \ D_x^i I_m = 0 \quad \dots (4)$$

Then $E_0 = (A/\mu, 0,0,0,0, T_h/\mu_m, 0)$

For positive equilibrium point $E^* = (S^*, I^*, R^*, H^*, V^*, {S_m}^*, {I_m}^*)$

$$S^* = \frac{(\gamma + \alpha + \mu)T_h}{\beta_1}, \qquad I^* = \frac{A\beta_1 - \mu(\gamma + \alpha + \mu)T_h}{\beta_1(\gamma + \alpha + \mu)},$$

$$R^* = \frac{\{\alpha(\delta + \mu) + \delta\gamma\}\{A\beta_1 - \mu(\gamma + \alpha + \mu)T_h\}}{(\delta + \mu)(\eta + \mu)(\gamma + \alpha + \mu)}, \qquad H^* = \frac{\gamma\{A\beta_1 - \mu(\gamma + \alpha + \mu)T_h\}}{(\delta + \mu)(\gamma + \alpha + \mu)},$$

$$V^* = \frac{\eta\{\alpha(\delta + \mu) + \delta\gamma\}\{A\beta_1 - \mu(\gamma + \alpha + \mu)T_h\}}{\mu(\delta + \mu)(\eta + \mu)(\gamma + \alpha + \mu)}, \qquad {S_m}^* = \frac{\mu_m T_h}{\beta_2},$$

$${I_m}^* = \frac{T_m \beta_2 - {\mu_m}^2 T_h}{\beta_2 \mu_m}$$

The Jacobian matrix J_{E_0} for system (1) for disease free equilibrium point is as follows

$$J_{E_0} = \begin{bmatrix} -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\gamma + \alpha + \mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & -(\eta + \mu) & \delta & 0 & 0 & 0 \\ 0 & \gamma & 0 & -(\delta + \mu) & \gamma & 0 & 0 \\ 0 & 0 & \eta & 0 & -\mu & 0 & 0 \\ 0 & -\beta_2 S_m/T_h & 0 & 0 & 0 & -\mu_m & 0 \\ 0 & \beta_2 S_m/T_h & 0 & 0 & 0 & 0 & -\mu_m \end{bmatrix}$$

The characteristic equation is $|J_{E_0} - \lambda I| = 0$

$$\begin{vmatrix} -(\mu+\lambda) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\gamma+\alpha+\mu+\lambda) & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & -(\eta+\mu+\lambda) & \delta & 0 & 0 & 0 \\ 0 & \gamma & 0 & -(\delta+\mu+\lambda) & \gamma & 0 & 0 \\ 0 & 0 & \eta & 0 & -(\mu+\lambda) & 0 & 0 \\ 0 & -\beta_2 S_m/T_h & 0 & 0 & 0 & -(\mu_m+\lambda) & 0 \\ 0 & \beta_2 S_m/T_h & 0 & 0 & 0 & 0 & -(\mu_m+\lambda) \end{vmatrix} = 0$$

$$\Rightarrow (\mu+\lambda)(\gamma+\alpha+\mu+\lambda)(\mu_m+\lambda)(\mu_m+\lambda)\{(\eta+\mu+\lambda)(\delta+\mu+\lambda)(\mu+\lambda)+\delta\eta\gamma\} = 0$$

The four Characteristic roots are $\lambda_1 = -\mu,\ \lambda_2 = -(\gamma+\alpha+\mu), \lambda_3 = -\mu_m,\ \lambda_4 = -\mu_m$ which are negative, and the cubic equation is

$$\lambda^3 + (\delta+3\mu+\eta)\lambda^2 + (3\mu^2+2\delta\mu+2\eta\mu+\eta\delta)\lambda + (\eta\delta\mu+\eta\mu^2+\delta\mu^2+\mu^3+\delta\eta\gamma) = 0$$

Compare this cubic equation with $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

$$\Rightarrow a_1 = (\delta+3\mu+\eta) > 0,$$

$$a_2 = (3\mu^2+2\delta\mu+2\eta\mu+\eta\delta) > 0,$$

$$a_3 = (\eta\delta\mu+\eta\mu^2+\delta\mu^2+\mu^3+\delta\eta\gamma) > 0$$

Here $a_1.a_2 - a_3 > 0$.

According to Routh array E_0 is stable.

Endemic equilibrium :

The Jacobian matrix J_{E^*} for system (1) for endemic equilibrium point is as follows

$$J_{E^*} = \begin{bmatrix} -\left(\mu+\frac{\beta_1 I_m{}^*}{T_h}\right) & 0 & 0 & 0 & 0 & 0 & -\frac{\beta_1 S^*}{T_h} \\ \frac{\beta_1 I_m{}^*}{T_h} & -(\gamma+\alpha+\mu) & 0 & 0 & 0 & 0 & \frac{\beta_1 S^*}{T_h} \\ 0 & \alpha & -(\eta+\mu) & \delta & 0 & 0 & 0 \\ 0 & \gamma & 0 & -(\delta+\mu) & 0 & 0 & 0 \\ 0 & 0 & \eta & 0 & -\mu & 0 & 0 \\ 0 & -\beta_2 S_m{}^*/T_h & 0 & 0 & 0 & -\beta_2 I^*/T_h - \mu_m & 0 \\ 0 & \beta_2 S_m{}^*/T_h & 0 & 0 & 0 & \beta_2 I^*/T_h & -\mu_m \end{bmatrix}$$

The characteristic equation is $|J_{E^*} - \lambda I| = 0$

$$\begin{vmatrix} -\left(\mu+\frac{\beta_1 I_m{}^*}{T_h}+\lambda\right) & 0 & 0 & 0 & 0 & 0 & -\frac{\beta_1 S^*}{T_h} \\ \frac{\beta_1 I_m{}^*}{T_h} & -(\gamma+\alpha+\mu+\lambda) & 0 & 0 & 0 & 0 & \frac{\beta_1 S^*}{T_h} \\ 0 & \alpha & -(\eta+\mu+\lambda) & \delta & 0 & 0 & 0 \\ 0 & \gamma & 0 & -(\delta+\mu+\lambda) & 0 & 0 & 0 \\ 0 & 0 & \eta & 0 & -(\mu+\lambda) & 0 & 0 \\ 0 & -\beta_2 S_m{}^*/T_h & 0 & 0 & 0 & -(\beta_2 I^*/T_h+\mu_m+\lambda) & 0 \\ 0 & \beta_2 S_m{}^*/T_h & 0 & 0 & 0 & \beta_2 I^*/T_h & -(\mu_m+\lambda) \end{vmatrix} = 0$$

$$\Rightarrow (\mu_m + \lambda)(\mu + \lambda)(\eta + \mu + \lambda)(\delta + \mu + \lambda)\left\{\left(\mu + \frac{\beta_1 I_m^*}{T_h} + \lambda\right)\left(\frac{\beta_2 I^*}{T_h} + \mu_m + \lambda\right)(\gamma + \alpha + \mu + \lambda) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}(\mu + \lambda)\right\} = 0$$

$$\Rightarrow (\mu_m + \lambda)(\mu + \lambda)(\eta + \mu + \lambda)(\delta + \mu + \lambda)\left[\lambda^3 + \left(2\mu + \frac{\beta_1 I_m^*}{T_h} + \frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha\right)\lambda^2 + \left\{\left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu) + \left(\frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha + \mu\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\right\}\lambda + \left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu)\left(\mu + \frac{\beta_1 I_m^*}{T_h}\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\mu\right] = 0$$

The four Characteristic roots are $\lambda_1 = -\mu_m,\ \lambda_2 = -\mu, \lambda_3 = -(\eta + \mu),\ \lambda_4 = -(\delta + \mu)$

which are negative, and the cubic equation is

$$\lambda^3 + \left(2\mu + \frac{\beta_1 I_m^*}{T_h} + \frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha\right)\lambda^2 + \left\{\left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu) + \left(\frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha + \mu\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\right\}\lambda + \left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu)\left(\mu + \frac{\beta_1 I_m^*}{T_h}\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\mu = 0$$

Compare this cubic equation with $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$

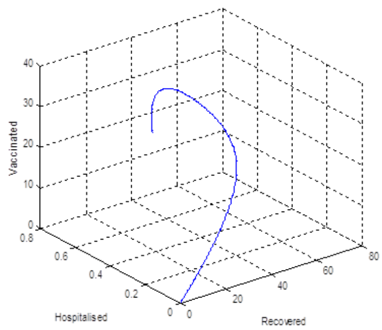
$$\Rightarrow a_1 = \left(2\mu + \frac{\beta_1 I_m^*}{T_h} + \frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha\right) > 0,$$

$$a_2 = \left\{\left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu) + \left(\frac{\beta_2 I^*}{T_h} + \mu_m + \gamma + \alpha + \mu\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\right\} > 0,$$

$$a_3 = \left(\frac{\beta_2 I^*}{T_h} + \mu_m\right)(\gamma + \alpha + \mu)\left(\mu + \frac{\beta_1 I_m^*}{T_h}\right) + \frac{\beta_2 S_m^*}{T_h}\frac{\beta_1 S^*}{T_h}\mu > 0$$

Here $a_1 . a_2 - a_3 > 0$.

According to Routh array E^* is stable.



Proposition :

(i) Routh Hurwitz criterion are satisfied for E_0 point , $a_1 > 0, a_2 > 0, a_3 > 0$ *and* $a_1.a_2 - a_3 > 0,$ *then* E_0 is locally asymptotically stable.

(ii) Routh Hurwitz criterion are satisfied for E^* point , $a_1 > 0, a_2 > 0, a_3 > 0$ *and* $a_1.a_2 - a_3 > 0,$ *then* E^* is locally asymptotically stable.

Conclusion : By using Routh-Hurwitz Criteria we find that the above model is stable in endemic equilibrium point. By simulation we observe stability of the model exist in endemic equilibrium point. In extension of this work we can also use harmonic mean type incidence rate for control the disease.

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