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Gaussian and Gamma Mixture Model Approach to Rainfall Analysis in Flood-Prone Regions of Karnataka

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Abstract:

Flood is one of the many natural disasters that affects people's social and economic well-being in India and around the world. The current flood analysis study focusses on Coastal Karnataka regions namely Dakshina Kannada, Udupi, Uttara Kannada, Chikkamagalur, Kodagu, and Shivamogga also known as flood prone regions of Karnataka due to heavy rainfall recorded every year. After being rainfall data collected for 57 years, the southwest monsoon rainfall data for these areas is examined and put through a Gaussian-Gamma mixture (GGM) model. This is because rainfall data has multimodal characteristics and does not follow a Gaussian distribution. Prior presenting the data to the GGM model, the data was normalized in order to do the comprehensive statistical analysis. The Maximum Likelihood estimator method is used to determine the parameters involved in the GGM model. The data is seen to follow the GGM distribution, and the model able to capture the data's moments. With this in view, level crossing statistics are calculated and computed for the data length used at 10% and 20% above normal values using GGM model and Gaussian model. The comparison shows that GGM model is able to capture the flood events of the regions considered for the current study better than the Gaussian model both at 10% and 20% above normal values. For Karnataka's flood-prone areas, this research provides vital insights into flood risk management, supporting proactive disaster mitigation, efficient resource planning, and climate adaption measures.

Keywords: Flood-prone regions, Heavy rainfall, Gaussian distribution, Gamma distribution, Mixture model, level crossing.

Introduction:

One of Karnataka's environmental problems is flooding, which has an impact on ecosystems, infrastructure, agriculture, and human life. The state's diverse terrain, which includes plateau, hilly, and coastal regions, raises the possibility of floods of varying severity, especially in areas that are prone to flooding like Dakshina Kannada, Udupi, Uttara Kannada, Chikkamagalur, Kodagu, and Shivamogga. Due to heavy monsoonal rainfall, river overflow, and shifting climatic patterns, these areas frequently flood, underscoring the necessity of effective flood risk assessment and management plans. To comprehend flood dynamics and lessen their effects, precise rainfall pattern modelling and forecast are essential. To capture the intricate features of rainfall data, including variability, skewness, and extreme events, sophisticated statistical techniques are required.

The Gaussian models often fail to represent the full range of rainfall behaviours, particularly under extreme conditions, necessitating a more comprehensive approach like the Gaussian and Gamma Mixture Model (GGM). When dealing with non-Gaussian time series data, defining it in terms of key moments and the power spectral density function is often preferred. Gaussian processes are widely used for this, and their transformations can serve as effective non-Gaussian modeling approaches. Navneet Kumar, Bernhard Tischbein, and Mirza Kaleem Beg explored rainfall and temperature trends in India's Upper Kharun Catchment using various trend detection methods, including an innovative Gaussian-linear trend detection test. Analyzing rainfall data from 1961 to 2011 revealed no significant trends except for an increase in peak monthly rainfall, while temperature data displayed slight monthly increases. Their work provides insights into local climate dynamics and resource management, aiding climate adaptation strategies. Kandula V. Subrahmanyam et al. examined heavy rainfall predictions over a peninsular Indian station using machine learning, emphasizing the limitations of traditional Numerical Weather Prediction models. They proposed Gaussian Process Regression (GPR) for predicting heavy and light rainfall days, utilizing 116 years of rainfall data from Sriharikota, India. The GPR model outperformed other methods, achieving low error rates (RMSE = 0.161; MAE = 0.126) and showing potential for spatial rainfall predictions across India. Supported by ISRO and IMD data, this study demonstrates GPR's value in heavy rainfall prediction. Md. Ashraful Alam, Craig Farnham, and Kazuo Emura studied maximum monthly rainfall in Bangladesh using Gaussian distributions. They found single Gaussian distributions to be the best fit for 51% of weather stations, while multiple-component Gaussian distributions (N2, N3, N5) were optimal for others. Return periods for extreme rainfall were calculated for policymaking, aiding disaster management and risk assessment for floods and landslides. C.A. Glasbey and I.M. Nevison developed a novel approach for modeling hourly rainfall using a monotonic transformation to achieve normality, creating a latent Gaussian variable for autocorrelation modeling. Validated against real data, the model demonstrated flexibility in forecasting, simulation, and fine-resolution data generation, offering a robust alternative to traditional methods.

Niharika Mishra and Ajay Kushwaha applied Gaussian Process Regression to improve rainfall prediction accuracy, achieving 95.4% accuracy using meteorological data from Raipur. Their work highlights machine learning's potential in water resource management amidst dynamic weather patterns. Zhengzheng Li et al. introduced the Gaussian Rainfall-Rate Estimator (GMRE) for radar-based rainfall-rate estimation. Based on the Gaussian model and Bayesian least squares estimation, GMRE demonstrated superior performance over existing techniques and adaptability across rain regimes, highlighting its potential for global applications. Pradeebane Vaittinada-Ayar et al. developed a high-resolution spatial rainfall model for hydrological applications in France's Ardèche catchment, combining an autoregressive meta-Gaussian process with weather pattern sub-sampling. The model excels in replicating rainfall statistics, offering potential for mountainous catchments. K'ufre-Mfon E. Ekerete et al. studied rainfall drop size distribution, focusing on its impact on satellite signal attenuation. They highlighted the limitations of standard models and proposed alternative Gaussian-based approaches for better accuracy and understanding. Moonhyuk Kwon et al. presented a multivariate stochastic soil moisture estimation approach using a Gaussian-nonstationary hidden Markov model (GM-NHMM) in South Korea. Rainfall significantly improved soil moisture predictions, outperforming ordinary regression models and maintaining spatial coherence. Amjad Hussein and Safaa K. Kadhem analyzed spatial variation in maximum monthly rainfall in Ireland (2018–2020) using Bayesian normal models. They calculated return periods for extreme rainfall, aiding long-term risk planning and highlighting data heterogeneity at weather stations. S. Ly, C. Charles, and A. Degré compared geostatistical and deterministic methods for spatial interpolation of rainfall in Belgium, finding Gaussian models to be the best fit for variogram modeling. Their work emphasizes the importance of spatial interpolation in hydrological modeling, particularly in hilly regions. Kumudha H. R. and Kokila Ramesh studied Indian monsoon rainfall, employing an ANN model to capture intra-seasonal and inter-annual variability. Their approach explained 94% of observed variability, demonstrating the utility of machine learning in rainfall forecasting.

Several studies have explained the dynamics of flooding and rainfall in Karnataka and other regions, providing valuable insights into flood risk assessment, modeling, and management. Bennihalla Basin Study (2019) by Basavaraj Hatti et al aimed to develop a flood hazard zonal map for the Bennihalla river basin in Gadag and Dharwad districts, Karnataka, using geo informatics technology. The study utilized satellite imagery (IRS-1D, LISS-III), SOI topo sheets, and GIS tools like MapInfo, ERDAS, and AutoCAD to integrate thematic layers such as lithology, drainage density, landform, soil, and rainfall. The results classified the area into high, moderate, and low flood hazard zones. Urban expansion in areas like Navalgund and Nargund has increased vulnerability, with residential, agricultural, and industrial activities encroaching into flood-prone zones. The maps aim to raise public awareness and assist authorities in flood mitigation and planning safe settlements. The study concluded that geo-informatics is a robust tool for flood hazard assessment, with potential for application in similar regions globally.

The review of these studies reveals key gaps, such as the under exploration of non-linear trends and extreme weather events. Addressing these gaps could enhance long-term climate projections and mitigation strategies. While Gaussian models are widely used, exploring alternatives like Gaussian and Gamma models could offer improved accuracy and insights for modeling rainfall and its impacts.

Data:

Karnataka is divided into three geographical subdivisions such as Coastal Karnataka, North Interior Karnataka, and South Interior Karnataka. The state experiences four distinct seasons namely the winter season during January to February, which contributes 1% of the annual rainfall, the pre-monsoon (PRM) season occurs from March to May, contributing 7% of annual rainfall and the Northeast Monsoon (NEM) or post-monsoon season (October to December), accounting for 12% of the annual rainfall, and the Southwest Monsoon (SWM) season during June to September, which dominates with 80% of the total rainfall. With the majority of rainfall occurring during the SWM, precise forecasting is particularly important for managing flood risks in vulnerable regions.

This study focuses on analyzing and modeling flood-prone regions of Karnataka, highlighted on the Karnataka map in Figure 1. The analysis concentrates on Southwest Monsoon (SWM) rainfall, which may be one of the reasons of floods in regions such as Dakshina Kannada, Udupi, Uttara Kannada, Chikkamagalur, Kodagu, and Shivamogga. For this purpose, 57 years of SWM rainfall data (1960–2016) has been analysed, obtained from the Indian Institute of Tropical Meteorology (IITM) (http://www.tropmet.res.in) and the Karnataka State Natural Disaster Monitoring Centre (KSNDMC) (https://www.ksndmc.org). Key descriptive statistics, including Long Term Average (LTA), Long Term Deviation (LTD), skewness, and kurtosis, are presented in Table 1 to provide insights into rainfall variability.

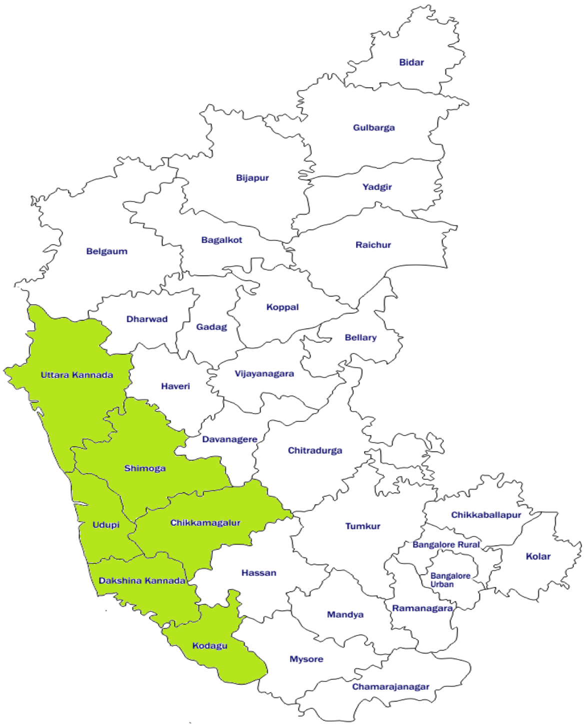

Figure 1: Karnataka map with regions having highlighted flood prone regions

Table 1: Descriptive statistics of SWM Rainfall Data of flood prone regions of Karnataka for the period of 57 years from IITM (1960-2017)

Region	LTA (m_z) (in cm)	LTD (σ_z) (in cm)	Skewness (S_z)	Kurtosis (K_z)
Dakshina Kannada	158.06	30.85	0.44	0.33
Udupi	107.15	17.04	0.18	-0.34
Uttara Kannada	270.80	35.60	0.37	0.47
Chikkamagalur	101.25	19.79	0.47	0.42
Kodagu	63.05	14.96	0.87	1.15
Shivamoga	98.63	24.28	0.42	-0.01

The correlation coefficient that reveals the strengths and weaknesses of links between regions has been created using the gathered data in order to identify relationships between regional characteristics. The pairwise correlation study of rainfall patterns across Karnataka's flood-prone areas is shown in Table 2. The analysis highlights the need for integrated flood risk assessments in **Kodagu, Chikkamagalur, Shivamogga, and parts of Uttara Kannada**, due to the correlations observed between these hilly regions, reflecting their interdependent rainfall dynamics. Coastal regions such as **Dakshina Kannada, Udupi, and Uttara Kannada**, exhibiting moderate correlations, require localized monitoring and tailored flood mitigation strategies to address variability in rainfall patterns effectively. The differences in rainfall patterns between regions with different landscapes highlight how complex rainfall behavior can be, showing the need for flood management strategies tailored to each region. This study offers important insights into how rainfall patterns are connected across Karnataka's flood-prone areas.

Table 2: The correlation coefficient matrix of flood prone regions of Karnataka

Region	Dakshina Kannada	Udupi	Uttara Kannada	Chikkamagalur	Kodagu	Shivamoga
Dakshina Kannada	1	0.69	0.56	0.52	0.57	0.44
Udupi	0.69	1	0.69	0.57	0.55	0.55
Uttara Kannada	0.56	0.69	1	0.50	0.43	0.65
Chikkamagalur	0.52	0.57	0.50	1	0.83	0.88
Kodagu	0.57	0.55	0.43	0.83	1	0.68
Shivamoga	0.44	0.55	0.65	0.88	0.68	1

The rainfall data R_i $(for\ i\ = 1,2, \dots, n)$ is normalized using its long-term average (m_{z_i}), using the following equation.

$$Z_i = \log\left(\frac{R_i}{m_Z}\right) \dots\dots..(1)$$

This transformation offers several analytical advantages, including the ability to represent data points on both the negative and positive sides of the axis, enhancing the flexibility of data interpretation without constraints. Key descriptive statistics of normalized data Z_i, such as the mean $\left(m_{Z_i}\right)$, standard deviation $\left(\sigma_{z_i}\right)$ skewness (S_{z_i}), and kurtosis (K_{z_i}), are tabulated in Table 2.

This approach provides a comprehensive view of the data distribution, aiding in the selection of a suitable distribution function for modeling. The transformation ensures that the mean of Z_i is designed to be approximately zero, allowing for accurate representation of the data's first four moments in the model. The fundamental statistics outlined in the table 3 serve as a foundation for gaining deeper insights into the underlying patterns and characteristics of the data.

Table 3: Descriptive Statistics of the normalized data of Z_i using equation (1)

Region	LTA (m_{z_i})	LTD (σ_{z_i})	Skewness (S_{z_i})	Kurtosis (K_{z_i})
Dakshina Kannada	-0.02	0.20	-0.17	0.07
Udupi	-0.01	0.16	-0.19	-0.50
Uttara Kannada	-0.01	0.13	-0.07	0.22
Chikkamagalur	-0.02	0.20	-0.14	0.05
Kodagu	-0.03	0.23	0.11	0.13
Shivamoga	-0.03	0.25	-0.17	-0.50

The rainfall data distribution, represented as histograms in Figure 2, illustrates the distribution of normalized Southwest Monsoon (SWM) rainfall data (Z_i) for the regions of Dakshina Kannada, Udupi, Uttara Kannada, Chikkamagaluru, Kodagu, and Shivamogga. Each histogram illustrates the transformed rainfall data, overlaid with a Gaussian (normal) distribution curve, to evaluate the data's alignment with a Gaussian distribution. These visualizations help assess the suitability of Gaussian models for representing rainfall variability in these regions. While the SWM rainfall data for each region exhibits some alignment with a Gaussian pattern, significant deviations are observed in several regions, particularly in the form of bimodal distributions. These bimodal patterns suggest that the rainfall data cannot be fully captured by a simple Gaussian distribution. The presence of such characteristics reflects the complexity of rainfall dynamics influenced by the diverse topography and climatic conditions of these regions, including coastal areas and the hilly terrain of the Western Ghats.

To address these limitations, a Gaussian and Gamma Mixture Model (GGM) has been introduced. This model combines the strengths of Gaussian and Gamma distributions to better represent both central tendencies and extreme variability in rainfall patterns. The GGM model effectively accounts for the bimodal characteristics observed in the data, providing a more accurate representation of rainfall distributions across the regions. By incorporating the GGM model, this study aims to capture the complex statistical properties of rainfall in flood-prone regions of Karnataka. This improved modeling approach enhances the understanding of rainfall variability, offering valuable insights for flood forecasting, risk management, and the development of region-specific strategies to mitigate the impacts of extreme weather events.

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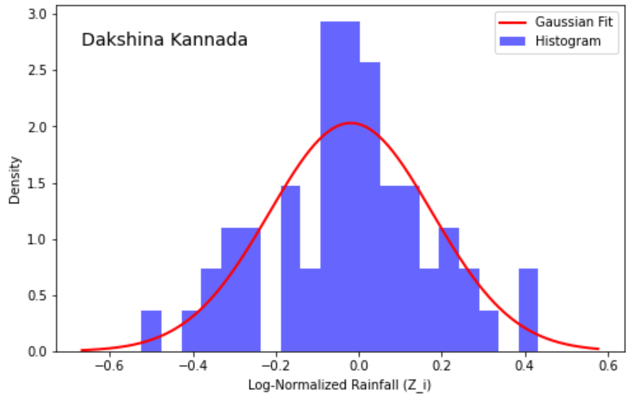
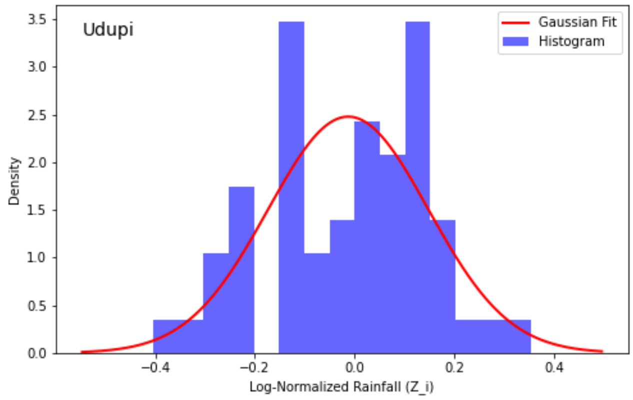
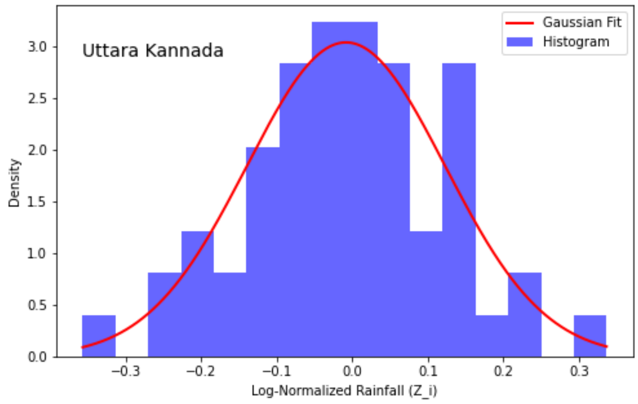
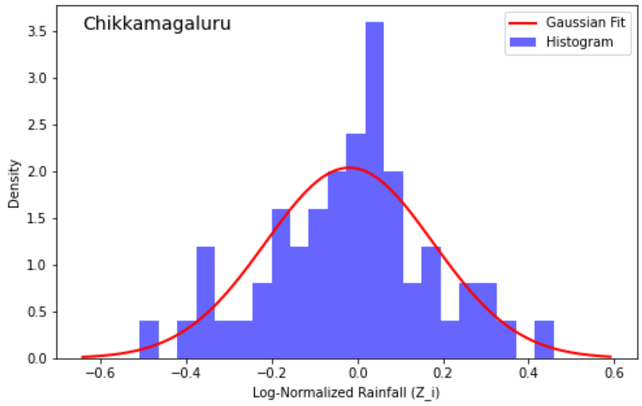








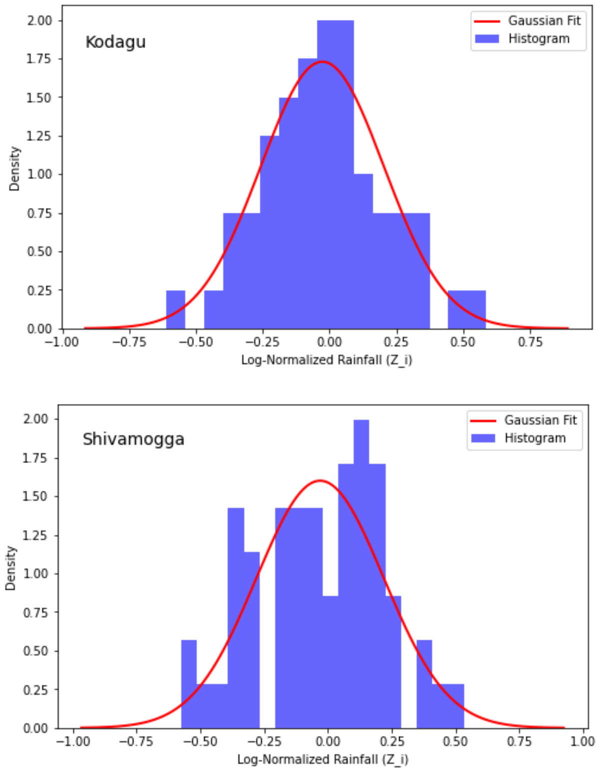


Figure 2: Normalized rainfall data distribution as histograms of the flood prone regions of Karnataka overlapping with Gaussian distribution comparison

Overview of the Gaussian and Gamma Model:

The histogram analysis in the previous section revealed that the rainfall data for this study deviates significantly from a Gaussian distribution, highlighting the need for a model capable of capturing its non-Gaussian characteristics. The work by Kokila and Iyengar emphasizes the importance of Gaussian mixture models for analyzing monsoon rainfall and subdivision regions in India. Building on this foundation, the **Gaussian and Gamma Mixture Model (GGM)** is introduced as an effective approach to address the unique characteristics of rainfall distributions observed in this study. In this paper, the objective is to model Indian monsoon rainfall in Karnataka's flood-prone regions using the Gaussian and Gamma Mixture Model. The transformed variable z is proposed to be modeled as a combination of Gaussian and Gamma random variables x and y, with a mixing proportion w. These variables are assumed to be independently and identically distributed. The combination is expressed as a proportion w_i, which determines the proportionate contribution of the two distributions.

The model for z is mathematically represented as:

$z = wx + (1 - w)y$ ------------------------- (2)

This formulation sets the foundation for capturing the statistical complexity of rainfall data, offering a framework for understanding and predicting rainfall patterns in Karnataka’s flood-prone regions. The mean and variance of z conditioned on u is given by:

$m_{z|u} = wm_x + (1 - w)m_y \text{ and } \sigma_z^2 = w^2\sigma_x^2 + (1 - w)^2\sigma_y^2$ ------------------------- (3)

The conditional probability density function of z given u is formulated as:

$$p(z) = \frac{w}{\sigma\sqrt{2\pi}} e^{-\frac{(z_i-\mu)^2}{2\sigma^2}} + (1-w) z_i^{\alpha-1} \frac{e^{-\frac{z_i}{\beta}}}{\beta^\alpha \Gamma(\alpha)} \text{------------------------} (4)$$

This equation integrates both Gaussian and Gamma components, with the Gaussian part capturing central tendencies and the Gamma part addressing the non-Gaussian characteristics observed in rainfall data. This approach is particularly effective for representing monsoon rainfall patterns in Karnataka's flood-prone regions.

The parameters μ, σ^2, α and β are determined using the Maximum Likelihood Estimation (MLE) method. These parameters encapsulate the essential features of the Gaussian and Gamma distributions associated with the random variables x and y, respectively. The likelihood function L for this model is expressed as follows:

$$L(\mu, \sigma^2, \alpha, \beta) = \prod_{i=1}^{n} [w_i * f_{gaussian}(x: \mu, \sigma^2) + (1-w_i) * f_{gamma}(y: \alpha, \beta)] \text{-------------} (5)$$

Maximizing the likelihood function directly can be computationally complex, so it is more practical to work with the logarithm of the likelihood function. The logarithmic transformation simplifies the calculations while preserving the relationships, ensuring that maximizing the log-likelihood yields the same result as maximizing the original likelihood.

The log-likelihood function is represented as:

$$\ln[L(\mu, \sigma^2, \alpha, \beta)] = \sum_{i=1}^{n} \ln\left[\frac{w}{\sigma\sqrt{2\pi}} e^{-\frac{(z_i-\mu)^2}{2\sigma^2}} + (1-w) z_i^{\alpha-1} \frac{e^{-\frac{z_i}{\beta}}}{\beta^\alpha \Gamma(\alpha)}\right] \text{------------------------} (6)$$

To determine the optimal estimates for each parameter, the partial derivatives of the log-likelihood function are calculated with respect to each parameter. These derivatives are then set to zero, and the resulting equations are solved to obtain the parameter values. This process provides the estimates for μ, σ^2, α and β, as presented in Table 4.

Table 4: The parameter values calculated using Maximum Likelihood method from equation (4)

Region	w	μ	σ^2	α	β
Dakshina Kannada	0.54	-0.16	0.02	0.01	0.42
Udupi	0.47	-0.15	0.01	0.03	0.01
Uttara Kannada	0.55	-0.10	0.07	0.01	0.02
Chikkamagalur	0.49	-0.17	0.02	0.04	0.53
Kodagu	0.54	-0.19	0.02	0.01	0.02
Shivamoga	0.54	-0.22	0.03	0.03	0.07

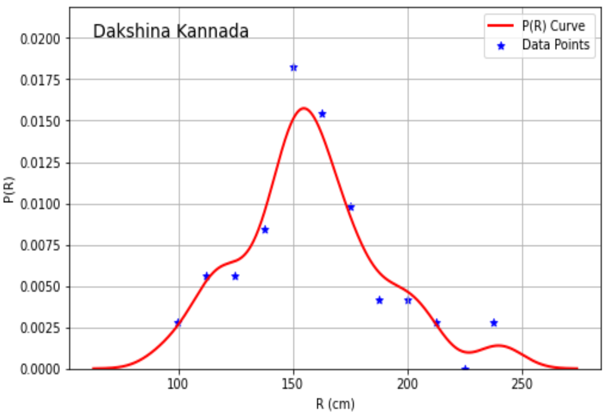
Equation (3) has been validated to meet all the criteria of a valid probability density function. This model is then applied to the original dataset using the transformation specified in Equation (6). The resulting probability density function for R is given as below.

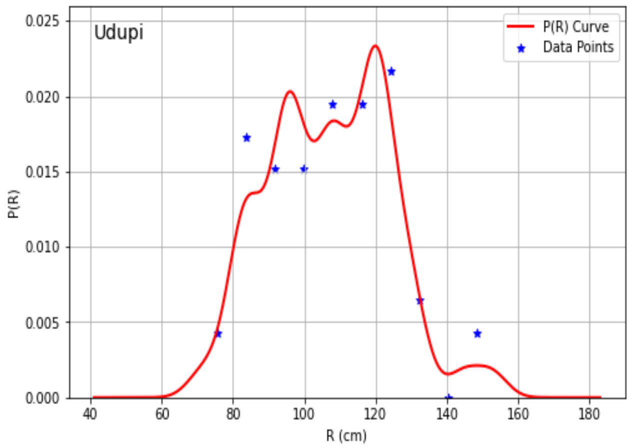
$$p(R) = \frac{1}{R}\left[\frac{w}{\sigma\sqrt{2\pi}}e^{-\frac{\left(\log\left(\frac{R_i}{m_Z}\right)-\mu\right)^2}{2\sigma^2}} + (1-w)z_i^{\alpha-1}\frac{e^{-\frac{\log()}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}\right] \text{------------------------} (7)$$

Figure 3 presents a comparison between the sample histogram of the observed data and the transformed Gaussian and Gamma Mixture (GGM) model. The visual analysis reveals that the GGM model aligns closely with the probability density function described in Equation (7), effectively capturing the overall data distribution. The first four moments of the GGM model, including the mean, variance, skewness, and kurtosis, have been computed and compared with those of the observed data, as shown in Table 5. If the data adhered to a Gaussian distribution, the skewness and kurtosis values respectively. However, the analysis shows significant deviations in these metrics across all cases, confirming that the observed data does not follow a Gaussian distribution. Further demonstrates that the GGM model successfully replicates the moments of the data, particularly the skewness and kurtosis, which are critical for understanding non-Gaussian behavior. This ability to accurately capture the key statistical properties of the data underscores the reliability of the GGM model for representing rainfall patterns. Its effectiveness in modeling the observed data highlights its suitability for analyzing complex distributions in this study.

Table 5: Comparison between the GGM model moments with the original data moments

	Actual Moments (R)				Model Moments (R)			
Region/ Subdivision	μ_R	σ_R	Sk_R	Ku_R	μ_{R_i}	σ_{R_i}	Sk_{R_i}	Ku_{R_i}
Dakshina Kannada	158.06	30.85	0.43	0.33	157.06	31.62	0.49	0.33
Udupi	107.15	17.04	0.18	-0.34	107.35	17.32	0.18	-0.33
Uttara Kannada	270.80	35.59	0.37	0.47	270.03	35.35	0.39	0.48
Chikkamagalur	101.25	19.79	0.47	0.42	101.49	19.49	0.48	0.45
Kodagu	63.05	14.96	0.86	1.15	62.57	14.14	0.82	1.19
Shivamoga	98.63	24.28	0.42	-0.01	98.60	24.50	0.45	-0.08




Udupi
P(R) Curve
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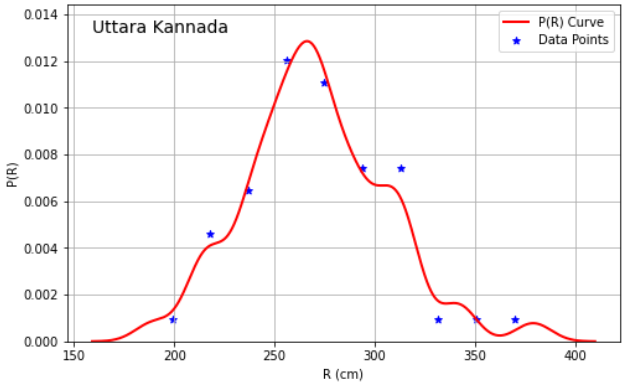
Uttara Kannada
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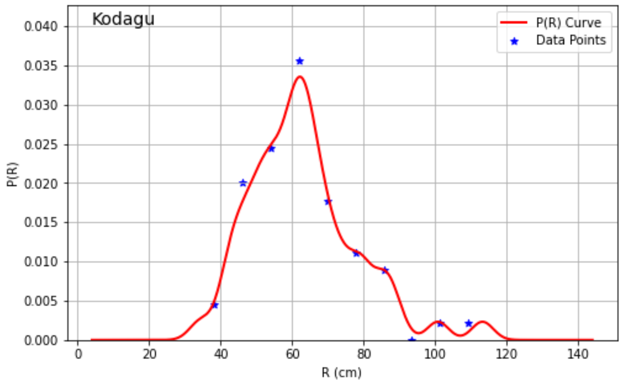
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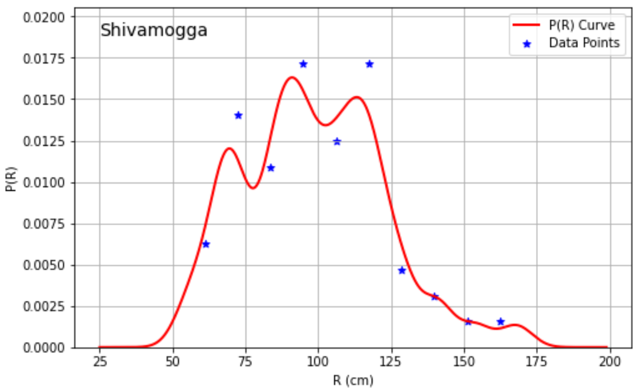
Shivamogga
P(R) Curve
Data Points
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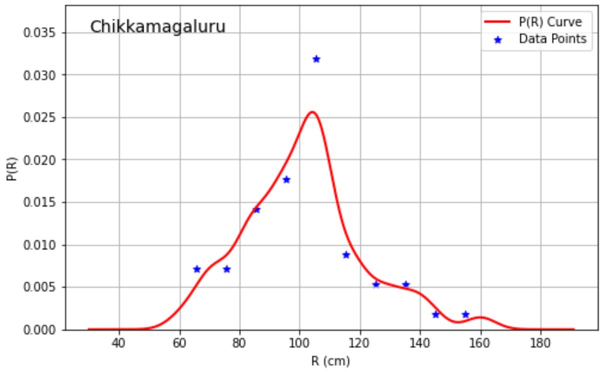

Figure 3: A comparison between the actual data distribution represented by (*) in blue colour and GGM represented by a solid red line.

Results:

In this study, the transformed rainfall data (z) has been successfully modeled using the Gaussian and Gamma Mixture Model (GGM), with the moments of the data being closely reproduced. This modeling approach enables a more accurate

representation of the statistical properties of rainfall data. As x *and* y are random variables within this model, their joint probability densities can be theoretically derived, offering a detailed understanding of the behavior and characteristics of the data.

The effort to develop and apply the Gaussian and Gamma Mixture Model is further justified if it shows an improved ability to capture the level crossing statistics of the transformed rainfall data (z) when compared to a simple Gaussian model. To evaluate this, the rainfall process is assumed to be continuous. The level crossing frequencies, representing the number of times the rainfall data crosses specific thresholds, are calculated for upward crossings at two critical levels. Thresholds of +10% relative to the normal value are used to identify stations experiencing above-normal rainfall (positive crossings). Similarly, thresholds of +20% represent more extreme conditions, with positive crossings signifying severe rainfall events and negative crossings indicating severe droughts.

The crossing statistics for these thresholds are computed for the transformed rainfall data (z) using both the Gaussian and Gamma Mixture Model and the simple Gaussian model for comparison. This comprehensive analysis is conducted across all regions and subdivisions considered in this study to ensure thorough evaluation. The expected rates of upward and downward level crossings are calculated based on these thresholds, offering a quantitative measure of the models' capabilities to capture the rainfall data's behavior at critical thresholds. These rates provide valuable insights into the effectiveness of the Gaussian and Gamma Mixture Model relative to the simpler Gaussian model, particularly in representing the data's behavior under extreme conditions.

The level crossing rates for upward and downward crossings are expressed using the following equations.

$$N_u^{Guass} = \int_0^{\infty} \dot{z} \frac{w}{2\sigma\dot{\sigma}\sqrt{2\pi}} \, e^{\left(-\frac{(z-\mu)^2}{2\sigma^2} + \frac{(\dot{z}-\dot{\mu})^2}{2\dot{\sigma}^2}\right)} \text{------------------------} \quad (8)$$

$N_u^{Gamma} = \int_0^{\infty} \dot{z} \frac{1-w}{\beta^{\alpha}\dot{\beta}^{\dot{\alpha}}\Gamma(\alpha)\dot{\Gamma}(\dot{\alpha})} z^{\alpha-1}\dot{z}^{\dot{\alpha}-1}e^{-\frac{\mu}{\beta}}e^{-\frac{\dot{z}}{\dot{\beta}}}$------------------------- (9)

The total upward level crossing rate is given as $N_u = N_u^{Guass} + N_u^{Gamma}$

The upward and downward level crossings for a specific threshold (a) is defined as:

$N^{+}(a) = \int_0^{\infty} \dot{z}\, p(a, \dot{z})\, d\dot{z}$------------------------- (10)

To evaluate the integrals in Equations (9), the joint density function $p(a, \dot{z})$ is required. The steps for deriving $p(a, \dot{z})$ is as follows,

$z = wx + (1-w)y$, then the derivatives is given by$\dot{z} = w\dot{x} + (1-w)\dot{y}$. The conditional mean and variance of $\dot{z}$, given w are derived as below.

$\mu_{\dot{z}|w} = w\mu_i + (1-w)\,\alpha_{\dot{y}}\beta_{\dot{y}}$ and $\sigma_{\dot{z}|w}^2 = w^2\sigma_{\dot{x}}{}^2 + (1-w)^2\left(\alpha_{\dot{y}}\beta_{\dot{y}}{}^2\right)$

The parameters $\mu_{\dot{z}}$, $\sigma_{\dot{z}}$, $\alpha_{\dot{z}}$ and $\beta_{\dot{z}}$ are estimated using Maximum Likelihood Estimation (MLE) method. The computed values of these parameters are presented in Table 6.

Table 6: The parameter values of a derivative $\dot{z}$ of the process z

Region	$\mu_{\dot{z}}$	$\sigma_{\dot{z}}$	$\alpha_{\dot{z}}$	$\beta_{\dot{z}}$
Dakshina Kannada	-0.12	0.02	66.99	-0.01
Udupi	-0.12	0.01	277.05	-0.01
Uttara Kannada	-0.08	0.01	180.87	-0.02
Chikkamagalur	-0.13	0.02	94.87	-0.01
Kodagu	-0.15	0.02	83.50	-0.01
Shivamoga	-0.17	0.02	64.29	-0.01

The unconditional joint density of $(z, \dot{z})$, essential for calculating level crossing statistics, is expressed as follows:

$p(z, \dot{z}) = \frac{w}{2\sigma\dot{\sigma}\sqrt{2\pi}} e^{\left(-\frac{(z-\mu)^2}{2\sigma^2} + \frac{(\dot{z}-\dot{\mu})^2}{2\dot{\sigma}^2}\right)} + \frac{1-w}{\beta^{\alpha}\dot{\beta}^{\dot{\alpha}}\Gamma(\alpha)\dot{\Gamma}(\dot{\alpha})} z^{\alpha-1}\dot{z}^{\dot{\alpha}-1}e^{-\frac{\mu}{\beta}}e^{-\frac{\dot{z}}{\dot{\beta}}}$ ----------------------(11)

At a given level a, the joint probability $p(z, \dot{z})$, required for determining level crossing statistics, is derived. Level crossings represent the total number of instances where the process (z) transitions across a specified threshold (a).

These crossings are computed using the joint probability density $p(z, \dot{z})$. They are critical for analyzing events like rainfall exceeding normal or critical values (positive crossings). By substituting Equation (10) into Equations (11), the crossing statistics are calculated and summarized in Table 7.

This analysis provides a quantitative understanding of the frequency and severity of these events, offering insights into the behavior of the rainfall process across critical thresholds.

Table 7: Number of upward level crossings

Region	10% above			20% above		
	Observed	Gaussian and Gamma	Gaussian	Observed	Gaussian and Gamma	Gaussian
Dakshina Kannada	15	15	8	7	6	2
Udupi	19	21	7	5	7	2
Uttara Kannada	12	12	10	3	3	2
Chikkamagalur	13	13	8	8	7	2
Kodagu	14	13	3	10	8	3
Shivamoga	20	18	7	11	9	2

Discussion

The statistical analysis of rainfall data from the flood-prone regions of Dakshina Kannada, Udupi, Uttara Kannada, Kodagu, Chikkamagalur, and Shivamogga revealed significant variations in rainfall patterns. Descriptive statistics, including Long Term Average (LTA), Long Term Deviation (LTD), skewness, and kurtosis, indicated that the rainfall distribution deviates from a Gaussian pattern. The data displayed a bimodal structure in several regions, highlighting the need for advanced modeling techniques, as visualized in the histogram plots (Figure 3). This deviation underscores the complexity of rainfall behavior in these regions, influenced by diverse topographical and climatic factors.

The Gaussian and Gamma Mixture Model (GGM) was applied to the transformed rainfall data (zz) to address the non-Gaussian characteristics. The model successfully captured the statistical properties of the data, as evidenced by the alignment of the GGM with the observed data in terms of the first four moments (mean, variance, skewness, and kurtosis). The comparison of observed and modeled distributions (Figure 4) demonstrated that the GGM closely replicates the probability density function of rainfall data, outperforming the simple Gaussian model. The ability of the GGM to account for bimodal patterns makes it particularly effective for analyzing complex rainfall distributions in the flood-prone regions of Karnataka. This feature is crucial for understanding the underlying dynamics of rainfall and developing accurate flood risk assessments.

Level crossing statistics were computed to evaluate the model's ability to predict extreme rainfall events. The analysis considered two thresholds: +10% and -10% of the normal value for above-normal rainfall and drought conditions, respectively, and +20% and -20% for severe rainfall and extreme drought conditions. The results revealed that the GGM consistently outperformed the Gaussian model in capturing the frequency of upward and downward crossings (Table 6). For instance, in **Shivamogga**, the GGM closely predicted observed upward crossings at the +20% threshold, whereas the Gaussian model significantly underestimated the counts. Similarly, for downward crossings in Kodagu, the GGM provided accurate estimates compared to the Gaussian model, which failed to capture extreme drought conditions effectively. These results highlight the robustness of the GGM in capturing the critical thresholds that define extreme weather events. The model's ability to represent both upward and downward level crossings demonstrates its potential for flood forecasting and drought risk assessment.

The findings reveal strong correlations among rainfall patterns in the hilly regions of Kodagu, Chikkamagalur, Shivamogga, and Uttara Kannada**,** reflecting their interconnected climatic dynamics in the Western Ghats. Coastal regions such as Dakshina Kannada and Udupi**,** while moderately correlated, exhibit localized rainfall variability, emphasizing the need for tailored flood mitigation strategies. The variation in correlation across distinct topographies highlights the importance of region-specific flood risk management approaches. The GGM's ability to accurately model rainfall data has significant implications for disaster preparedness and resource management. The model's reliability in representing rainfall extremes offers a valuable tool for policymakers to plan flood mitigation strategies, allocate resources, and improve early warning systems. Additionally, the insights from level crossing statistics can aid in designing robust infrastructure and agricultural practices to withstand extreme weather events.

Conclusion

This study demonstrates the effectiveness of the Gaussian and Gamma Mixture Model (GGM) in addressing the limitations of traditional Gaussian models for analyzing rainfall data in Karnataka's flood-prone regions. The GGM successfully captures the complex statistical properties of rainfall, including skewness, kurtosis, and level crossing frequencies, which are critical for understanding and predicting extreme weather events such as floods. The model's ability to represent bimodal and non-Gaussian behavior observed in the data ensures a more accurate depiction of rainfall distributions compared to simpler approaches. The application of the GGM provides valuable insights into the regional dynamics of rainfall patterns, particularly in diverse topographical regions such as Dakshina Kannada, Udupi, Uttara Kannada, Kodagu, Chikkamagalur, and Shivamogga**.** The model's robustness in predicting upward level crossings at critical thresholds enhances its utility for flood forecasting and risk assessment. These findings emphasize the importance of integrating advanced statistical models into disaster preparedness and resource management strategies, aiding policymakers in planning for extreme weather events and improving regional resilience. This research provides the way for future studies by suggesting the incorporation of additional climatic variables, such as soil moisture and temperature, and expanding the model's application to other flood-prone regions. The GGM thus serves as a reliable tool for enhancing flood risk mitigation and climate adaption efforts.

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