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Solution of Ebola Virus Disease Outbreaks with Novel Technique Conformal Fractional Differential Transform Method

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Abstract:

Differential Transform Method (DTM) is the new technique to solve the linear and nonlinear differential equations with initial and boundary conditions which provides the closest form of the exact solution. Conformable Fractional Differential Transform Method (CFDTM) is formulated for the fractional order power series in the same way. This paper deals with Ebola Virus Disease (EVD) epidemiological model of differential equations and applied CFDTM to the SIRDP (Susceptible-Infected-Recovered-Deceased-Pathogens) model for different values of order ζ. By graphical representation, we can conclude that by taking immense measure in controlling the EVD by isolating the infected individuals from the environment and careful burial of deceased bodies. Mathematical calculations are done using Maple software to form the iterative series.

2020 Mathematics Subject Classifications: 65L06, 92D25, 92D30.

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1. Introduction

Ebola Virus Disease (EVD) is a severe disease caused by Ebola Virus, a member of the filovirus family, which occurs in humans and other primates. The disease emerged in 1976 in almost simultaneous outbreaks in the Democratic Republic of the Congo (DRC) and Sudan (now South Sudan). The largest outbreak took place in West Africa between March 2014 and June 2016, affecting primarily Guinea, Liberia and Sierra Leone over 28,000 cases were recorded. There are 6 species of Ebola Virus and 4 of which have caused disease in humans.

(1) Zaire Ebola Virus (EBOV)

(2) Sudan Ebola Virus (SUDV)

(3) Tai Forest (TAFV) (formerly known as Ebola Ivory Coast)

(4) Bundibugyo Ebola Virus (BDBV)

The first human case is an Ebola outbreak is acquired through contact with blood, secretion organs or other body fluids of an infected animal. EVD has been documented in people who handled infected chimpanzees, gorillas and forest antelopes, both dead and alive in cote d'lvoire the Republic of Congo and Gabon. The virus is then transmitted from person to person through direct contact with blood, secretions, organs or other bodily fluids of infected through contact with objects such as needles or soiled clothing, that have been contaminated with infected secretions have been fuelled by traditional practices of burial, in which mourners have direct contact with the bodies of the deceased. Hospital workers have fully been infected during Ebola outbreaks through close contact with infected patients and insufficient use of correct infection control precautions and barrier nursing procedures. The primary symptoms of Ebola often includes fever, weakness and fatigue, sore throat, loss of appetite unexplained harmouaging, bleeding or bruising gastrointestinal symptoms including abdominal pain diarrhoea and vomiting and other symptoms include red eyes, skin rashes and hiccups (late stage) [2–8, 10–13, 16].

A few of the numerical techniques for determining the solution of differential equations is the Differential Transform Method (DTM). Zhou is the first one to establish the DTM. He employed DTM to tackle linear and nonlinear initial value problems in electric circuit analysis [23]. A new analytical technique for solving fractional differential equations (FDEs) has recently been devised. Conformable Fractional Differential Transform Method (CFDTM) is a technique that formulates fractional power series in the same way as DTM formulates Taylor series. The conformable fractional differential transform method is being used to analyse fractional differential equations, fractional partial differential equations, fractional-order integro-differential equations and fractional differential-algebraic equations [1, 14, 15, 17-21].

In this paper, SIRDP(Susceptible-Infected-Recovered-Deceased-Pathogens) model is framed to get a numerical solution using Conformable Fractional Differential Transform Method (CFDTM) with their initial condition and parametric values.

2. Conformable Fractional Differential Transform Method

In order to expand the analytical and continuous function, $f(t)$ in terms of a fractional power series, we consider the basic definition and its properties of the conformable fractional one-dimensional differential transform method and studied by Ünal and Gökdoğan [22].

Definition 2.1. [22] If we suppose that the function $f(t)$ is infinitely $\alpha -$ differentiable function for some $\alpha \epsilon (0, 1]$, then the conformable fractional differential transform of $f(t)$ is defined as follows:

$$F_\alpha(k) = \frac{1}{\alpha^k\, k!}\left[\left(T_\alpha^{t_0} f\right)^{(k)}(t)\right]_{t=t_0}, \qquad (1)$$

where $\left(T_\alpha^{t_0} f\right)^{(k)}(t)$ denotes the k^{th} iterate of the fractional derivative $\left(T_\alpha^{t_0} f\right)(t)$ for a function $f: (t_0,\ \infty) \to \mathbb{R}$ given by

$$\left(T_\alpha^{t_0} f\right)(t) \coloneqq \lim_{\varepsilon \to 0}\left\{\frac{f\left(t+\varepsilon(t-t_0)^{1-\alpha}\right)-f(t)}{\varepsilon}\right\}, \qquad t \ge t_0 \ge 0; \quad 0 < \alpha \le 1. \qquad (2)$$

Definition 2.2. [22] If $F_\alpha(k)$ denotes the conformable fractional differential transform of the function $f(t)$ given by Definition 2.1, then the inverse fractional differential transform of $F_\alpha(k)$ is defined by

$$f(t) = \sum_{k=0}^{\infty} F_\alpha(k)\,(t-t_0)^{\alpha\,k} = \sum_{k=0}^{\infty} \frac{1}{\alpha^k\,k!}\left[\left(T_\alpha^{t_0}\,f\right)^{(k)}\,(t)\right]_{t=t_0}(t-t_0)^{\alpha\,k}. \qquad (3)$$

By applying Definition 2.1 and 2.2, we are led to Definition 2.3 below:

Definition 2.3. [22] The conformable fractional differential transform (CFDT) of the initial conditions for integer-order derivatives are defined as follows:

$$F_\alpha(k) = \begin{cases} \frac{1}{(\alpha\,k)!}\left[\frac{d^{\alpha\,k}\{f(t)\}}{dt^{\alpha\,k}}\right]_{t=t_0}; & \alpha\,k \in \mathbb{N} \\ 0; & \alpha\,k \notin \mathbb{N} \end{cases} \quad \text{for } k = 0, 1, \cdots, \left[\frac{n}{\alpha}\right] - 1, \qquad (4)$$

where $\mathbb{N}$ denotes the set of positive integers and n is the order of the corresponding fractional differential equation.

For properties one can refer [9, 14, 22].

3. Mathematical Formulation

The Ebola virus in the habitat is classified into five compartments: susceptible population $s(t)$, infected individuals $\mathfrak{i}(t)$, recovered $q(t)$, infected deceased $w(t)$ and the Ebola virus pathogen in the environment is classified as $\mathfrak{g}(t)$. Various equations have been used to explain the model and its behavior, including [15]

$$\begin{aligned}
\frac{ds(t)}{dt} &= \alpha_1 - [\beta_1 \mathfrak{i}(t) + \beta_2\, w(t) + \gamma\, \mathfrak{g}(t)]s(t) - \varsigma\, s(t), \\
\frac{d\mathfrak{i}(t)}{dt} &= \alpha_2 + [\beta_1\, \mathfrak{i}(t) + \beta_2\, w(t) + \gamma\, \mathfrak{g}(t)]s(t) - (\varsigma + \theta + \epsilon)\mathfrak{i}(t), \\
\frac{dq(t)}{dt} &= \epsilon\, \mathfrak{i}(t) - \varsigma\, q(t) \qquad (5) \\
\frac{dw(t)}{dt} &= (\varsigma + \theta)\, \mathfrak{i}(t) - \omega\, w(t), \\
\frac{d\mathfrak{g}(t)}{dt} &= \nu + \chi\, \mathfrak{i}(t) + \rho\, w(t) - \xi\, \mathfrak{g}(t)
\end{aligned}$$

Table 1. Parametric values for Ebola Virus Disease (EVD) [15].

Variable	Description	Values	Variable	Description	Values
α_1	New Births	10	γ	Polluted environment	0.01
α_2	Infected neonate	3	θ	Infectious human death	0.05
β_1	Susceptible infectious	0.006	ρ	Deceased person rate	0.04
β_2	Diseased individuals	0.012	ς	Deceases person	0.5

				accumulated	
ν	Polluting environment	0	ϵ	Recovery rate	0.06
ω	Rate of human burial	0.8	χ	Contagious rate	0.04

4. Mathematical Modeling of the Conformable Fractional EVD Model

The conformable fractional model of real development of outbreak of infection in a huge population is given by the following conformable fractional differential system,

$$\begin{aligned} T_\zeta\, s(t) &= \alpha_1 - [\beta_1 i(t) + \beta_2\, w(t) + \gamma\, g(t)]s(t) - \varsigma\, s(t), \\ T_\zeta i(t) &= \alpha_2 + [\beta_1\, i(t) + \beta_2\, w(t) + \gamma\, g(t)]s(t) - (\varsigma + \theta + \epsilon)i(t), \\ T_\zeta q(t) &= \epsilon\, i(t) - \varsigma\, q(t), \\ T_\zeta w(t) &= (\varsigma + \theta)\, i(t) - \omega\, w(t), \\ T_\zeta g(t) &= \nu + \chi\, i(t) + \rho\, w(t) - \xi\, g(t). \end{aligned} \tag{6}$$

$$\begin{cases} s(0) = 3470 \\ i(0) = 3317 \\ q(0) = 2287 \\ w(0) = 1171 \\ g(0) = 0 \end{cases} \tag{7}$$

4.1. Application of Conformable Fractional Differential Transform Method.

Equation (6) can be rewritten as follows:

$$\zeta(p+1)\mathcal{S}_\zeta(p+1) = \alpha_1\delta(p) - \sum_{r=0}^{p} \mathcal{S}_\zeta(r)\left(\beta_1\mathfrak{I}_\zeta(p-r) + \beta_2\mathcal{W}_\zeta(p-r) + \gamma\mathcal{G}_\zeta(p-r)\right) - \varsigma\mathcal{S}_\zeta(p)$$

$$\zeta(p+1)\mathfrak{I}_\zeta(p+1) = \alpha_2\,\delta(p) - \sum_{r=0}^{p} \mathcal{S}_\zeta(r)\left(\beta_1\mathfrak{I}_\zeta(p-r) + \beta_2\mathcal{W}_\zeta(p-r) + \gamma\mathcal{G}_\zeta(p-r)\right) - (\varsigma + \theta + \epsilon)\mathfrak{I}_\zeta(p),$$

$$\zeta(p+1)\mathcal{Q}_\zeta(p+1) = \epsilon\,\mathfrak{I}_\zeta(p) - \varsigma\,\mathcal{Q}_\zeta(p), \tag{8}$$

$$\zeta(p+1)\mathcal{W}_\zeta(p+1) = (\varsigma + \theta)\mathfrak{I}_\zeta(p) - \omega\,\mathcal{W}_\zeta(p),$$

$$\zeta(p+1)\mathcal{G}_\zeta(p+1) = \nu\,\delta(p) + \chi\,\mathfrak{I}_\zeta(p) + \rho\,\mathcal{W}_\zeta(p) - \xi\,\mathcal{G}_\zeta(p).$$

Hence the recurrence relation is obtained as

$$\mathcal{S}_\zeta(p+1) = \frac{1}{\zeta\,(p+1)}\left[\alpha_1\delta(p) - \sum_{r=0}^{p}\mathcal{S}_\zeta(r)\left(\beta_1\mathfrak{T}_\zeta(p-r)+\beta_2\mathcal{W}_\zeta(p-r)+\gamma\mathcal{G}_\zeta(p-r)\right) - \varsigma\mathcal{S}_\zeta(p)\right]$$

$$\mathfrak{T}_\zeta(p+1) = \frac{1}{\zeta\,(p+1)}\left[\alpha_2\delta(p) + \sum_{r=0}^{p}\mathcal{S}_\zeta(r)\left(\beta_1\mathfrak{T}_\zeta(p-r)+\beta_2\mathcal{W}_\zeta(p-r)+\gamma\mathcal{G}_\zeta(p-r)\right) - (\varsigma+\theta+\epsilon)\mathfrak{T}_\zeta(p)\right]$$

$$\mathcal{Q}_\zeta(p+1) = \frac{1}{\zeta\,(p+1)}\left[\epsilon\,\mathfrak{T}_\zeta(p) - \varsigma\,\mathcal{Q}_\zeta(p),\right] \qquad (9)$$

$$\mathcal{W}_\zeta(p+1) = \frac{1}{\zeta\,(p+1)}\left[(\varsigma+\theta)\mathfrak{T}_\zeta(p) - \omega\,\mathcal{W}_\zeta(p),\right]$$

$$\mathcal{G}_\zeta(p+1) = \frac{1}{\zeta\,(p+1)}\left[\nu\,\delta(p) + \chi\,\mathfrak{T}_\zeta\,(p) + \rho\,\mathcal{W}_\zeta(p) - \xi\,\mathcal{G}_\zeta(p)\,\right].$$

Applying initial conditions (7) and parameter values from the table (1) we obtain the series for the classical order $\zeta = 1$, upto certain order by inverse differential transform method as follows :

$$s(t) = \sum_{l=0}^{\infty}\mathcal{S}_\zeta(k)t^{k\zeta} = 3470 - 116075.3800\,t + 714521.3145\,t^2 + 24337984.34\,t^3 + \cdots$$

$$i(t) = \sum_{l=0}^{\infty}\mathfrak{T}_\zeta(k)t^{k\,\zeta} = 3317 + 115800.0100\,t - 778859.1625\,t^2 - 24060529.42\,t^3 + \cdots$$

$$q(t) = \sum_{l=0}^{\infty}\mathcal{Q}_\zeta(k)t^{k\,\zeta} = 2287 - 944.4800000\,t + 3710.120300\,t^2 - 16195.53663\,t^3 + \cdots \quad (10)$$

$$w(t) = \sum_{l=0}^{\infty}\mathcal{W}_\zeta(k)t^{k\,\zeta} = 1171 + 887.5500000\,t + 31489.98275\,t^2 - 151188.1752\,t^3 + \cdots$$

$$g(t) = \sum_{l=0}^{\infty}\mathcal{G}_\zeta(k)t^{k\,\zeta} = 179.5200000\,t + 2331.058400\,t^2 - 9988.232980\,t^3 + \cdots$$

5. Results and Discussion

The system of differential equations has been solved numerically to get approximate solutions.

The solution is plotted using graphical representation.

It is noted that the effectiveness of the EVD in the environment is due to the contamination from the infected people in the environment. There are a few preventive measures that can be adopted for the further spread of Ebola Virus Disease. Some of them are by

(1) Avoiding contact with blood and body fluids such as urine, feces, saliva, sweat, vomit, breast milk, amniotic fluid, vaginal fluids of people who are sick.

(2) Avoiding contact with semen from a man who has recovered from EVD until negative results.

(3) Avoiding contact with an infected person's blood and fluids in clothes, bedding, needles and medical equipment's.

(4) Avoiding funeral or burial practices that involve touching the body of someone who died from EVD or suspect EVD.

(5) Avoiding contact with bats forest antelopes and non-human primates and by avoiding bushmeat.

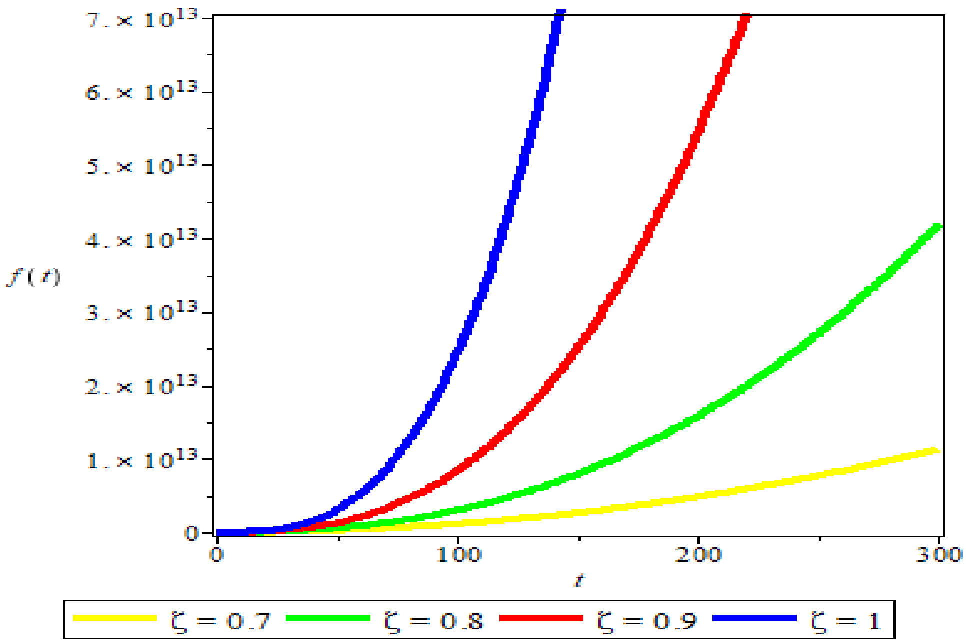

Figure 1. Graphical representation for vulnerable population $s(t)$ at time t by CFDTM for different values of ζ.

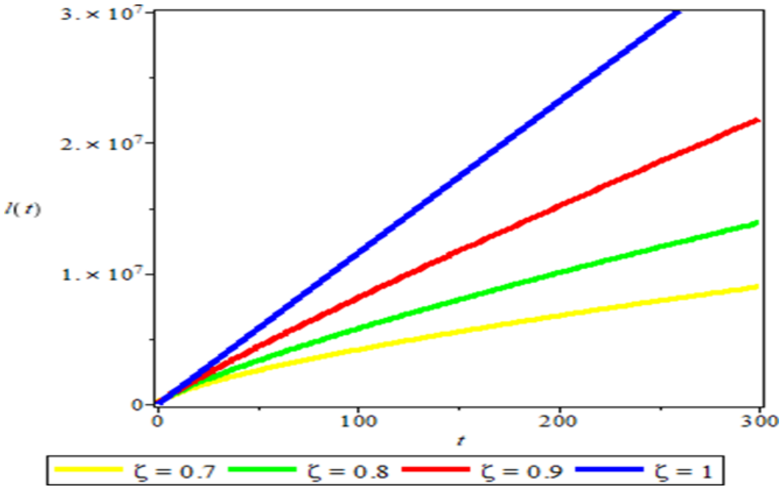

Figure 2. Graphical representation for vulnerable population $i(t)$ at time t by CFDTM for different values of ζ.

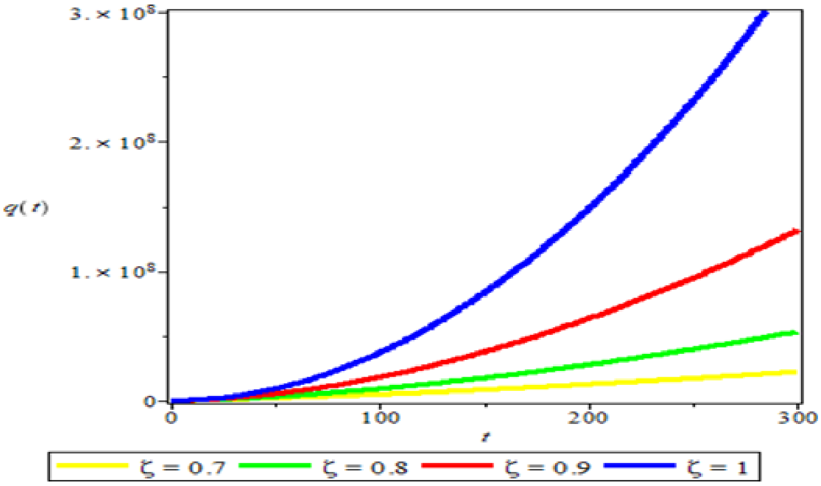

Figure 3. Graphical representation for vulnerable population $q(t)$ at time t by CFDTM for different values of ζ.

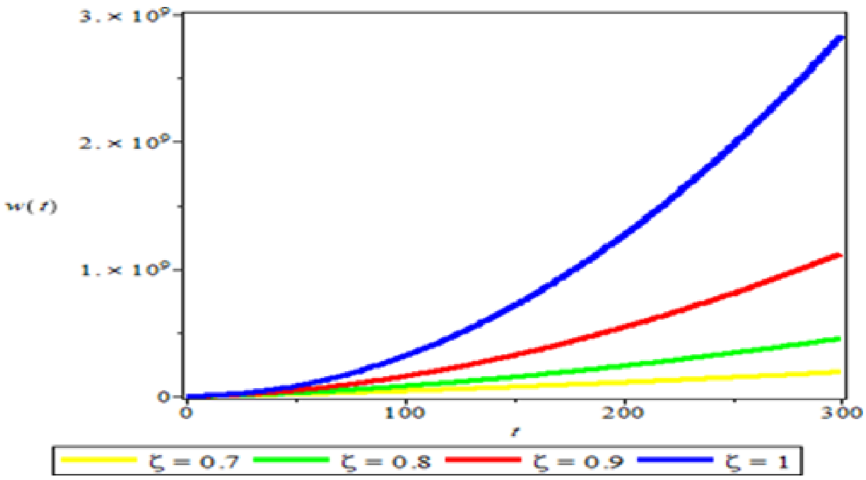

Figure 4. Graphical representation for vulnerable population $q(t)$ at time t by CFDTM for different values of ζ.

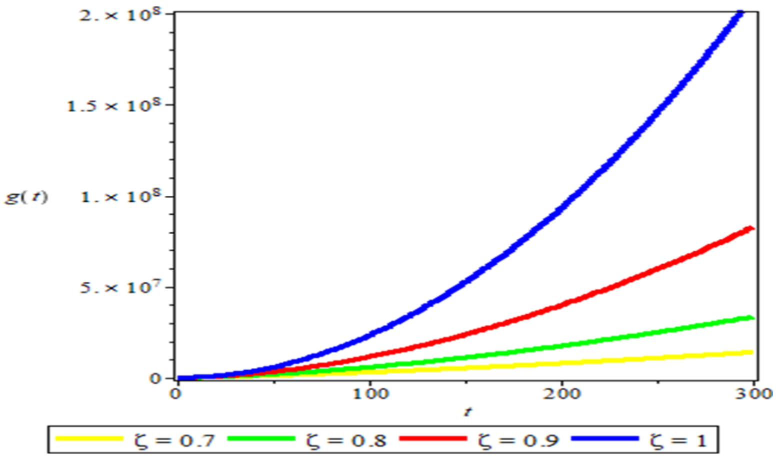

Figure 5. Graphical representation for vulnerable population $\mathcal{G}(t)$ at time t by CFDTM for different values of ζ.

6. Conclusion

A conformable fractional mathematical model for the spread of EVD has been proposed of presented. We have found out approximate solutions with the numerical method called CFDTM for the SIRDP model. This method is based on conformable derivative which is extremely popular in the last few years. The result reveals that to bring the Ebola Virus Disease under control infected person should be isolated from the environment and also by avoiding funeral and burial practices that involve touching the body of the died EVD person and also the careful burial of them.

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