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Non-Homogeneous Fuzzy Differential Equation with Triangular Fuzzy Number as Initial condition

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Abstract:
A solution of First Order Nonhomogeneous Linear Fuzzy Differential Equation with initial condition as Triangular Fuzzy Number is discussed in this paper and to obtained a general solution, we have used a method of interval arithmetic on α-cut interval. Here, we have proposed a solution Non-Homogeneous Fuzzy Differential Equation by considering four different possible cases of real valued functions involved in differential equations. At the last, an example of first order nonhomogeneous linear fuzzy differential equation under fuzzy initial condition is being solved to verify the result.

Keywords: Fuzzy Differential Equation, Triangular Fuzzy number, Support and Core of Fuzzy set, Interval arithmetic.

1. Introduction

Fuzzy differential equation is a model of varying situations under vague conditions has an important role to various fields of Engineering and Science. The concept of Fuzzy Differential elution was first introduced by Kandel, A. and Byatt, W.J [1] in 1978. Followed by in 1982 Dubois and Prade [2], [3], [4] discuss about derivative of crisp function at fuzzy point and derivative of fuzzy function at crisp point. In 1985, Kaleva, O. [6] mention a differentiability and integrability of fuzzy valued function with exitance and uniqueness its solution. Seikkala, S. [5] has conveyed a use of extension principle and extremal solution of initial value problems to solve fuzzy differential equation in 1987. In the year 1990, Baidosov, V. [21] and Hullermeier, E [22] has introduced a differential system with fuzzy parameter known as fuzzy differential inclusion. In year 1999, Oberguggenberger, M. [23] used zadeh's extension principle get solution to fuzzy differential equation.

In year 2000, Buckley, et.al. [9] acknowledge a new solution of first order fuzzy initial value problem depends on various categories of derivatives of fuzzy function. Park, J. Han, H. [10] discussed a unique fuzzy solution of differential equation under fuzzy circumference applying successive approximation method . In 2008, Chaco-Cano, Y. and Romain-Flores, H. [11], [24] Proposed a new solution to the Fuzzy differential equation based in generalised H-differentiability and π- derivative of function. In 2010, Bede. B. et.al [12], [13] used L-R type fuzzy number as Starting situation of fuzzy differential equation and also proposed two general solutions for fuzzy differential equation with existence. Duraisamy, C. and Usha,B. [14] solved a fuzzy initial value problem of first order by using third order Runge-Kutta method with Trapezoidal fuzzy number as initial condition. Salahshour, S. [15] In 2011, has promotes uniqueness and existence of solution of Nth-order fuzzy differential equation generalised differentiability in Banach space. Plotnikov, A.

with Skripnik, N. [25] In 2012, emphasized on new concept of fuzzy function generalized derivative and existence of a fuzzy differential equation. In year 2013, Mondal, S. et.al [16], [18] has discussed a solution of homogeneous first order linear fuzzy differential equation using method of Lagrange multiplier and intuitionistic Triangular fuzzy number and in year 2014, they [27] have promotes solution of Nonhomogeneous Fuzzy differential equation using Lagrange multiplier method. In year 2020 Alamin, A. et.al. [26] analyse a Solution of non-homogeneous difference equation and in 2023, Padmapriya, V., & Kaliyappan, M. [28] has discussed a solution of system of non-homogeneous fuzzy fractional differential equations. In 2024, Bawane D.P. and Ladke L.S. [29] has discus a solution for the linear fuzzy differential equation of first order using interval arithmetic techniques.

2. Preliminaries

Definition 2.1: Fuzzy Set

A set A define on universe of discourse X is said to be fuzzy if $\forall\ t \in X$, membership grade of belongings is lies between $[0,1] \subset \mathbb{R}$ and it is denoted and define as
$A = \left\{\frac{\mu_A(t)}{t} \middle| \, t \in X \, , \; \mu_A(t) \in [0,1] \right\}$. Where, $\mu_A : X \to [0,1]$ is the membership grade.

Definition 2.2: α-cut of Fuzzy set

A crisp set obtained from a fuzzy set which containing all elements whose membership grade is higher than or equal to the state value $\alpha \in [0,1]$ is known as α-cut of Fuzzy set. It is represented as as $\mathrm{A}^{\alpha} = \{t_k | t_k \in X \text{ and } \mu_A(t_k) \geq \alpha\}$.

Definition 2.3: Fuzzy Set Support

A crisp set obtained from a fuzzy set with all members whose membership grade is more than 0 is known Fuzzy set support. It is represented as, $\mathrm{Supp}(A) = \{t \;\; | \; t \in X \text{ and } \mu_A(t) > 0\}$.

Definition 2.4: Fuzzy Set Core

It is Classical Set of elements $t \in X$ whose membership grade of belongingness in fuzzy set A equal to 1 and it is denoted as follows
$\mathrm{Core}(A) = \{t \;\; | \, t \in X \text{ and } \mu_A(t) = 1\}$.

Definition 2.5: Arithmetic's on closed Intervals

Consider $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are two closed intervals explain on real number set $\mathbb{R}$ in such a way that $a_1 \leq a_2$ and $b_1 \leq b_2$, the arithmetic's such as subtraction and addition are as follows.

a) For Subtraction: $A - B = [a_1, a_2] - [b_1, b_2] = [(a_1 - b_2) \; , \; (a_2 - b_1)]$.
b) For Addition: $A + B = [a_1, a_2] + [b_1, b_2] = [(a_1 + b_1) \; , \; (a_2 + b_2)]$

Definition 2.6: Fuzzy Number

A fuzzy set A defines on set of real number in $\mathbb{R}$ is said to be a fuzzy number as long as it fulfil the circumstances as below

(i) Set A is normalize fuzzy set i.e. core of fuzzy set is non-void

(ii) Support of fuzzy set A is constrained.

i.e. for real numbers $\lambda_1, \; \lambda_2 \in \mathbb{R}$ there exist $t \in (-\infty, \lambda_1)$ & $(\lambda_2 \, , \infty)$ such that $\mu_A(\mathrm{t}) = 0$.

(iii) Every α-cut set are closed intervals.

i.e. for every $\alpha \in [0,1]$, $A_\alpha = [a_\alpha,\ b_\alpha]$ is closed interval.

Definition 2.7: Triangular Fuzzy Number (TNF)

A Triangular Fuzzy Number 'A' characterize with three points $[a_1\ ,\ a_2,\ a_3]$ with membership function

$$\mu_A(t) = \begin{cases} 0, & if\ t < a_1\ and\ \ t > a_3 \\ \dfrac{t - a_1}{a_2 - a_1}, & if\quad a_1 \le t \le a_2 \\ \dfrac{a_3 - t}{a_3 - a_2}, & if\quad a_2 \le t \le a_3 \end{cases}$$

and α-cut set is $A_\alpha = [a_1 + \alpha(a_2 - a_1),\ \ a_3 - \alpha(a_3 - a_2)] = [a_1 + \alpha\mathcal{L},\ a_3 - \alpha\mathcal{R}]$,
here $\mathcal{L} = (a_2 - a_1) > 0$ & $\mathcal{R} = (a_3 - a_2) > 0$.
If $\mathcal{L} = \mathcal{R}$, triangular fuzzy number A $= [a_1\ ,\ a_2,\ a_3]$ is symmetric in nature.

Definition 2.8: First Order Homogeneous Fuzzy Differential Equation

Equation $\left(t, y,\ \frac{dy}{dt}\right) = 0$, $y(t_0) = y_0$ is homogeneous fuzzy differential equation of first order, if initial value y_0 is fuzzy valued number and hence its solution is also a fuzzy solution.

Definition 2.9: Nonhomogeneous First Order Fuzzy Differential Equation

Equation $\left(t, y,\ \frac{dy}{dt}\right) \neq 0$, $y(t_0) = y_0$ is nonhomogeneous fuzzy differential equation of first order, where initial value y_0 is fuzzy valued number and hence its solution is also a fuzzy solution.

Definition 2.10: Strong and Weak Solution

Let $f\left(t, y,\ \frac{dy}{dt}\right) = 0\ \neq 0$, $y(t_0) = y_0$ is the fuzzy differential equation of first order, where initial condition y_0 is fuzzy value then solution $y(t)$ is strong solution provided it's α-cut, $y(t,\alpha) = [y_1(t,\alpha)\ , y_2(t,\alpha)]$ satisfy following conditions (I) $\frac{\partial y_1(t,\alpha)}{\partial \alpha} > 0$ and (II) $\frac{\partial y_2(t,\alpha)}{\partial \alpha} < 0$, otherwise it is a weak solution.

3. Case Study

Consider a Non-Homogeneous First Order Linear Fuzzy Differential Equation

$$\frac{dy(t)}{dt} + P(t)y(t) = Q(t) \tag{1}$$

with $\tilde{y}(t) = y(t_0) = [a_1,\ a_2\ , a_3]$ Is TFN. Here $P(t)$ and $Q(t)$ are any functions of real valued variable t. The solution of equation (1) is value of dependent variable $y(t)$ whose α-cut interval is denoted as $y(t) = [\ y_1(t,\alpha),\ y_2(t,\alpha)\ ]$ and the α-cut interval of given initial condition is denoted as $y(t_0) = [a_1 + \alpha\mathcal{L},\ a_3 - \alpha\mathcal{R}] = [\ y_1(t_0,\alpha),\ y_2(t_0,\alpha)\ ]$

where, $\mathcal{L} = (a_2 - a_1) > 0$ and $\mathcal{R} = (a_3 - a_2) > 0$.

<u>Case-I: If $P(t)$ and $Q(t)$ are any real valued functions</u>

An α-cut of differential equation (1) is

$$\left[\frac{dy_1(t,\propto)}{dt},\frac{dy_2(t,\propto)}{dt}\right]+P(t)[y_1(t,\propto),y_2(t,\propto)]=Q(t)$$

i.e. $\frac{dy_1(t,\propto)}{dt}+P(t)y_1(t,\propto)=Q(t)$ (2)

$\frac{dy_2(t,\propto)}{dt}+P(t)y_2(t,\propto)=Q(t)$ (3)

Since $P(t)$ is real valued function, the integrating factor is $I(t)=e^{\int P(t)dt}$ And solutions are

$y_1(t,\propto)\ I(t)=\int Q(t)\ I(t)dt\ +C_1$ (4)

$y_2(t,\propto)\ I(t)=\int Q(t)\ I(t)dt\ +C_2$ (5)

at $t=t_0$, $C_1=y_1(t_0,\propto)\ I(t_0)-(\int Q(t)\ I(t)dt\ )_{t=t_0}$ and

$C_2=y_2(t_0,\propto)\ I(t_0)-(\int Q(t)\ I(t)dt\ )_{t=t_0}$

and from equation (4) and (5) we get

$y_1(t,\propto)\ =\left(I(t)\right)^{-1}\{\int Q(t)\ I(t)dt\ +(a_1\ +\alpha\mathcal{L})\ I(t_0)-(\int Q(t)\ I(t)dt\ )_{t=t_0}\}$ (6)

$y_2(t,\propto)\ =\left(I(t)\right)^{-1}\{\int Q(t)\ I(t)dt\ +(a_3\ -\alpha\mathcal{R})\ I(t_0)-(\int Q(t)\ I(t)dt\ )_{t=t_0}\}$ (7)

are satisfying a condition of strong solution i.e. $\frac{\partial y_1(t,\alpha)}{\partial\alpha}=\mathcal{L}I(t_0)\left(I(t)\right)^{-1}>0$

and $\frac{\partial y_2(t,\alpha)}{\partial\alpha}=-\mathcal{R}I(t_0)\left(I(t)\right)^{-1}<0$.

Hence, based on α-cut interval arithmetic, the solution of differential equation (1) as follows $y(t)=[y_1(t,\propto),\ \ y_1(t,\propto),\ \ y_2(t,\propto),\ y_2(t,\propto)]$

$=\left(I(t)\right)^{-1}\{\int Q(t)\ I(t)dt\ +y(t_0)\,I(t_0)-(\int Q(t)\ I(t)dt\ )_{t=t_0}\}$ (8)

Case-II: If $P(t)$ is real valued function and $Q(t)$ is fuzzy function

Let $Q(t)$ is fuzzy valued function such that

$inf.Q(t)=Q_1(t)$ and $sup.Q(t)=Q_2(t)$

Then, α-cut of differential equation (1) is

$$\left[\frac{dy_1(t,\propto)}{dt},\frac{dy_2(t,\propto)}{dt}\right]+P(t)[y_1(t,\propto),y_2(t,\propto)]=[Q_1(t,\propto),Q_2(t,\propto)]$$

i.e. $\frac{dy_1(t,\propto)}{dt}+P(t)y_1(t,\propto)=Q_1(t,\propto)$ (9)

$\frac{dy_2(t,\propto)}{dt}+P(t)y_2(t,\propto)=Q_2(t,\propto)$ (10)

Since $P(t)$ is real valued function, the integrating factor is $I(t)=e^{\int P(t)dt}$ And solutions are

$y_1(t,\propto)\ I(t)=\int Q_1(t,\propto)\ I(t)dt\ +C_1$ (11)

$y_2(t,\propto)\ I(t)=\int Q_2(t,\propto)\ \ I(t)dt\ +C_2$ (12)

at $t=t_0$, $C_1=y_1(t_0,\propto)\ I(t_0)-(\int Q_1(t,\propto)\ I(t)dt\ )_{t=t_0}$ and

$C_2 = y_2(t_0,\propto)\ I(t_0) - (\int Q_2(t,\propto)\ I(t)dt\ )_{t=t_0}$

and from equation (11) and (12) we get

$$y_1(t,\propto)\ = \left(I(t)\right)^{-1}\left\{\int Q_1(t,\propto)\ I(t)dt\ + (a_1\ + \alpha\mathcal{L})\ I(t_0) - (\int Q_1(t,\propto)\ I(t)dt\ )_{t=t_0}\right\} \quad (13)$$

$$y_2(t,\propto)\ = \left(I(t)\right)^{-1}\left\{\int Q_2(t,\propto)\ I(t)dt\ + (a_3\ - \alpha\mathcal{R})\ I(t_0) - (\int Q_2(t,\propto)\ I(t)dt\ )_{t=t_0}\right\} \quad (14)$$

are satisfying a condition of strong solution i.e. $\frac{\partial y_1(t,\alpha)}{\partial\alpha} > 0$ and $\frac{\partial y_2(t,\alpha)}{\partial\alpha} < 0$.

Hence, based on α-cut interval arithmetic, the solution of differential equation (1) as follows $y(t) = [y_1(t,\propto)\ ,\quad y_1(t,\propto),\quad y_2(t,\propto),\ y_2(t,\propto)]$

$$= \left(I(t)\right)^{-1}\left\{\int [Q_1(t), Q_2(t)\ ]\ I(t)dt\ + y(t_0)\, I(t_0) - (\int [Q_1(t), Q_2(t)\ ]\ \ I(t)dt\ )_{t=t_0}\right\} \quad (15)$$

Where, $Q_1(t) = Min\{Q_1(t,0), Q_1(t,1), Q_2(t,0), Q_2(t,1)\}$ and

$Q_2(t) = Max\{Q_1(t,0), Q_1(t,1), Q_2(t,0), Q_2(t,1)\}$

Case-III: If $P(t)$ is fuzzy function and $Q(t)$ is real function

Let $P(t)$ is fuzzy valued function such that $inf.P(t) = P_1(t)$ and $sup.P(t) = P_2(t)$

Then, α-cut of differential equation (1) is

$$\left[\frac{dy_1(t,\propto)}{dt}, \frac{dy_2(t,\propto)}{dt}\right] + [P_1(t,\propto), P_2(t,\propto)]\ [y_1(t,\propto), y_2(t,\propto)] = Q(t)$$

i.e. $$\frac{dy_1(t,\propto)}{dt} + P_1(t,\propto)y_1(t,\propto) = Q(t) \quad (16)$$

$$\frac{dy_2(t,\propto)}{dt} + P_2(t,\propto)y_2(t,\propto) = Q(t) \quad (17)$$

Since $P(t)$ is fuzzy valued function, the integrating factors are $I_1(t,\propto) = e^{\int P_1(t,\propto)dt}$ and $I_2(t,\propto) = e^{\int P_2(t,\propto)dt}$ and hence solutions are

$$y_1(t,\propto)\ I_1(t,\propto) = \int Q(t)\ \ I_1(t,\propto)dt\ + C_1 \quad (18)$$

$$y_2(t,\propto)\ I_2(t,\propto) = \int Q(t)\ \ I_2(t,\propto)dt\ + C_2$$
(19)

at $t = t_0$, $C_1 = y_1(t_0,\propto)\ \ I_1(t_0,\propto) - (\int Q(t)\ \ I_1(t,\propto)dt\ )_{t=t_0}$ and

$C_2 = y_2(t_0,\propto)\, I_2(t_0,\propto) - (\int Q(t)\ \ I_2(t,\propto)dt\ )_{t=t_0}$

and from equation (18) and (19) we get

$$y_1(t,\propto)\ = \left(I_1(t,\propto)\right)^{-1}\left\{\int Q(t)\ I_1(t,\propto)dt\ + (a_1\ + \alpha\mathcal{L})\ I_1(t_0\,,\propto) - (\int Q(t)\ \ I_1(t,\propto)dt\ )_{t=t_0}\right\}$$
(20)

$$y_2(t,\propto)\ = \left(I_2(t,\propto)\right)^{-1}\left\{\int Q(t)\, I_2(t,\propto)dt\ + (a_3\ - \alpha\mathcal{R})\, I_2(t_0\,,\propto) - (\int Q(t)\ I_2(t,\propto)dt\ )_{t=t_0}\right\}$$
(21)

are satisfying a condition of strong solution i.e. $\frac{\partial y_1(t,\alpha)}{\partial \alpha} > 0$ and $\frac{\partial y_2(t,\alpha)}{\partial \alpha} < 0$.

Hence, based on α-cut interval arithmetic, the solution of differential equation (1) as follows $y(t) = [y_1(t,\propto),\ \ y_1(t,\propto),\ \ y_2(t,\propto),\ y_2(t,\propto)]$

$$= \left(I(t,\propto)\right)^{-1}\left\{\int Q(t)\ I(t,\propto)dt\ + y(t_0)\, I(t_0,\propto) - (\int Q(t)\, I(t,\propto)dt\,)_{t=t_0}\right\} \tag{22}$$

Where, $\left(I(t,\propto)\right) = \left[\left(I_1(t,\propto)\right), \left(I_2(t,\propto)\right)\right]$

Here, $I_1(t) = Min\{I_1(t,0), I_1(t,1), I_2(t,0), I_2(t,1)\}$ and

$I_2(t) = Max\{I_1(t,0), I_1(t,1), I_2(t,0), I_2(t,1)\}$

Case-IV: If $P(t)$ and $Q(t)$ are fuzzy function

Let $Q(t)$ is fuzzy valued function such that $inf.Q(t) = Q_1(t)$ and $sup.Q(t) = Q_2(t)$

Let $P(t)$ is fuzzy valued function such that $inf.P(t) = P_1(t)$ and $sup.P(t) = P_2(t)$

Then, α-cut of differential equation (1) is

$$\left[\frac{dy_1(t,\propto)}{dt}, \frac{dy_2(t,\propto)}{dt}\right] + [P_1(t,\propto), P_2(t,\propto)]\,[y_1(t,\propto), y_2(t,\propto)] = [Q_1(t,\propto), Q_2(t,\propto)]$$

i.e. $$\frac{dy_1(t,\propto)}{dt} + P_1(t,\propto)y_1(t,\propto) = Q_1(t,\propto) \tag{23}$$

$$\frac{dy_2(t,\propto)}{dt} + P_2(t,\propto)y_2(t,\propto) = Q_2(t,\propto) \tag{24}$$

Since $P(t)$ is fuzzy valued function, the integrating factors are $I_1(t,\propto) = e^{\int P_1(t,\propto)dt}$ and $I_2(t,\propto) = e^{\int P_2(t,\propto)dt}$ and hence solutions are

$$y_1(t,\propto)\ I_1(t,\propto) = \int Q_1(t,\propto)\ I_1(t,\propto)dt\ + C_1 \tag{25}$$

$$y_2(t,\propto)\ I_2(t,\propto) = \int Q_2(t,\propto)\ I_2(t,\propto)dt\ + C_2 \tag{26}$$

at $t = t_0$, $C_1 = y_1(t_0,\propto)\ \ I_1(t_0,\propto) - (\int Q_1(t,\propto)\, I_1(t,\propto)dt\,)_{t=t_0}$ and

$C_2 = y_2(t_0,\propto)\, I_2(t_0,\propto) - (\int Q_2(t,\propto)\ I_2(t,\propto)dt\,)_{t=t_0}$

and from equation (18) and (19) we get

$$y_1(t,\propto)\ = \left(I_1(t,\propto)\right)^{-1}\left\{\int Q_1(t,\propto)\ I_1(t,\propto)dt\ + (a_1\ + \alpha\mathcal{L})\ I_1(t_0,\propto) - (\int Q_1(t,\propto)\ \ I_1(t,\propto)dt\,)_{t=t_0}\right\} \tag{27}$$

$$y_2(t,\propto)\ = \left(I_2(t,\propto)\right)^{-1}\left\{\int Q_2(t,\propto)\, I_2(t,\propto)dt\ + (a_3\ - \alpha\mathcal{R})\, I_2(t_0,\propto) - (\int Q_2(t,\propto)\, I_2(t,\propto)dt\,)_{t=t_0}\right\} \tag{28}$$

are satisfying a condition of strong solution i.e. $\frac{\partial y_1(t,\alpha)}{\partial \alpha} > 0$ and $\frac{\partial y_2(t,\alpha)}{\partial \alpha} < 0$.

Hence, based on α-cut interval arithmetic, a solution of differential equation (1) is as follows $y(t) = [y_1(t,\propto),\ \ y_1(t,\propto),\ \ y_2(t,\propto),\ y_2(t,\propto)]$

$$= \left(I(t,\propto)\right)^{-1}\left\{\int Q(t)\ I(t,\propto)dt\ + y(t_0)\, I(t_0,\propto) - (\int Q(t)\, I(t,\propto)dt\,)_{t=t_0}\right\} \tag{29}$$

here, $\left(I(t,\propto)\right)=\left[\left(I_1(t,\propto)\right),\left(I_2(t,\propto)\right)\right]$ $\quad Q(t)=[Q_1(t),Q_2(t)]$

Here, $I_1(t)=Min\{I_1(t,0),I_1(t,1),I_2(t,0),I_2(t,1)\}$,

$I_2(t)=Max\{I_1(t,0),I_1(t,1),I_2(t,0),I_2(t,1)\}$,

$Q_1(t)=Min\{Q_1(t,0),Q_1(t,1),Q_2(t,0),Q_2(t,1)\}$ and

$Q_2(t)=Max\{Q_1(t,0),Q_1(t,1),Q_2(t,0),Q_2(t,1)\}$.

Solved Example:

Ex.: Consider non homogeneous differential equation $\frac{dy(t)}{dt}+2y(t)=t$ with starting condition

$y_0=y(t_0)=y(0)=[0,1,2]$ is TFN.

Here $P(t)=2$ and $Q(t)=t$ are real valued functions

Then, an integrating factor is $I(t)=e^{\int 2dt}=e^{2t}$ and at $t=t_0=0,\ I(t_0)=e^0=1$

and hence its solution from (8) is

$$y(t)=\left(I(t)\right)^{-1}\left\{\int Q(t)\ I(t)dt\ +y(t_0)\,I(t_0)-\left(\int Q(t)\ I(t)dt\ \right)_{t=t_0}\right\}$$

$$=\ e^{-2t}\{\int t\ e^{2t}dt\ +[0,1,2]\ -\left(\int t\ e^{2t}dt\ \right)_{t=0}\}$$

$$=\ e^{-2t}\left\{\left(\frac{t}{2}-\frac{1}{4}\right)\ e^{2t}\ +[0,1,2]\ -\left(\left(\frac{t}{2}-\frac{1}{4}\right)\ e^{2t}\ \right)_{t=0}\right\}$$

$$=\ e^{-2t}\left\{\left(\frac{t}{2}-\frac{1}{4}\right)\ e^{2t}\ +[0,1,2]+\frac{1}{4}\right\}$$

$$=\ e^{-2t}\left\{\left(\frac{t}{2}-\frac{1}{4}\right)\ e^{2t}\ +\left[\frac{1}{4},\frac{5}{4},\frac{9}{4}\right]\right\}$$ is the required solution

4. Conclusion

This paper promotes a solution of Nonhomogeneous First Order Linear Fuzzy Differential Equation with starting value condition as Triangular Fuzzy Number. Here, we have considered four different possible cases of functions involved in differential equations. In this paper we solve a random example with triangular fuzzy number as initial condition and obtained a strong solution by case-I. Here we propose that for other cases of functions involved in differential equations, the method is appropriate and may get strong solution. Further we propose to solve first order Nonhomogeneous Differential Equation under different fuzzy numbers as Initial value

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