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Application of Neutrosophic Sets in Multi Attribute Decision Making

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Abstract: The selection of an optimal hospital for healthcare services is a complex decision-making problem due to the involvement of multiple, often conflicting, criteria such as quality of care, infrastructure, staff expertise, cost, and patient satisfaction. This paper presents an innovative approach for hospital selection using a hybrid decision-making framework combining neutrosophic sets, entropy weight, and the Multi-Attribute Decision-Making (MADM) method. Neutrosophic sets are employed to handle the inherent uncertainty and indeterminacy in hospital evaluation criteria, allowing for more flexible and accurate representation of data that may not be fully precise or certain. The entropy weight method is applied to objectively determine the significance of each evaluation criterion, ensuring that the decision process is not biased by subjective judgments. Finally, the MADM technique integrates the weighted criteria into a comprehensive decision model to rank hospitals based on their overall performance. The proposed model is tested with a real-world dataset of hospitals, demonstrating its effectiveness in selecting the most suitable healthcare facility while considering diverse factors. The results highlight the potential of combining neutrosophic sets with advanced decision-making techniques for improving decision accuracy and reliability in complex healthcare decisions.

Keywords: Neutrosophic Sets; Entropy; MADM.

1. Introduction

Atanassov (1986, 2000) first proposed the use of intuitionistic fuzzy sets as a means of representation of data information that makes use of ordered pairs, i.e., 2-tuples, which may account for the membership and the non-membership degrees. According to Atanassov (1989), the sum of the membership and non-membership degrees of each 2-tuple in an IFS must be less than or equal to 1. Although fuzzy sets (FSs), first proposed by Zadeh in 1965, have shown promise in various domains, accurately evaluating their membership functions in practice can be difficult, since they are constrained to real values between zero and one. To overcome this difficulty, Atanassov and Gargov (1989) introduced interval-valued intuitionistic fuzzy sets (IVIFS) by extending the concept to represent the membership and non-membership degrees using intervals instead of precise values. This way is very useful in the complex environment, owing to a lot of facilitation in improving the flexibility of fuzzy sets, which was proposed to deal with ambiguity and uncertainty (Zhou et al., 2014a; Meng and Chen, 2015; Wu and Chiclana, 2014; Onar et al., 2015; Hu et al., 2015; Zhou et al., 2016).

IFSs and IVIFSs can deal with incomplete information. However, the information at the hands of a decision-maker is often fuzzy and inconsistent, and IFSs and IVIFSs are incapable of dealing with it;

therefore, Smarandache (1999, 2003) developed the neutrosophic sets (NSs), primarily by a philosophical perspective. NSs simultaneously consider truth membership, indeterminacy membership, and falsity membership, each independent of the others. Despite their theoretical attractiveness, NSs remain challenging to be applied in practical science and engineering applications. There are three significant branches of fuzzy theory research: entropy, similarity measures, and cross-entropy. They have a lot of broad applications in medical diagnosis, information fusion systems, and so on. Entropy is one of the important elements to quantify uncertain information, which attracts much attention since its emergence (Wei et al., 2011). The concept of fuzzy entropy was first proposed by Zadeh in 1968 to quantify fuzziness in information for decision-making. Luca and Termini (1972) afterwards gave a formal definition of the entropy of a FS and pointed out the axioms that fuzzy entropy should fulfill. Szmidt and Kacprzyk (2001) proposed a non-probabilistic entropy measure for IFSs. The idea was to use the ratio of intuitionistic fuzzy cardinalities in defining the axiomatic conditions for intuitionistic fuzzy entropy. Ye created an entropy-weighted approach to calculate entropy weights and suggested two entropy measures for interval-valued intuitionistic fuzzy sets (IVIFSs) in 2010. Jin et al. (2014) investigated an interval-valued intuitionistic fuzzy continuous weighted entropy to solve multi-criteria decision-making (MADM) problems using the continuous ordered weighted averaging (COWA) operator. An entropy function was created by Majumdar and Samanta (2014) to quantify the degree of uncertainty in a single-valued neutrosophic set (SVNV).

Similarity measures and cross-entropy are mainly used to evaluate the discrimination information. Great amounts of work have been carried out for such topics (Grzegorzewski, 2004; Zhou and Chen, 2013; Hung and Yang, 2007; Zhou et al., 2014b; Ye, 2017). Liu (1992) proposed axiomatic definitions for entropy, distance measures and similarity measures of FSs, and revealed their basic interrelations deeply. Vlachos and Sergiadis (2007) firstly proposed the concept of intuitionistic fuzzy cross-entropy and investigated the relationships of cross-entropy and entropy. Beliakov et al. (2014) proposed a novel method to define the similarity measures for IFSs, under which each similarity measure consists of two parts: one representing the similarity aspect and another handling hesitancy. Based on Jaccard, Dice, and cosine similarity measures in vector space, Ye (2014a) presented three vector similarity measures for SVNSs to determine the ranking of alternatives in MADM problems. With a view to overcoming the limitation of the similarity measure in Ye (2014a), Ye (2015) developed modified cosine similarity measures for SVNSs based on the cosine function. Based on the distance between two SVNSs, Majumdar and Samanta (2014) proposed some similarity measures for SVNSs and discussed their properties. In a single-valued neutrosophic environment, Ye (2014b) initiated a cross-entropy method to determine MADM methods. The relation among entropy, similarity measures, and cross-entropy has gained much attention. Zhang et al. (2009) and Zeng and Li (2006) proved that the entropy and similarity measures of interval-valued fuzzy sets (IVFSs) can be transformed into each other

The process of selecting the best hospital is a multifaceted decision that demands careful consideration of various factors, such as medical expertise, infrastructure, patient care quality, and operational efficiency. Given the complexities involved in hospital assessment, decision-makers often face significant challenges in making an informed choice. Traditional methods of hospital selection primarily rely on qualitative assessments or limited quantitative data, which may not fully capture the underlying uncertainties and subjectivity inherent in the evaluation process.

To address these challenges, this paper proposes an advanced decision-making framework that combines Neutrosophic Set Theory, Entropy Weighting, and Multi-Attribute Decision-Making (MADM) techniques to facilitate a more accurate and robust hospital selection process. Neutrosophic Set Theory offers a flexible and powerful way to model and manage imprecision, inconsistency, and indeterminacy in hospital performance data. The Entropy Weighting method is introduced to objectively assign weights to different selection criteria based on their importance, ensuring that the decision-making process aligns with real-world priorities. Finally, the MADM technique is applied to evaluate and rank hospitals based on the aggregated performance across multiple dimensions. This integrated approach is designed to provide healthcare decision-makers with a reliable tool for choosing the best hospital, offering a more comprehensive and systematic analysis than traditional methods.

The significance of this approach lies in its ability to combine theoretical robustness with practical applicability, addressing key limitations of existing methods in hospital selection. By incorporating these advanced techniques, we aim to deliver a framework that can be applied to various healthcare settings, ensuring that patients and stakeholders make well-informed decisions in selecting healthcare providers.

Entropy Weight Method:

The Entropy Weight method is a well-established technique used to assign weights to different criteria based on their level of information or uncertainty. The concept of entropy comes from information theory and measures the level of disorder or uncertainty in a system. In the entropy weighting method, the entropy of each criterion is computed, and the weights are assigned inversely proportional to the entropy values. Criteria with higher uncertainty or variability are assigned lower weights, while those with more stable or consistent data receive higher weights. This method is particularly useful for ensuring that the most informative and significant criteria are given more emphasis in the decision-making process.

Multi-Attribute Decision-Making (MADM) Methods:

A collection of decision-making strategies known as MADM approaches seeks to assess options according to a number of factors. These techniques are especially helpful when decision-makers are faced with a range of options, each of which performs differently across a number of qualities. Among the popular MADM techniques are the Analytical Hierarchy Process (AHP), the Weighted Sum Model (WSM), and the Weighted Product Model (WPM). Using entropy-based weights and neutrosophic set representations, we employ a MADM technique in this study to aggregate hospital performance across multiple parameters, offering a thorough assessment and ranking of options.

Together, these techniques create a robust decision-making framework that is capable of handling the uncertainty, imprecision, and complexity inherent in hospital selection, while offering a systematic approach to identify the best hospital based on a set of predefined criteria.

Definition 1.1.1. In 1998, Smarandache published the single-valued Neutrosophic Set (NS) [38, 39, 40]. Assume that U is a discourse universe and that $A_{NS} \subset U$. Then, A_{NS}={ $<x, \mathcal{T}_A(x)), \mathcal{I}_A(x), \mathcal{F}_A(x)>|$

x ∊ U}, where $\mathcal{T}_A(x)$, $\mathcal{I}_A(x)$ and $\mathcal{F}_A(x)$: U → [0, 1] stand for the degrees of membership, indeterminacy, and non-membership, respectively, with $0 \le \mathcal{T}_A(x)\,(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \le 3$.

2. Operators in Neutrosophic Sets

Definition 2.1: Let γ, δ are two NSs. Then operational relations are defined as,

1. $\gamma \oplus \delta = <\mathcal{T}_\gamma(x) + \mathcal{T}_\delta(z) - \mathcal{T}_\gamma(x) \,.\, \mathcal{T}_\delta(z), \mathcal{I}_\gamma(x) \,.\, \mathcal{I}_\delta(z), \mathcal{F}_\gamma(x) \,.\, \mathcal{F}_\delta(z)>$.
2. $\gamma \otimes \delta = < \mathcal{T}_\gamma(x).\mathcal{T}_\delta(z), \mathcal{I}_\gamma(x) + \mathcal{I}_\delta(z) - \mathcal{I}_\gamma(x) \,.\, \mathcal{I}_\delta(z), \mathcal{F}_\gamma(x) + \mathcal{F}_\delta(z) - \mathcal{F}_\gamma(x) \,.\, \mathcal{F}_\delta(z) >$
3. $\lambda.\gamma = < 1 - \left(1 - \mathcal{T}_\gamma(x)\right)^\lambda, \left(\mathcal{I}_\gamma(x)\right)^\lambda, \left(\mathcal{F}_\gamma(x)\right)^\lambda >, where\ \lambda > 0.$
4. $\gamma^{\lambda} = < (\mathcal{T}_\gamma(x))^\lambda, 1 - (1 - \mathcal{I}_\gamma(x))^\lambda, 1 - \left(1 - \mathcal{F}_\gamma(z)\right)^\lambda >, where\ \lambda > 0.$

Theorem 2.2: Let $\gamma_i = (\mathcal{T}_\gamma(x_i), \mathcal{I}_\gamma(x_i), \mathcal{F}_\gamma(x_i)), i = 1,2,3.\dots n$ be a collection of NS, then Neutrosophic Weighted Aggregation (NWA) operator value is also a NS and $NSWA(\gamma_1, \gamma_2, \gamma_3 \,.\,.\,.\, \gamma_n) = \left(1 - \prod_{i,j=1}^{n}\left(1 - \mathcal{T}_\gamma(x_i)\right)^{w_j}, \ \prod_{i,j=1}^{n}\left(\mathcal{I}_\gamma(x_i)\right)^{w_j}, \prod_{i,j=1}^{n}\left(\mathcal{F}_\gamma(x_i)\right)^{w_j}\right)$ …………….. (1) where $w = (w_1, w_2, w_3, \dots\ w_n)^T$ be the weight vector of $\gamma_j (j = 1,2,3, \dots.n)$ and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Proof. We prove Equation (1) by mathematical induction on n.

If n = 2, then we have $NWA(\gamma_1, \gamma_2) = w_1\gamma_1 \oplus w_2\gamma_2$

By Definition 2.1, we can see that both $w_1\gamma_1\ and\ w_2\gamma_2$ are NS and the value of $w_1\gamma_1 \oplus w_2\gamma_2$ is also a NS. From the operational laws of NS, we have

$w_1\gamma_1 = \left(1 - \left(1 - \mathcal{T}_\gamma(x_1)\right)^{w_1}, \left(\mathcal{I}_\gamma(x_1)\right)^{w_1}, \left(\mathcal{F}_\gamma(x_1)\right)^{w_1}\right);$

$w_2\gamma_2 = \left(1 - \left(1 - \mathcal{T}_\gamma(x_2)\right)^{w_2}, \left(\mathcal{I}_\gamma(x_2)\right)^{w_2}, \left(\mathcal{F}_\gamma(x_2)\right)^{w_2}\right)$

Then, $NWA(\gamma_1, \gamma_2) = w_1\gamma_1 \oplus w_2\gamma_2 =< 2 - \left(1 - \mathcal{T}_\gamma(x_1)\right)^{w_1} - \left(1 - \mathcal{T}_\gamma(x_2)\right)^{w_2} -$
$\left(1 - \left(1 - \mathcal{T}_\gamma(x_1)\right)^{w_1}\right)\left(1 - \left(1 - \mathcal{T}_\gamma(x_2)\right)^{w_2}\right), \left(\mathcal{I}_\gamma(x_1)\right)^{w_1}\left(\mathcal{I}_\gamma(x_2)\right)^{w_2}, \left(\mathcal{F}_\gamma(x_1)\right)^{w_1}\left(\mathcal{F}_\gamma\right)(x_2)^{w_2} >$
$=< 1 - \left(1 - \mathcal{T}_\gamma(x_1)\right)^{w_1}\left(1 - \mathcal{T}_\gamma(x_2)\right)^{w_2}, \left(\mathcal{I}_\gamma(x_1)\right)^{w_1}\left(\mathcal{I}_\gamma(x_2)\right)^{w_2},$
$\left(\mathcal{F}_\gamma(x_1)\right)^{w_1}\left(\mathcal{F}_\gamma(x_2)\right)^{w_2} >.$

If n = k, then Equation (1) holds, i.e.,

$NWA(\gamma_1, \gamma_2, \gamma_3 \dots \gamma_n) = w_1\gamma_1 \oplus w_2\gamma_2 \dots \dots \dots \oplus w_k\gamma_k$

$$=< 1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_i)\right)^{wj},\prod_{j=1}^{k}\left(\mathcal{I}_{\gamma}(\mathrm{x}_i)\right)^{wj},\prod_{j=1}^{k}\left(\mathcal{F}_{\gamma}(\mathrm{x}_i)\right)^{wj} >$$

and the aggregated value is a NS, then when n = k + 1, by the operational laws of NS, we have

$N\mathrm{WA}(\gamma_1,\gamma_2,\gamma_3 \dots \gamma_{\mathrm{k}+1}) = w_1\gamma_1 \oplus w_2\gamma_2 \dots \dots \dots \oplus w_k\gamma_k \oplus w_{k+1}\gamma_{k+1}$

$= \langle 1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_i)\right)^{wj} + \left(1-\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_{k+1})\right)^{w_{k+1}}\right) - \left(1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_i)\right)^{wj}\right)\Big(1-$
$\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_{k+1})\right)^{w_{k+1}}\Big), \prod_{j=1}^{k+1}\left(\mathcal{I}_{\gamma}(\mathrm{x}_i)\right)^{wj}, \prod_{j=1}^{k+1}(\mathcal{F}_{\gamma})(\mathrm{x}_i))^{wj}\rangle$

$=< 1-\prod_{j=1}^{k+1}\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_i)\right)^{wj}, \prod_{j=1}^{k+1}\left(\mathcal{I}_{\gamma}(\mathrm{x}_i)\right)^{wj}, \prod_{j=1}^{k+1}\left(\mathcal{F}_{\gamma}(\mathrm{x}_i)\right)^{wj} >.$

By which aggregated value is also a NS, Therefore, when n = k + 1, Equation (1) holds. Thus, by steps 1 and 2, we recognize that Equation (1) sustain for all n.

Theorem 2.3: Let $\gamma_i = (\mathcal{T}_{\gamma}(x_i),\ \mathcal{I}_{\gamma}(x_i), \mathcal{F}_{\gamma}(x_i)), i = 1,2,3.\dots n$ be a collection of NS, then Neutrosophic Weighted geometric (NWG) operator value is also a NS and

$N\mathrm{WG}(\gamma_1,\gamma_2,\gamma_3 \ . \ . \ . \gamma_n) =$

$\left(\prod_{i,j=1}^{n}(\mathcal{T}_{\gamma}(x_i))^{w_j},\ 1-\prod_{i,j=1}^{n}(1-\mathcal{I}_{\gamma}(x_i))^{w_j},\ \prod_{i,j=1}^{n}(1-\mathcal{F}_{\gamma}(x_i))^{w_j}\right)$ …………….. (2)

where $w = (w_1, w_2, w_3, \dots\ w_n\ )^T$ be the weight vector of $\gamma_j (j = 1,2,3, \dots . n)$ and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Proof. We prove Equation (1) by mathematical induction on n.

If n = 2, then we have $N\mathrm{WG}(\gamma_1,\gamma_2) = w_1\gamma_1 \otimes w_2\gamma_2$

By Definition 2.1, we can see that both $w_1\gamma_1\ and\ w_2\gamma_2$ are NS and the value of $w_1\gamma_1 \oplus w_2\gamma_2$ is also a NS. From the operational laws of NS, we have

$w_1\gamma_1 = \left(\left(\mathcal{T}_{\gamma}(\mathrm{x}_1)\right)^{w_1}, 1-\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_1)\right)^{w_1}, 1-\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_1)\right)^{w_1}\right);$

$w_2\gamma_2 = \left(\left(\mathcal{T}_{\gamma}(\mathrm{x}_2)\right)^{w_2}, 1-\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_2)\right)^{w_2}, 1-\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_2)\right)^{w_2}\right)$

Then, $N\mathrm{WG}(\gamma_1,\gamma_2)\ = w_1\gamma_1 \otimes w_2\gamma_2 =< \left(\mathcal{T}_{\gamma}(\mathrm{x}_1)\right)^{w_1} (\mathcal{T}_{\gamma})(\mathrm{x}_2))^{w_2},\ 2-\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_1)\right)^{w_1} -$
$\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_2)\right)^{w_2} - \left(1-\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_1)\right)^{w_1}\right)\left(1-\left(1-\mathcal{I}_{\gamma}(\mathrm{x}_2)\right)^{w_2}\right), 2-\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_1)\right)^{w_1} -$
$\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_2)\right)^{w_2} - \left(1-\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_1)\right)^{w_1}\right)\left(1-\left(1-\mathcal{F}_{\gamma}(\mathrm{x}_2)\right)^{w_2}\right) >$
$=< \left(\mathcal{T}_{\gamma}(\mathrm{x}_1)\right)^{w_1}\left(\mathcal{T}_{\gamma}(\mathrm{x}_2)\right)^{w_2}, 1-\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_1)\right)^{w_1}\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_2)\right)^{w_2}, 1-\left(1-\mathcal{T}_{\gamma}(\mathrm{x}_2)\right)^{w_1}\Big(1-$
$\mathcal{T}_{\gamma}(\mathrm{x}_2)\Big)^{w_2} >$.

If n = k, then Equation (1) holds, i.e.,

$NWG(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_n) = w_1\gamma_1 \otimes w_2\gamma_2 \ldots \ldots \ldots \otimes w_k\gamma_k$

$$=< \prod_{j=1}^{k} \left(\mathcal{T}_\gamma(x_i)\right)^{wj}, 1 - \prod_{j=1}^{k} \left(1 - \mathcal{I}_\gamma(x_i)\right)^{wj}, 1 - \prod_{j=1}^{k} \left(1 - \mathcal{F}_\gamma(x_i)\right)^{wj} >$$

and the aggregated value is a NSHSS, then when n = k + 1, by the operational laws of NSHSS, we have

$NWA(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_{k+1}) = w_1\gamma_1 \otimes w_2\gamma_2 \ldots \ldots \ldots \otimes w_k\gamma_k \otimes w_{k+1}\gamma_{k+1}$

$= \langle \prod_{j=1}^{k+1} \left(\mathcal{T}_\gamma(x_i)\right)^{wj}, 1 - \prod_{j=1}^{k} \left(1 - \mathcal{I}_\gamma(x_i)\right)^{wj} + \left(1 - \left(1 - \mathcal{I}_\gamma(x_{k+1})\right)^{w_{k+1}}\right) - \Big(1 - \prod_{j=1}^{k} \left(1 - \mathcal{I}_\gamma(x_i)\right)^{wj}\Big)\left(1 - \left(1 - \mathcal{I}_\gamma(x_{k+1})\right)^{w_{k+1}}\right), 1 - \prod_{j=1}^{k} \left(1 - \mathcal{F}_\gamma(x_i)\right)^{wj} + \Big(1 - \left(1 - \mathcal{F}_\gamma(x_{k+1})\right)^{w_{k+1}}\Big) - \left(1 - \prod_{j=1}^{k} \left(1 - \mathcal{F}_\gamma(x_i)\right)^{wj}\right)\left(1 - \left(1 - \mathcal{F}_\gamma(x_{k+1})\right)^{w_{k+1}}\right)\rangle$

$=< \prod_{j=1}^{k+1} \left(\mathcal{T}_\gamma(x_i)\right)^{wj}, 1 - \prod_{j=1}^{k+1} \left(1 - \mathcal{I}_\gamma(x_i)\right)^{wj}, 1 - \prod_{j=1}^{k+1} \left(1 - \mathcal{F}_\gamma(x_i)\right)^{wj} >.$

By which aggregated value is also a NS, Therefore, when n = k + 1, Equation (1) holds. Thus, by steps 1 and 2, we recognize that Equation (1) sustain for all n.

Definition 2.3. Let $\gamma_i = \{ (\mathcal{T}(x_i), \mathcal{I}(x_i), \mathcal{F}(x_i)) | x_i \in X\}$ be a NS set on X. Then the entropy of γ_i is defined as,

$E(\gamma_i) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_\gamma(x_i) + \mathcal{I}^2{}_\gamma(x_i)) \left|\left(4\mathcal{F}^2{}_\gamma(x_i) - 1\right)\right| + (\mathcal{T}^2{}_\gamma(x_i) + \mathcal{F}^2{}_\gamma(x_i))\left|\left(4\mathcal{I}^2{}_\gamma(x_i) - 1\right)\right| + (\mathcal{F}^2{}_\gamma(x_i) + \mathcal{I}^2{}_\gamma(x_i))\left|(4\mathcal{T}^2{}_\gamma(x_i) - 1)\right|) \quad \ldots\ldots (2)$

Theorem 2.4. The proposed entropy on NS(X) fulfilling the conditions:

1. $E(\gamma_i) = 0$, if γ is a crisp set ie., $\gamma_i = \left(\mathcal{T}_{\gamma(y)}(x_i), \mathcal{I}_{\gamma(y)}(x_i), \mathcal{F}_{\gamma(y)}(x_i)\right) = (1,0,0)$ or $\gamma_i = \left(\mathcal{T}_\gamma(x_i), \mathcal{I}_\gamma(x_i), \mathcal{F}_\gamma(x_i)\right) = (0,0,1)$ for all $x_i \in X$.
2. $E(\gamma_i) = 1$, if $\gamma_i = \{\left(x_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) | x_i \in X\}$
3. $E(\gamma_i) = E(\gamma_i{}^C)$, for all $\gamma_i \in NSHSS(X)$;
4. $E(\gamma_i) \le E(\delta_i)$ if either $\mathcal{T}_\gamma(x_i) \le \mathcal{T}_\delta(x_i), \mathcal{I}_\gamma(x_i) \le \mathcal{I}_\delta(x_i), \mathcal{F}_\gamma(x_i) \le \mathcal{F}_\delta(x_i)$ when $max(\mathcal{T}_\delta(x_i), \mathcal{I}_\delta(x_i), \mathcal{F}_\delta)(x_i)) \le \frac{1}{2}$ *or* $\mathcal{T}_\gamma(x_i) \ge \mathcal{T}_\delta(x_i)$, $\mathcal{I}_\gamma(x_i) \ge \mathcal{I}_\delta(x_i), \mathcal{F}_\gamma(x_i) \ge \mathcal{F}_\delta(x_i)$ when $min\left(\mathcal{T}_\delta(x_i), \mathcal{I}_\delta(x_i), \mathcal{F}_\delta(x_i)\right) \ge \frac{1}{2}$.

Proof:

1. Let $\gamma_i = \left(\mathcal{T}_{\gamma(y)}(x_i), \mathcal{I}_{\gamma(y)}(x_i), \mathcal{F}_{\gamma(y)}(x_i)\right) = (1,0,0)$ for all $x_i \in X$. Then,

$E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{F}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{F}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{I}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|(4\mathcal{T}^2{}_\gamma(\mathrm{x}_i) - 1)\right|)$

$$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((1^2+0)|(4\times 0-1)| + (1^2+0)|(4\times 0-1)| + (0+0)|(4\times 1-1)|)$$

$$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m} 2 = 1 - \frac{1}{2m}(2m) = 1 - 1 = 0$$

Similarly, If $\gamma_{\mathrm{i}} = \left(\mathcal{T}_\gamma(\mathrm{x}_i), \mathcal{I}_\gamma(\mathrm{x}_i), \mathcal{F}_\gamma(\mathrm{x}_i)\right) = (0,1,0)$ and $\left(\mathcal{T}_\gamma(\mathrm{x}_i), \mathcal{I}_\gamma(\mathrm{x}_i), \mathcal{F}_\gamma(\mathrm{x}_i)\right) = (0,0,1)$

$\forall x_i \in X$, then $E(\gamma_{\mathrm{i}}) = 0$.

2. Let $\gamma_{\mathrm{i}} = \left\{\left(x_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \middle| x_i \in X\right\}$. Then

$E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{F}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{F}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{I}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|(4\mathcal{T}^2{}_\gamma(\mathrm{x}_i) - 1)\right|)$

$$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}\left(\left(\frac{1}{2^2}+\frac{1}{2^2}\right)\right)\left|\left(4\left(\frac{1}{2^2}\right)-1\right)\right| + \left(\frac{1}{2^2}+\frac{1}{2^2}\right)\left|\left(4\left(\frac{1}{2^2}\right)-1\right)\right| + \left(\frac{1}{2^2}+\frac{1}{2^2}\right)\left|\left(4\left(\frac{1}{2^2}\right)-1\right)\right|$$

$$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}\left(\left(\frac{1}{4}+\frac{1}{4}\right)|(1-1)| + \left(\frac{1}{4}+\frac{1}{4}\right)|(1-1)| + \left(\frac{1}{4}+\frac{1}{4}\right)|(1-1)|\right)$$

$$= 1 - 0 = 1.$$

3. Let $E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{F}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{F}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{I}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|(4\mathcal{T}^2{}_\gamma(\mathrm{x}_i) - 1)\right|)$

$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{T}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{T}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{I}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| + (\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{F}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right|) = E(\gamma_{\mathrm{i}}{}^C)$.

4. Let $\mathcal{T}_\gamma(\mathrm{x}_i) \le \mathcal{T}_\delta(\mathrm{x}_i), \mathcal{I}_\gamma(\mathrm{x}_i) \le \mathcal{I}_\delta(\mathrm{x}_i), \mathcal{F}_\gamma(\mathrm{x}_i) \le \mathcal{F}_\delta(\mathrm{x}_i)$ and

$max\left(\mathcal{T}_\delta(\mathrm{x}_i), \mathcal{I}_\delta(\mathrm{x}_i), \mathcal{F}_\delta(\mathrm{x}_i)\right) \le \frac{1}{2}$.

Then, $1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i))\left|(4\mathcal{F}^2{}_\gamma(\mathrm{x}_i)) - 1)\right| + (\mathcal{T}^2{}_\gamma(\mathrm{x}_i) + \mathcal{F}^2{}_\gamma(\mathrm{x}_i))\left|\left(4\mathcal{I}^2\mathcal{I}_\gamma(\mathrm{x}_i) - 1\right)\right| + \left(\mathcal{F}^2{}_\gamma(\mathrm{x}_i) + \mathcal{I}^2{}_\gamma(\mathrm{x}_i)\right)\left|\left(4\mathcal{T}^2{}_\gamma(\mathrm{x}_i) - 1\right)\right| \le$

$1-\frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_{\delta}(\mathrm{x}_i)+\mathcal{I}^2{}_{\delta}(\mathrm{x}_i))|(4\mathcal{F}^2{}_{\delta}(\mathrm{x}_i)-1)|+(\mathcal{T}^2{}_{\delta}(\mathrm{x}_i)+\mathcal{F}^2{}_{\delta}(\mathrm{x}_i))|(4\mathcal{I}^2{}_{\delta}(\mathrm{x}_i)-1)|+$
$(\mathcal{F}^2{}_{\delta}(\mathrm{x}_i)+\mathcal{I}^2{}_{\delta}(\mathrm{x}_i))|(4\mathcal{T}^2{}_{\delta}(\mathrm{x}_i)-1)|)$

When $max\big(\mathcal{T}_{\delta}(\mathrm{x}_i),\mathcal{I}_{\delta}(\mathrm{x}_i),\mathcal{F}_{\delta}(\mathrm{x}_i)\big)\leq\frac{1}{2}$.

3. Neutrosophic Sets in MADM

The selection of an optimal hospital for healthcare services is a complex decision-making problem due to the involvement of multiple, often conflicting, criteria such as quality of care, infrastructure, staff expertise, cost, and patient satisfaction. This approach neutrosophic methods in handling uncertainty and imprecision through the NS model. The proposed methodology benefits customers by enabling informed choices, making the review process both comprehensive and reliable.

3.1 Algorithm: Multi-Attribute Decision-Making Using NS and Entropy

Step 1: Initialize the Process

To evaluate each criterion, it is necessary to first identify the decision-makers

$\kappa=\{\kappa_1,\kappa_2,.\ldots..\kappa_n\}$ criteria $\nabla=\{\nabla_1,\nabla_2,\ldots..\nabla_n\}$, and alternatives $\Omega=\{\Omega_1,\Omega_2\ldots.,\Omega_n\}$, then gather input data from the decision-makers.

Step 2: Selection of Criteria and Decision Makers

Select the decision-makers $\kappa=\{\kappa_1,\kappa_2,\ldots\ldots_n\}$ who will assess the criteria

$\nabla=\{\nabla_1,\nabla_2,\ldots..\nabla_n\}$ and alternatives after it has been established which criteria bear on the decision-making process.

Step 3: Formulate Propositions Using NSHSS

Utilise attributes and sub-attributes to create power sets of criteria and an NSHSS architecture. The evaluation of each alternative is then represented as follows:

$\gamma_{\mathrm{i}}=\big(\mathcal{T}_{\gamma}(\mathrm{x}_i),\mathcal{I}_{\gamma}(\mathrm{x}_i),\mathcal{F}_{\gamma}(\mathrm{x}_i)\big)$.

Step 4: Linguistic Evaluation of Each Criterion

- Decision-makers' linguistic evaluations for each of the NSs criteria $\nabla=\{\nabla_1,\nabla_2,\ldots..\nabla_n\}$. To denote membership ($\mathcal{T}$), indeterminacy ($\mathcal{I}$) and non-membership ($\mathcal{F}$), propositions are converted into neutrosophic values.

Step 5: Utilising the proposed entropy method, determine the entropy value, weight of the criteria and degree of divergence of each criterion.

Apply the entropy formula to compute the weight wi for each criterion Ci $E(\gamma_{\mathrm{i}})=1-\frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}^2{}_{\gamma}(\mathrm{x}_i)+\mathcal{I}^2{}_{\gamma}(\mathrm{x}_i))\left|\left(4\mathcal{F}^2{}_{\gamma}(\mathrm{x}_i)-1\right)\right|+(\mathcal{T}^2{}_{\gamma}(\mathrm{x}_i)+\mathcal{F}^2{}_{\gamma}(\mathrm{x}_i))\left|\left(4\mathcal{I}^2{}_{\gamma}(\mathrm{x}_i)-1\right)\right|+(\mathcal{F}^2{}_{\gamma}(\mathrm{x}_i)+\mathcal{I}^2{}_{\gamma}(\mathrm{x}_i))\left|(4\mathcal{T}^2{}_{\gamma}(\mathrm{x}_i)-1)\right|)$ such that $\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{w_i}=1$.

Step 6: Aggregate the PFS using NWA

Use NWA_w and NWG_w aggregation operator to combine the evaluations of all criteria for each alternative

$$NWA_w(\alpha_1,\ \alpha_2 \ldots\ldots \alpha_n) = \left(1 - \prod_{i,j=1}^{n}(1 - \mathcal{T}_\alpha(x_i))^{w_j},\ \prod_{i,j=1}^{n}(\mathcal{I}_\alpha(x_i))^{w_j},\ \prod_{i,j=1}^{n}(\mathcal{F}_\alpha(x_i))^{w_j}\right) \qquad \_\_\_\_\_\_\_\_(1)$$

and

$$NWG_w(\alpha_1,\ \alpha_2 \ldots\ldots \alpha_n) = \left(\prod_{i,j=1}^{n}(\mathcal{T}_\alpha(x_i))^{w_j}, 1 - \prod_{i,j=1}^{n}(1 - \mathcal{I}_\alpha(x_i))^{w_j}, 1 - \prod_{i,j=1}^{n}(1 - \mathcal{F}_\alpha(x_i))^{w_j}\right)$$

________(2)

***Step 7*:** Evaluate Alternatives Using a Score Function $S = \frac{2+\mathcal{T}_a-\mathcal{I}_a-\mathcal{F}_a}{3}$ ___________(3)

Step 8: Rank the alternatives $A = \{A_1, A_2, \ldots. A_n\}$ based on their scores.

Step 9: Select the Best Alternative(s)

Step 10: End the Process

3.2 Flow Chart for Multi Attributes Decision Making Using Neutrosophic Set

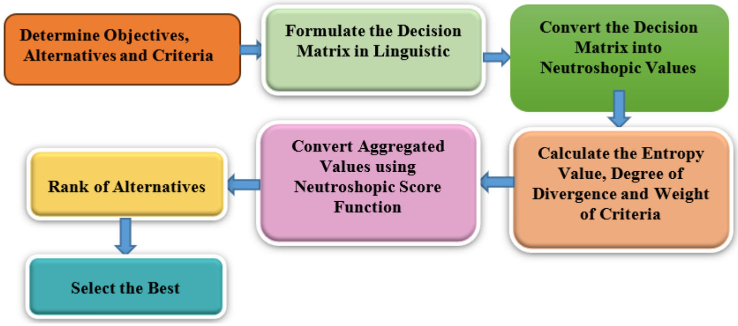

Figure 1: Flow Chart for Multi Attributes Decision Making Using Neutrosophic Set.

Evaluation Criteria:

When selecting the best hospital, multiple factors must be considered to ensure a comprehensive evaluation that meets the needs of patients and healthcare stakeholders. In the context of Multi-Attribute Decision-Making (MADM), these factors (also called criteria) serve as the key decision-making parameters used to evaluate and rank hospitals. MADM methods enable the comparison of alternative hospitals based on several criteria, leading to an informed, systematic, and data-driven decision.

The criteria involved in selecting the best hospital can vary depending on the specific healthcare needs, but some core and commonly used criteria include:

1. Quality of Care (ξ1)

One of the most important criteria in selecting the best hospital is the **quality of care** provided. This encompasses a variety of factors such as the effectiveness of medical treatments, the experience of healthcare professionals, patient safety, and overall patient satisfaction.

2. Cost and Affordability (ξ2)

The **cost** of healthcare services is another critical criterion for hospital selection. Hospitals often provide a wide range of services, and their pricing can vary depending on factors such as location, facilities, and specialization. Cost must be balanced with the quality of services provided.

3. Hospital Infrastructure and Facilities (ξ3)

A hospital's **infrastructure** and **facilities** play a crucial role in determining its overall performance. High-quality hospitals tend to have state-of-the-art medical equipment, clean and comfortable facilities, and modern technology that can improve patient care.

4. Staff Competency and Training (ξ4)

The competency and training of the hospital's healthcare providers are essential to ensuring that patients receive the best possible care. Hospitals that invest in ongoing training and professional development for their medical staff are likely to maintain high standards of care.

5. Patient Services and Satisfaction (ξ5)

The overall **patient experience** is another vital criterion. This encompasses the interactions between patients and hospital staff, waiting times, the quality of communication, and the comfort of hospital facilities.

Evaluation Process:

Table 1: The way that the DMs evaluated the alternatives through experts' opinions, in which linguistic variables are justifiable for assessment of the alternatives under prefixed standards. It also shows a way of converting linguistic inputs from experts to NSs

Table 1: Linguistic Terms and corresponding NS values

Linguistic Term	Notation	$(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$
Extremely Weak	Ξ1	(0, 100, 100)
Very Weak	Ξ2	(20, 90, 80)
Weak	Ξ3	(30, 80, 70)
Slightly Weak	Ξ4	(40, 70, 70)
Below Average	Ξ5	(50, 60, 60)
Average	Ξ6	(60, 60, 50)
Above Average	Ξ7	(70, 40, 40)
Slightly Strong	Ξ8	(80, 30, 30)
Strong	Ξ9	(85, 20, 30)

Very Strong	Ξ10	(90, 20, 20)
Extremely Strong	Ξ11	(100, 10, 10)

The decision matrix is presented in Table 2, where evaluations of the decision-makers on each alternative (ϑ1, ϑ2, ϑ3 and ϑ4) are carried out with respect to identified criteria and sub-criteria. Evaluations will be made using pre-defined linguistic phrases in order to express the different levels of performance as depicted below.

Table 2: Decision Matrix in Linguistic Terms

Criteria	Θ1	Θ2	Θ3	Θ4
$\mathcal{A}1$	Ξ1	Ξ2	Ξ4	Ξ3
$\mathcal{A}2$	Ξ2	Ξ4	Ξ3	Ξ3
$\mathcal{A}3$	Ξ1	Ξ2	Ξ3	Ξ4
$\mathcal{A}4$	Ξ10	Ξ9	Ξ7	Ξ6
$\mathcal{A}5$	Ξ2	Ξ4	Ξ3	Ξ1

Table 3: Decision matrix in neutrosophic values. Here each alternative ϑ1, ϑ2, ϑ3, ϑ4 is evaluated w.r.t. criteria. Each evaluation will be made by using three components: Truth ($\mathcal{T}$), Indeterminacy ($\mathcal{I}$) and Falsity ($\mathcal{F}$). This will provide a complete description of the extent to which a criterion is satisfied, uncertain, or unsatisfied for each alternative.

Table 3: Decision Matrix in Neutrosophic Values

Criteria	Θ1 $(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$	Θ2 $(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$	Θ3 $(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$	Θ4 $(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$
$\mathcal{A}1$	(0,100,100)	(20, 90, 80)	(40, 70, 70)	(30, 80, 70)
$\mathcal{A}2$	(20, 90, 80)	(40, 70, 70)	(30, 80, 70)	(30, 80, 70)
$\mathcal{A}3$	(0,100,100)	(20, 90, 80)	(30, 80, 70)	(40, 70, 70)
$\mathcal{A}4$	(90, 20, 20)	(85, 20, 30)	(70, 40, 40)	(60, 60, 50)
$\mathcal{A}5$	(20, 90, 80)	(40, 70, 70)	(30, 80, 70)	(0,100,100)

Using the proposed entropy we will get the entropy, Degree of divergence and weight of the criteria in Table 4.

Table 4: Criteria Weights

Methods / Criteria	Quality of Care	Cost and Affordability	Hospital Infrastructure and Facilities	Staff Competency and Training	Patient Services and Satisfaction
Entropy value	-0.9996	-0.2907	-0.9996	0.458075	-0.9996

Degree of divergence D_j^p	1.9996	1.2907	1.9996	0.541925	1.9996
weight of the criteria W_j^p	0.25533	0.16481	0.25533	0.069199	0.25533

Table 5 The aggregating results by the NWA_w and NWG_woperators

Hospital	$NWA_w(\mathcal{T},\mathcal{I},\mathcal{F})$	$NWG_w(\mathcal{T},\mathcal{I},\mathcal{F})$
Θ1	(0.35,0.57,0.72)	(0.34,0.77,0.73)
Θ2	(0.44,0.65,0.66)	(0.44,0.66,0.66)
Θ3	(0.41,0.70,064)	(0.40,0.71,0.65)
Θ4	(0.35,0.76,0.69)	(0.32,0.78,0.72)

In Table 5, The aggregating values found by using NWA_w operator and NWG_w operator

Table 6 The score functions

Hospital	NWA_w	NWG_w
Θ1	0.2931	0.2810
Θ2	0.3778	0.3733
Θ3	0.3562	0.3475
Θ4	0.3002	0.2739

In table 6, we find the score values by using aggregating values from table 5.

Table 7 Ordering of Hospitals

	Ranking
NWA_w	$\Theta 2 > \Theta 3 > \Theta 4 > \Theta 1$
NWG_w	$\Theta 2 > \Theta 3 > \Theta 1 > \Theta 4$

Table 7, represented the ordering of Hospitals. From this table we conclude $\Theta 2$ is the best Hospital comparing with others.

3.3 Visualization of Entropy and Ranks

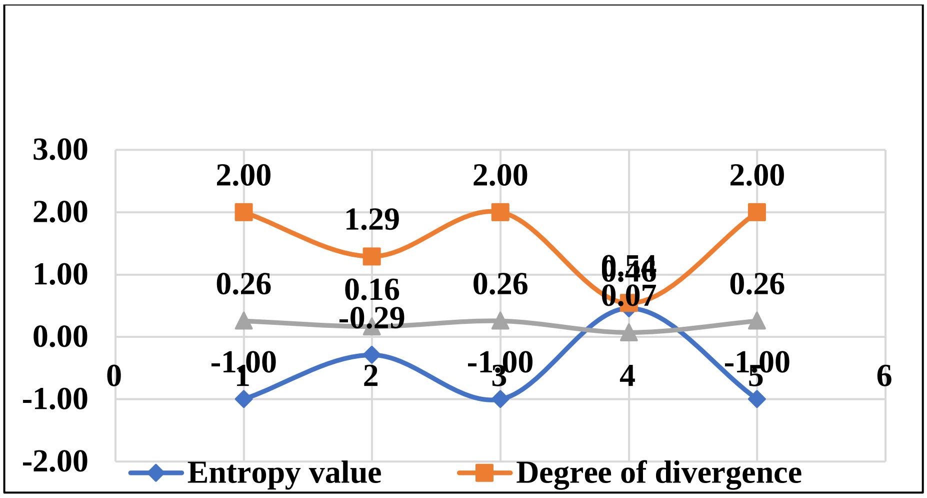

Figure 2: Flow Chart for Multi Attributes Decision Making Using Neutrosophic Set.

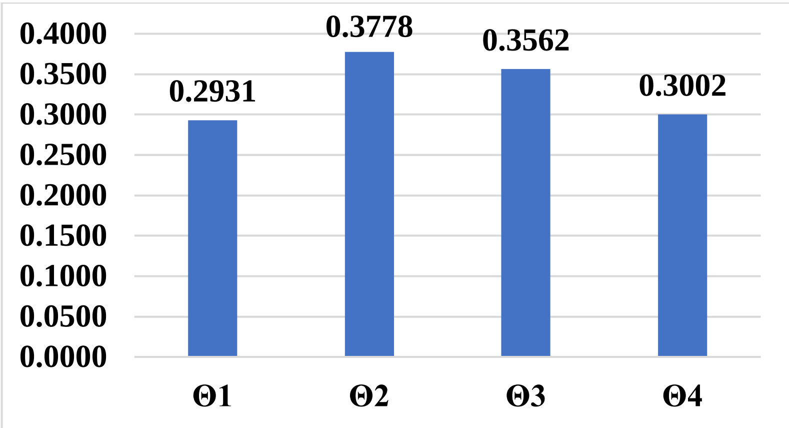

Figure 3 Ranking of Hospitals by N*WAw*

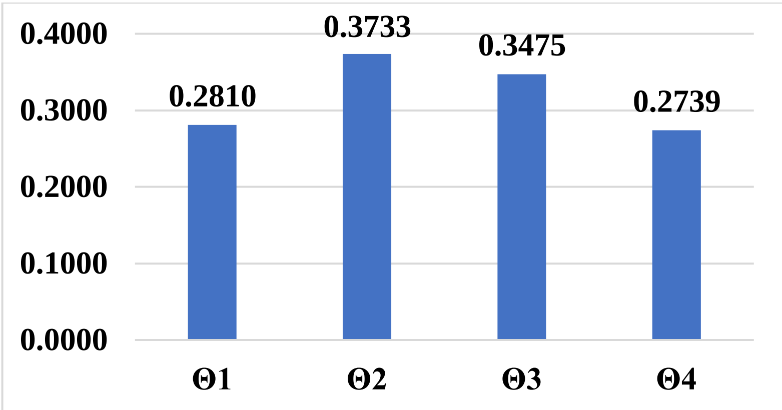

Figure 4 Ranking of Hospitals by N*W*G*w*

Figure 5 Comparison Ranking of Hospitals by *PFWAw* and *PFWGw*

4. Conclusions

In this study, we aimed to develop a robust methodology for selecting the best hospital from a set of available options by leveraging a combination of Neutrosophic Sets, Entropy, and Multi-Attribute Decision Making (MADM). The integration of these techniques provides a comprehensive and flexible approach to address the inherent uncertainty, vagueness, and imprecision involved in hospital selection, a process influenced by numerous qualitative and quantitative criteria. Neutrosophic Sets enabled us to model the uncertainty and indeterminacy present in the data, accommodating incomplete, inconsistent, or contradictory information. The inclusion of neutrosophic elements allowed for a more nuanced evaluation of hospital attributes, considering degrees of truth, falsity, and indeterminacy. he Entropy method was employed to quantify the uncertainty and variability in the decision-making criteria, helping to derive objective weights for each attribute. This ensured that the criteria were ranked appropriately based on their importance in the decision-making process, accounting for their inherent uncertainty. Multi-Attribute Decision Making (MADM) was utilized to combine all the gathered information and facilitate a well-rounded evaluation of each hospital. The MADM process enabled the synthesis of various criteria—such as medical expertise, infrastructure, accessibility, and patient satisfaction—into a single decision matrix, allowing for an optimal selection.

By applying this integrated framework, we were able to rank hospitals more effectively, overcoming challenges such as missing data, conflicting opinions, and diverse performance measures. This methodology not only provides a structured approach to decision-making but also enhances transparency and robustness in hospital selection, particularly in contexts where multiple decision-makers or stakeholders are involved.

In conclusion, the combination of Neutrosophic Sets, Entropy, and MADM offers a powerful tool for hospital selection, providing decision-makers with a comprehensive, flexible, and data-driven framework that improves the accuracy and reliability of the selection process. Future work could further refine this methodology by incorporating additional factors like time and dynamic changes in hospital performance, extending its applicability to real-time decision-making scenarios in healthcare management.

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