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A Method for Solving Non-Linear Initial Value Problems by Adomian Decomposition

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Abstract:

To solve first and second-order nonlinear initial value problems involving variable system coefficients, we provide the Adomian decomposition position approach in this article. We look at different approaches for the Adomian decomposition series and the series of Adomian polynomials to find the answers to first- and second-order nonlinear initial value problems. To consistently compute the Taylor expansion series of the solution using easily integrable terms, we have introduced a novel modified recursion method, which slows down the Adomian decomposition series significantly. We demonstrate the appropriate nonlinear recurrence relations for the coefficients of the solutions. We then go on to study the errors and acceleration of convergence as they correspond to the sequence of solution approximations. We conclude by looking at many illustrative cases to show how quickly the two sets of data converge.

Keywords: Initial Value Problems, Adomian Polynomials, Non-linear Ordinary differential Equations.

1. Introduction

Whether the mathematical model is linear or nonlinear, there are numerous numerical approaches to solve it. Several mathematical models have been proposed in the literature to address the problems of integral, partial differential, and ordinary differential equations. Among the many classical numerical methods are the following: the variable iteration method, the finite difference method, the finite element method, the Homotopy perturbation method, and the wavelet methods. Adomian decomposition is the topic of this chapter. A number of types of differentials, algebraic, difference, integral, and integro-differential equations can be solved using the Adamian Decomposition Method. The recursion scheme for the ADM is better than and clearly different from the Picard iterative technique [1]-[5]. It has been demonstrated that, rather than the traditional Taylor series expansion about a constant, i.e. the initial point, the Adomian decomposition series is comparable to a Banach-space analog of the Taylor series expansion about the initial solution component function [6].A method's efficacy in numerical analysis is highly dependent on its precision, consistency, convergence, and stability (zero, weak, and absolute stability). When comparing the accuracy qualities of different techniques, they are usually evaluated according to their convergence order, truncation error coefficients, computational simplicity, affordability of the procedure, and effectiveness for a large variety of ODEs. Researchers have developed several numerical strategies to address initial value problems [7]-[13]. George Adomian, the chair of the Centre for Applied Mathematics at the University of Georgia, pioneered the method during the 1970s and 1990s. This method finds the solution as a series whose terms are obtained via a recursive connection using the Adomian polynomials. Anyone who has used the ADM has likely mentioned its many advantages, as stated in [10]. In a short amount of time, the Adomian decomposition method yields an accurate numerical answer. It gives a rapid analytical solution to the nonlinear differential equation without using linearization or perturbation methods, and it does so accurately. Nevertheless, other academics have worked to improve the ADM's accuracy or expand its usefulness since its launch, leading to a plethora of technique updates. Review the sources cited in [14]- [18]. What follows is an explanation of ADM theory and principles grounded in numerous numerical examples.

2. Instruments and Tools

To illustrate the Adomian Decomposition technique for locating initial values for solutions of nonlinear ordinary differential equations, we demonstrate its implementation in MATLAB through the creation of graphs.

3. Methods

3.1 Adomian Decomposition Procedure Overview

There are many kinds of ordinary differential equations that the Adomian Decomposition Method can solve. Consider these equations:

$$L(z)+N(z)+R(z) = f(x) \qquad (1)$$

the linear component is denoted by R(z), the non-linear component is denoted by N(z), and the linear operator is L. We can determine if the inverse operator of L exists by defining it as L^{-1}.

$$z = L^{-1}\big(f(x)\big)-L^{-1}\big(N(z)\big)-L^{-1}\big(R(z)\big) \qquad (2)$$

An infinite series of the sort can explain the unknown function z, in which case the Adomian Decomposition Method can be applied.

$$z = \sum_{n=0}^{\infty} z_n$$

$$z = z_0+z_1+z_2+z_3+\ldots.. \qquad (3)$$

This will be determined iteratively as z_n components.

The nonlinear term is further defined by the method using the Adomian polynomials. The ADM requires the decomposition of the nonlinear operator N(z) using an infinite series of polynomials.

$$N(z) = \sum_{n=0}^{\infty} A_n$$

$$N(z) = A_0+A_1+A_2+.. . . \qquad (4)$$

in which the Adomian's polynomials are specified as $A_0,A_1,A_2\ldots$ Substituting (3) and (4) into equation (2) and using the fact that R is a linear operator we obtain

$$\sum_{n=0}^{\infty} z_n = L^{-1}\big(f(x)\big)-L^{-1}\left(\sum_{n=0}^{\infty} A_n\right)- L^{-1}\left(\sum_{n=0}^{\infty} R(z_n)\right)$$

$$z_0+z_1+z_2+z_3+\ldots= L^{-1}\big(f(x)\big)-L^{-1}(A_0+A_1+A_2+.. .)-L^{-1}\big(R(z_0)+R(z_1)+R(z_2)+\ldots\big) \qquad (5)$$

Examine the non-linear function g(z) .Next, we generate an infinite series around the initial function z_0 through the application of Taylor's series expansion for z(x).

$$g(z) = g(z_0)+g'(z_0)(z-z_0)+\frac{1}{2\,!}g''(z_0)(z-z_0)^2+\ldots \qquad (6)$$

By substitution (3) in (6), we have:

$$g(z) = g(z_0)+g'(z_0)\ (z_1+z_2+z_3+\ldots)+\frac{1}{2\,!}g''(z_0)\ (z_1+z_2+z_3+\ldots)^2+\ldots \qquad (7)$$

All terms of order n are often included in A_n. Consequently, the following is a list of the first five Adomian polynomial terms:

$$A_0 = g(z_0)$$

$$A_1 = g'(z_0)\ z_1$$

$$A_2 = g'(z_0)\ z_2+\frac{1}{2!}\ g''(z_0)\ z_1^2$$

$$A_3 = g'(z_0)\ z_3+\frac{2}{2!}\ g''(z_0)\ z_1z_2+\frac{1}{3!}\ g'''(z_0)\ z_1^3$$

$$A_4 = g'(z_0)\ z_4+\frac{1}{2!}\ g''(z_0)\ (2z_1z_3+z_2^2)+\frac{3}{3!}\ g'''(z_0)\ z_1^2z_2+\frac{1}{4!}\ g^{(4)}(z_0)z_1^4$$

$$A_5 = g'(z_0)\ z_5+\frac{1}{2!}\ g''(z_0)\ (2z_1z_4+2z_2z_3)+\frac{1}{3!}\ g'''(z_0)\ (3z_1^2z_3+3z_1z_2^2)$$
$$+\frac{4}{4!}\ G^{(4)}(z_0)z_1^3z_2+\frac{1}{5!}\ G^{(5)}(z_0)\ z_1^5 \qquad (8)$$

Adomian initially presented the Adomian polynomial A_n and defined it using the general formula.

$$A_n(z_0,z_1,z_2,z_3,\ldots.) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{n} z_k\lambda^k\right)\right]_{\lambda=0}, \qquad n = 0,1,2,3\ldots \qquad (9)$$

3.2 Decomposition of a First-Order Non-Linear ODE with an Initial Condition

Concentrate on the first order non-linear beginning value issue

$$\frac{dz}{dx}+R(z)+N(z) = f(x) \quad \text{with} \quad z(a) = b$$

To apply the ADM for solving the above equation, now let us look at the operator form of the following general equation:

$$L(z) + R(z) + N(z) = f(x) \qquad \text{with initial condition} \quad z(a) = b \qquad (10)$$

The non-linear terms are denoted by N(z), and the highest-order derivative of the equation is the linear differential operator L(z).

The expression $L = \frac{d}{dx}$ describes a first-order operator L. (11)

Next, if we presume that L can be inverted, we get the inverse operator L^{-1} by

$$L^{-1}(.)= \int_a^x (.)\ dx \qquad (12)$$

such that $L^{-1}(L(z(x))) = z(x)-z(a)$

So, in order to put the ADM into action, we start by rearranging the terms in equation (10) and then applying L^{-1} to both sides.

$$z(x) = z(a)+L^{-1}(f(x)) - L^{-1}(R(z)) - L^{-1}(N(z)) \qquad (13)$$

An infinite series of the sort can explain the unknown function z,

$$z = \sum_{n=0}^{\infty} z_n \tag{14}$$

The ADM requires the decomposition of the nonlinear operator N(z) using an infinite series of polynomials.

$$N(z) = \sum_{n=0}^{\infty} A_n \tag{15}$$

Substituting (14) and (15) in (13) we get

$$\sum_{n=0}^{\infty} z_n = z(a) + L^{-1}(f(x)) - L^{-1}\left(R\left(\sum_{n=0}^{\infty} z_n\right)\right) - L^{-1}\left(N\left(\sum_{n=0}^{\infty} A_n\right)\right) \tag{16}$$

Finding the different parts z_n of the solution z is as easy as pie when you use the recursive relation.

$$z_0 = z(a) + L^{-1}(f(x)) \tag{17}$$

$$z_{n+1} = -L^{-1}(R(z_n)) - L^{-1}(N(A_n)) \quad , \quad n \geq 0$$

$$z_1 = -L^{-1}(R(z_0)) - L^{-1}(N(A_0))$$

$$z_2 = -L^{-1}(R(z_1)) - L^{-1}(N(A_1))$$

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0 + z_1 + z_2 + z_3 + \ldots.. \tag{18}$$

3.3 Decomposition of Second Order Non-Linear ODE with Initial Conditions

The second order non-linear ODE with initial conditions

$$\frac{d^2z}{dx^2} + R(z) + N(z) = f(x) \text{ with } z(a) = b1 , z'(a) = b2$$

To apply the ADM for solving the above equation, now let us look at the operator form of the following general equation:

$$L(z) + R(z) + N(z) = f(x) \text{ with initial conditions } z(a) = b1 , z'(a) = b2 \tag{19}$$

The non-linear terms are denoted by N(z) and the highest-order derivative of the equation is the linear differential operator L(z).

The expression $L = \frac{d^2}{dx^2}$ describes a second order operator L. (20)

If we presume that L can be inverted, we get the inverse operator L^{-1} by

$$L^{-1}(.) = \int_a^x \int_a^x (.)\ dx\, dx \tag{21}$$

so that $L^{-1}(L(z(x))) = z(x) - z(a) - (x-a)\, z'(a)$

we start by rearranging the terms in equation (19) and then applying L^{-1} to both sides.

$$z(x) = z(a)+(x-a)\, z'(a)+L^{-1}(f(x)) - L^{-1}(R(z)) -L^{-1}(N(z)) \tag{22}$$

An infinite series of the sort can explain the unknown function z,

$$z = \sum_{n=0}^{\infty} z_n \tag{23}$$

The ADM requires the decomposition of the nonlinear operator N(z) using an infinite series of polynomials.

$$N(z)=\sum_{n=0}^{\infty} A_n \tag{24}$$

Substituting (23) and (24) in (22) we get

$$\sum_{n=0}^{\infty} z_n = z(a)+(x-a)\, z'(a)+L^{-1}(f(x))-L^{-1}\left(R\left(\sum_{n=0}^{\infty} z_n\right)\right)-L^{-1}\left(N\left(\sum_{n=0}^{\infty} A_n\right)\right)$$

The various components $z_n$of the solution z can be easily determined by using the recursive relation

$$z_0= z(a)+(x-a)\, z'(a)+L^{-1}(f(x)) \tag{25}$$

$$z_{n+1}= -L^{-1}(R(z_n))-L^{-1}(N(A_n)) \quad , \quad n \geq 0$$

$$z_1= -L^{-1}(R(z_0))-L^{-1}(N(A_0))$$

$$z_2= -L^{-1}(R(z_1))-L^{-1}(N(A_1))$$

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0+z_1+z_2+z_3+\ldots.. \tag{26}$$

4. Numerical Examples and Results

The present study has successfully resolved non-linear problems of first and second order using known correct solutions for the starting point conditions. Figures show comparisons between the numerical solution, correct solution, and absolute errors at the node positions. We contrast the outcomes of this study with the precise answers to the issues found using the Adomian Decomposition Method.

Example 1: Study at a first-order non-linear differential equation with variable coefficients and the initial value difficulty it provides.

$$\frac{dz}{dx}+xe^{-z} = 1 \ , \quad z(0)= 0 \ , \quad 0 \leq x \leq 1$$

The analytical solution is : $z(x) = \log(x+1)$

Here : $N(z) = xe^{-z}$, $R(z) = 0$, $f(x) =1$

The following determines the initial components:

$$z_0 = x$$

$$z_1 = exp(-x)\,(x + 1)-1$$

$$z_2 = -\frac{exp(-2x)\,(x - exp(x) + 1)^2}{2}$$

.

.

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0+z_1+z_2+z_3+\ldots..$$

A series version of the solution is thus provided by

$$z(x) = -3.442e\text{-}22exp(-10.0x)(2.905e{+}21x + 1.634e{+}22exp(2.0x) + 1.017e{+}23exp(4.0x)$$

$$+ 6.537e{+}22exp(8.0x) - 2.905e + \ldots\ldots..$$

Using ADM with ten iterations, the absolute error for Example 1 is computed, and it clearly converges to the exact answer.

TABLE 3.1: In this case, h=0.1, the Absolute Error and ADM values from Example 1

x_i	Exact	ADM	ADM Absolute Error
0	0	0	0
0.1	0.09531	0.09531	6.16E-17
0.2	0.182322	0.182322	1.68E-16
0.3	0.262364	0.262364	1.27E-16
0.4	0.336472	0.336472	4.86E-15
0.5	0.405465	0.405465	3.19E-13
0.6	0.470004	0.470004	9.04E-12
0.7	0.530628	0.530628	1.39E-10
0.8	0.587787	0.587787	1.38E-09
0.9	0.641854	0.641854	9.72E-09
1	0.693147	0.693147	5.26E-08

Thus, the max-absolute error for the Adomian Decomposition Method is
$\omega = 0.00000005264627749$

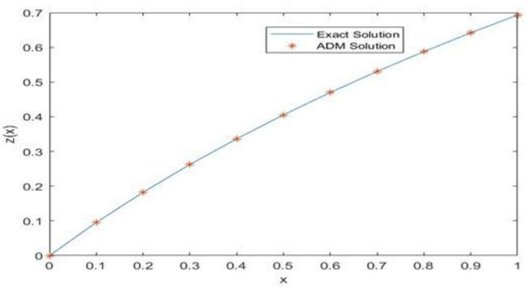

FIGURE (3.1). An analytical solution and a solution using Adomian Decomposition

Method.

Example 2: Study at a first-order non-linear differential equation with variable coefficients and the initial value difficulty it provides

$$\frac{dz}{dx}\text{-}(2x+1)z^2 = (2x+1)\,, \quad z(0) = 0\,, \quad 0 \le x \le \frac{\pi}{6}$$

The analytical solution is : $z(x) = \tan\left(x^2+x\right)$

Here : $N(z) = -(2x+1)\,z^2$, $R(z) = 0$, $f(x) = (2x+1)$

The following determines the initial components:

$$z_0 = x\,(x+1)$$

$$z_1 = \frac{x^3(x+1)^3}{3}$$

$$z_2 = \frac{2x^5(x+1)^5}{15}$$

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0+z_1+z_2+z_3+\ldots..$$

A series version of the solution is thus provided by

$$z(x) = 0.3333x^3(x + 1.0)^3 + 0.1333x^5(x + 1.0)^5 + 0.05397x^7(x + 1.0)^7 +$$
$$0.02187x^9(x + 1.0)^9\ 0.008863x^{11}(x + 1.0)^{11} + \ldots\ldots$$

Using ADM with ten iterations, the absolute error for Example 2 is computed, and it clearly converges to the exact answer.

TABLE 3.2: In this case, $h = \frac{\pi}{60}$, the Absolute Error and ADM values from Example 2

x_i	Exact	ADM	ADM Absolute Error
0	0	0	0
0.05236	0.055157	0.055157	1.08E-19
0.10472	0.116205	0.116205	2.17E-19
0.15708	0.183782	0.183782	0
0.20944	0.258865	0.258865	8.67E-19
0.261799	0.342903	0.342903	3.55E-16
0.314159	0.43803	0.43803	6.15E-14
0.366519	0.547414	0.547414	5.42E-12
0.418879	0.675858	0.675858	2.91E-10
0.471239	0.830908	0.830908	1.07E-08
0.523599	1.025023	1.025023	2.93E-07

Thus, the max-absolute error for the Adomian Decomposition Method is $\omega = 0.0000002928865575$

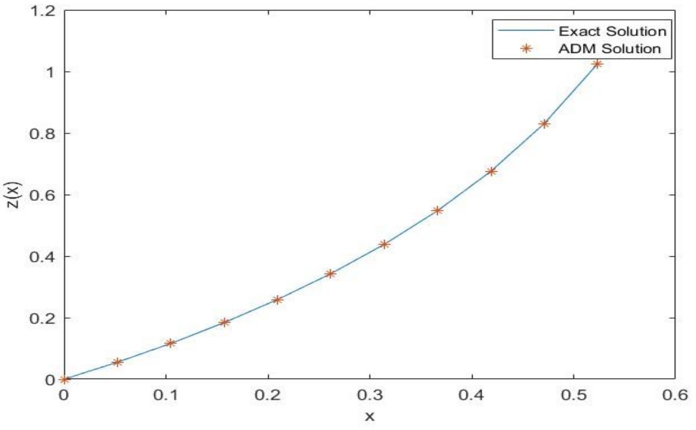

FIGURE (3.2). An analytical solution and a solution using Adomian Decomposition Method.

Example 3: Study at a second order non-linear differential equation with variable coefficients and the initial value difficulty it provides

$$\frac{d^2z}{dx^2} - x\frac{dz}{dx} + z^2 = x^4 + 3 \ , \quad z(1) = 2 \ , \ z'(1) = 2; \ 1 \leq x \leq 2$$

The analytical solution is : $z(x) = x^2 + 1$

Here: $R(z) = -x\frac{dz}{dx}$, $N(z) = z^2$, $f(x) = x^4 + 3$

The following determines the initial components:

$$z_0 = \frac{x^6}{30} + \frac{3x^2}{2} - \frac{6x}{5} + \frac{5}{3}$$

$$z_1 = -\frac{x^{14}}{163800} - \frac{x^{10}}{900} + \frac{x^9}{900} + \frac{x^8}{630} - \frac{3x^6}{40} + \frac{9x^5}{50} - \frac{43x^4}{150} + \frac{7x^3}{15} - \frac{25x^2}{18} + \frac{33791x}{16380} - \frac{1513}{1575}$$

.

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0 + z_1 + z_2 + z_3 + \ldots..$$

A series version of the solution is thus provided by

$$z(x) = 1.835e\text{-}41x^{86} + 3.056e\text{-}38x^{82} - 3.585e\text{-}38x^{81} - 9.558e\text{-}38x^{80} + 2.491e\text{-}35x^{78} - 6.194e\text{-}35x^{77}$$

$$- 8.007e\text{-}35x^{76} + 9.93e\text{-}35x^{75} + \ldots\ldots$$

Using ADM with ten iterations, the absolute error for Example 3 is computed, and it clearly converges to the exact answer

TABLE 3.3: In this case, h=0.1, the Absolute Error and ADM values from Example 3

x_i	**Exact**	**ADM**	**ADM** **Absolute Error**
1	2	2	0
1.1	2.21	2.21	0
1.2	2.44	2.44	9.71E-17
1.3	2.69	2.69	2.86E-14
1.4	2.96	2.96	1.76E-12
1.5	3.25	3.25	4.52E-11
1.6	3.56	3.56	6.49E-10
1.7	3.89	3.89	6.15E-09
1.8	4.24	4.24	4.25E-08
1.9	4.61	4.61	2.27E-07
2	5	5.000001	9.73E-07

Thus, the max-absolute error for the Adomian Decomposition Method is $\omega = 0.0000009734324069$

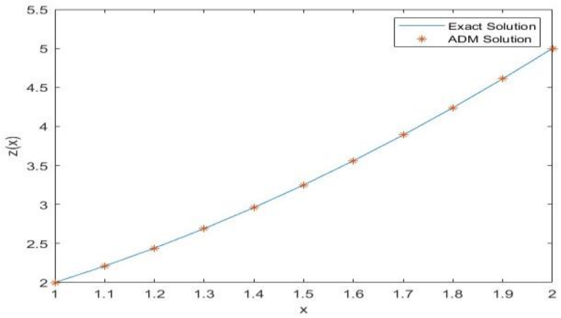

FIGURE (3.3). An analytical solution and a solution using Adomian Decomposition Method

Example 4: Study at a second order non-linear differential equation with constant coefficients and the initial value difficulty it provides

$$\frac{d^2z}{dx^2}+\frac{dz}{dx}+16z^4-4z^2 = (x-1)^8-(x-1)^4+x \ , \quad z(1) = 0 \ , \ z'(1) = 0; \ 1 \le x \le 2$$

The analytical solution is : $z(x) = \frac{(x-1)^2}{2}$

Here: $R(z) = \frac{dz}{dx}$, $N(z) = 16z^4-4z^2$, $f(x) = (x-1)^8-(x-1)^4+x$

The following determines the initial components:

$$z_0 = \frac{\left((x - 1)^2\left(x^8 - 8x^7 + 28x^6 - 56x^5 + 67x^4 - 44x^3 + 10x^2 + 19x + 28\right)\right)}{90}$$

$$z_1 = - 1.416e\text{-}10x^{42} + 5.948e\text{-}9x^{41} - 1.219e\text{-}7x^{40} + 1.626e\text{-}6x^{39} - 1.585e\text{-}5x^{38} + \ldots\ldots$$

.

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0 + z_1 + z_2 + z_3 + \ldots..$$

A series version of the solution is thus provided by

$$z(x) = 3.933e\text{-}422\,(x - 1.0)^2(6.876e{+}341x^{328} - 2.255e{+}344x^{327} + 3.688e{+}346x^{326} + \ldots\ldots\ldots$$

Using ADM with ten iterations, the absolute error for Example 4 is computed, and it clearly converges to the exact answer

TABLE 3.4: In this case, h=0.1, the Absolute Error and ADM values from Example 4

x_1	Exact	ADM	ADM Absolute Error
1	0	0	0
1.1	0.005	0.005	0
1.2	0.02	0.02	1.08E-19
1.3	0.045	0.045	1.93E-17
1.4	0.08	0.08	4.45E-15
1.5	0.125	0.125	3.94E-13
1.6	0.18	0.18	1.49E-11
1.7	0.245	0.245	3.73E-10
1.8	0.32	0.32	8.58E-09
1.9	0.405	0.405	2.03E-07
2	0.5	0.500005	5.28E-06

Thus, the max-absolute error for the Adomian Decomposition Method is $\omega = 0.000005283669566$

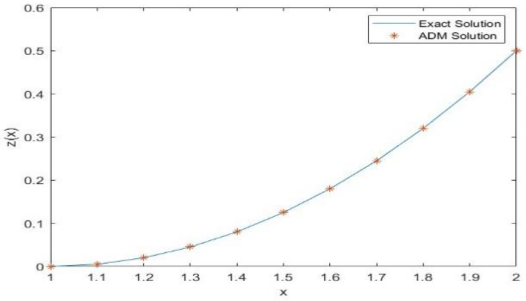

FIGURE (3.4). An analytical solution and a solution using Adomian Decomposition Method

Example 5: Study at a second-order non-linear differential equation with variable coefficients and the initial value difficulty it provides

$$\frac{d^2z}{dx^2} - z + xz\frac{dz}{dx} - xz^2 = 0 \ , \quad z\left(\frac{-1}{2}\right) = exp\left(\frac{1}{2}\right) \ , \ z'\left(\frac{-1}{2}\right) = exp\left(\frac{1}{2}\right) \quad ; \quad \frac{-1}{2} \leq x \leq \frac{1}{2}$$

The analytical solution is : $z(x) = e^{(x+1)}$

Here: $R(z) = -z \quad , \ N(z) = xz\frac{dz}{dx} - xz^2, \ f(x) = 0$

The following determines the initial components:

$z_0 = 1.649x + 2.473$

$z_1 = 0.03435(2.0x + 1.0)^2(2.0x + 7.0) + 0.002832(2.0x + 1.0)^3\left(6.0x^2 + 11.0x - 6.0\right)$

$z_2 = 1.512e\text{-}36\ (2.0x + 1.0)^3\left(5.146e{+}32x^6 + 4.411e{+}32x^5 - 3.04e{+}33\ x^4 + \ldots\right)$

$$z(x) = \sum_{n=0}^{\infty} z_n = z_0 + z_1 + z_2 + z_3 + \ldots..$$

A series version of the solution is thus provided by

$z(x) = 1.276e\text{-}14\ x^{41} - 4.378e\text{-}13x^{40} + 2.732e\text{-}12\ x^{39} + 2.701e\text{-}11\ x^{38} - 1.633e\text{-}10\ x^{37} - \ldots\ldots..$

Using ADM with ten iterations, the absolute error for Example 5 is computed, and it clearly converges to the exact answer

TABLE 3.5: In this case, h=0.1, the Absolute Error and ADM values from Example 5

x_i	Exact	ADM	ADM Absolute Error
-0.5	1.648721	1.648721	4.86E-17
-0.4	1.822119	1.822119	5.20E-17
-0.3	2.013753	2.013753	1.87E-16
-0.2	2.225541	2.225541	3.04E-14
-0.1	2.459603	2.459603	5.82E-13
0	2.718282	2.718282	3.14E-12
0.1	3.004166	3.004166	5.47E-12
0.2	3.320117	3.320117	5.42E-12
0.3	3.669297	3.669297	4.63E-12
0.4	4.0552	4.0552	6.75E-11
0.5	4.481689	4.481689	1.33E-09

Thus, the max-absolute error for the Adomian Decomposition Method is
$\omega = 0.000000001327041463$

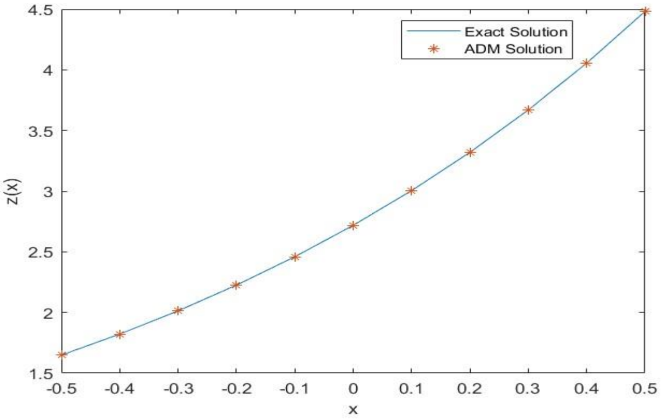

FIGURE (3.5). An analytical solution and a solution using Adomian Decomposition Method

5. Discussion

We can observe that the Adomian decomposition approach is near to the exact solution from the tables above. In addition, the results show that the approach is accurate, dependable, and converges quickly.

6. Conclusion

It was noted that adding more terms to our decomposition series improves accuracy, and that the given equations' solutions are stable and constant across the interval.

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