Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 9s (2025)

Various Types of Translation in Bipolar Valued Multi I-Fuzzy Normal Subrings of a Ring

[1]K.Vairamuthu & [2]S. Loganathan

[1] Department Of Mathematics, Sethupathy Government Arts College(Affiliated To Alagappa University, Karaikudi), Ramanathapuram -623 502, Tamilnadu, India. Email: Vairammathi83@Gmail.Com

[2]Department Of Mathematics, Sethupathy Government Arts College(Affiliated To Alagappa University, Karaikudi), Ramanathapuram -623 502, Tamilnadu, India. Email: Logaamaths2010@Gmail.Com

Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract:

In this paper, various types of translation in B_V MIFNSR of a ring are studied and dealt. Some theorems are given and,they are proved.

Keywords: Interval valued fuzzy subset,bipolar valued fuzzy subset, 〖B〗_V MIFS, 〖B〗_V MIFSR, 〖B〗_V MIFNSR and translations.

INTRODUCTION.

Zadeh [14]had introduced the fuzzy subset in 1965. It is one of the generalizations of crisp set. *Group was generalized as fuzzy group by* Azriel Rosenfeld [3]. *After, Different types of fuzzy were introduced by various authors*. In 1994, bipolar valued fuzzy subset was introduced by W.R.Zhang[15]. Bipolar valued multi I-fuzzy subring has been introduced by K.Vairamuthu, S.Loganathan[11]. The following papers [1], [2], [4], [5], [6], [7], [8], [9], [10], [12] and [13] were useful to write the this paper. In this paper, *various types of translation in* $B_V MIFNSR$ *of a ring are introduced* and established some results.

1.PRELIMINARIES.

Definition 1.1 [14] *A map* $\mathfrak{R}: \mathbb{M} \to \mathbb{D}[0,1]$ *is said to be an interval valued fuzzy subset of* $\mathbb{M}$, *where* $\mathbb{D}[0,1]$ *means collection of all closed subinterval of* $[0,1]$.

Definition 1.2 [15] *The ordered structure* $\mathfrak{T} = \{(\mathfrak{z}, \mathfrak{T}^{+}(\mathfrak{z}), \mathfrak{T}^{-}(\mathfrak{z})) : \mathfrak{z} \in \mathbb{W}\}$ *is called* a bipolar *valued fuzzy subset of* $\mathbb{w}$, *where* $\mathfrak{T}^{+}: \mathbb{w} \to [0,1]$ *is a positive membership* map and $\mathfrak{T}^{-}: \mathbb{w} \to [-1,0]$ is a negative membership map.

Definition 1.3 [11] *The ordered structure* $\mathfrak{T} = \{(\mathfrak{z}, \mathfrak{T}_1^{+}(\mathfrak{z}), \mathfrak{T}_2^{+}(\mathfrak{z}), \ldots, \mathfrak{T}_n^{+}(\mathfrak{z}), \mathfrak{T}_1^{-}(\mathfrak{z}), \mathfrak{T}_2^{-}(\mathfrak{z}), \ldots, \mathfrak{T}_n^{-}(\mathfrak{z})) : \mathfrak{z} \in \mathbb{W}\}$ *is called a bipolar valued multi* $I-$*fuzzy subset* $(B_V MIFS)$ of $\mathbb{w}$, where $\mathfrak{T}_i^{+}: \mathbb{w} \to \mathbb{D}[0,1]$ *is a positive membership* map and $\mathfrak{T}_i^{-}: \mathbb{w} \to \mathbb{D}[-1,0]$ is a negative membership map.

Definition 1.4 [11] $A\ B_V MIFS\ \ \text{Ӊ} = \langle\, \text{Ӊ}_1^+,\ \text{Ӊ}_2^+,\ldots,\text{Ӊ}_n^+,\ \text{Ӊ}_1^-, \text{Ӊ}_2^-,\ldots,\text{Ӊ}_n^- \rangle\ \ of\ a\ ring\ \text{Ȫ}\ is\ said$ to $be\ a$ bipolar valued multi I − fuzzy subring of Ȫ $(B_V MIFSR)\ if\ \text{Ӊ}\ has, for\ all\ i,$

(i) $\text{Ӊ}_i^+(\text{ℴ} - \text{ꝺ}) \geq rmin\{\text{Ӊ}_i^+(\text{ℴ}), \text{Ӊ}_i^+(\text{ꝺ})\},$

(ii) $\text{Ӊ}_i^+(\text{ℴꝺ}) \geq rmin\{\text{Ӊ}_i^+(\text{ℴ}), \text{Ӊ}_i^+(\text{ꝺ})\},$

(iii) $\text{Ӊ}_i^-(\text{ℴ} - \text{ꝺ}) \leq rmax\{\text{Ӊ}_i^-(\text{ℴ}), \text{Ӊ}_i^-(\text{ꝺ})\},$

(iv) $\text{Ӊ}_i^-(\text{ℴꝺ}) \leq rmax\{\text{Ӊ}_i^-(\text{ℴ}), \text{Ӊ}_i^-(\text{ꝺ})\}, for\ all\ \text{ℴ}, \text{ꝺ} \in \text{Ȫ},$

where $rmin\{[\text{ɼ}, \text{ą}], [\text{ъ}, \text{ꝺ}]\} = [\min\{\text{ɼ}, \text{ъ}\}, \min\{\text{ą}, \text{ꝺ}\}]$ and

$rmax\{[\text{ɼ}, \text{ą}], [\text{ъ}, \text{ꝺ}]\} = [\max\{\text{ɼ}, \text{ъ}\}, \max\{\text{ą}, \text{ꝺ}\}].$

Example 1.5 Let $R = \mathbb{Z}_3 = \{0, 1, 2\}\ be\ a\ ring\ with \oplus_3\ and \otimes_3. Then\ \mathfrak{C} = \{(0, [0.72, 0.81], [0.91, 1], [0.52, 0.63], [-0.91, -0.81], [-1, -0.91], [-0.81, -0.72]), (1, [0.51, 0.61], [0.71, 0.81], [0.31, 0.41], [-0.71, -0.61], [-0.61, -0.51], [-0.51, -0.41]), (2, [0.51, 0.61], [0.71, 0.81], [0.31, 0.41], [-0.71, -0.61], [-0.61, -0.51], [-0.51, -0.41])\}$ is a $B_V MIFSR$ of R.

Definition 1.6 $A\ B_V MIFSR\ \ \text{ꜩ} = \langle\, \text{ꜩ}_1^+,\ \text{ꜩ}_2^+,\ldots,\text{ꜩ}_n^+,\ \text{ꜩ}_1^-, \text{ꜩ}_2^-,\ldots,\text{ꜩ}_n^- \rangle\ \ of\ a\ ring\ \text{Ꞓ}\ is\ said$ to $be\ a$ bipolar valued multi I − fuzzy normal subring of R $(B_V MIFNSR)\ if\ \text{ꜩ}\ has, for$

$all\ i,$

(i) $\text{ꜩ}_i^+(\text{ŋ}\mathfrak{w}) = \text{ꜩ}_i^+(\mathfrak{w}\text{ŋ}),$

(ii) $\text{ꜩ}_i^-(\text{ŋ}\mathfrak{w}) = \text{ꜩ}_i^-(\mathfrak{w}\text{ŋ}), for\ all\ \text{ŋ}, \mathfrak{w} \in \text{Ꞓ}.$

Definition 1.7 [11] Let $\text{Œ} = \langle\, \text{Œ}_1^+,\ \text{Œ}_2^+,\ldots,\text{Œ}_n^+,\ \text{Œ}_1^-, \text{Œ}_2^-,\ldots,\text{Œ}_n^- \rangle$ and $\text{Ч} = \langle\, \text{Ч}_1^+,\ \text{Ч}_2^+,\ldots,\text{Ч}_n^+,\ \text{Ч}_1^-, \text{Ч}_2^-,\ldots,\text{Ч}_n^- \rangle$ be $two\ B_V MIFSs\ with\ degree\ n\ of\ a\ set\ \mathcal{W}. Then$

(i) $\text{Œ} \subset \text{Ч}\ if\ and\ only\ if\ \forall\ i, \text{Œ}_i^+(\text{ç}) \leq \text{Ч}_i^+(\text{ç})$ and $\text{Œ}_i^-(\text{ç}) \geq \text{Ч}_i^-(\text{ç}),\ \forall \text{ç} \in \mathcal{W}.$

(ii) $\text{Œ} \cap \text{Ч} = \{\langle \text{ç}, \text{rmin}(\text{Œ}_1^+(\text{ç}), \text{Ч}_1^+(\text{ç})), \text{rmin}(\text{Œ}_2^+(\text{ç}), \text{Ч}_2^+(\text{ç})), \ldots, \text{rmin}(\text{Œ}_n^+(\text{ç}), \text{Ч}_n^+(\text{ç})), \text{rmax}(\text{Œ}_1^-(\text{ç}), \text{Ч}_1^-(\text{ç})), \text{rmax}(\text{Œ}_2^-(\text{ç}), \text{Ч}_2^-(\text{ç})),\ldots, \text{rmax}(\text{Œ}_n^-(\text{ç}), \text{Ч}_n^-(\text{ç}))\rangle\ /\ \text{ç} \in \mathcal{W}\}.$

Definition 1.8. $Let\ \text{Ж} = \langle\, \text{Ж}_1^+,\ \text{Ж}_2^+,\ldots,\text{Ж}_n^+,\ \text{Ж}_1^-, \text{Ж}_2^-,\ldots,\text{Ж}_n^- \rangle$ be $B_V MIFS$ of the set $\mathcal{W}$. The transformations are defined as, ∀ i = 1, 2, …, n,

(i) $\text{ꝋ}\,(\text{Ж}) = \langle\, \text{ꝋ}(\text{Ж}_1^+), \text{ꝋ}(\text{Ж}_2^+), \ldots, \text{ꝋ}(\text{Ж}_n^+), \text{ꝋ}(\text{Ж}_1^-), \text{ꝋ}(\text{Ж}_2^-), \ldots,\text{ꝋ}(\text{Ж}_n^-) \rangle$, where $\text{ꝋ}(\text{Ж}_i^+)(\varrho) = rmin\ \{[½, ½], \text{Ж}_i^+(\varrho)\}$ and $\text{ꝋ}(\text{Ж}_i^-)(\varrho) = rmax\ \{[-½, -½], \text{Ж}_i^-(\varrho)\},\ \forall\ \varrho \in \mathcal{W}.$

(ii) $\bowtie (\text{Ж}) = \langle \bowtie(\text{Ж}_1^+), \bowtie(\text{Ж}_2^+), \ldots, \bowtie(\text{Ж}_n^+), \bowtie(\text{Ж}_1^-), \bowtie(\text{Ж}_2^-), \ldots,\bowtie(\text{Ж}_n^-) \rangle$, where $\bowtie(\text{Ж}_i^+)(\varrho) = rmax\ \{[½, ½], \text{Ж}_i^+(\varrho)\}$ and $\bowtie(\text{Ж}_i^-)(\varrho) = rmin\ \{[-½, -½], \text{Ж}_i^-(\varrho)\},\ \forall\ \varrho \in \mathcal{W}.$

(iii) $\mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}) = \langle\ \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_1^+), \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_2^+), \ldots, \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_n^+), \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_1^-), \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_2^-), \ldots, \mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_n^-) \rangle$, where $\mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_i^+)(\varrho) = rmin\ \{\varpi_i, \text{Ж}_i^+(\varrho)\}$ and $\mathfrak{Q}_{(\varpi,\varsigma)}(\text{Ж}_i^-)(\varrho) = rmax\ \{\varsigma_i, \text{Ж}_i^-(\varrho)\}$, $\forall\ \varrho \in \mathcal{W}, \varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \ldots, \varsigma_n), \varpi_i \in D[0, 1]\ \ and\ \varsigma_i \in D[-1, 0].$

(iv) $\mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}) = \langle\ \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_1^+), \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_2^+), \ldots, \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_n^+), \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_1^-), \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_2^-), \ldots, \mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_n^-) \rangle$, where $\mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_i^+)(\varrho) = rmax\ \{\varpi_i, \text{Ж}_i^+(\varrho)\}$ and $\mathfrak{R}_{(\varpi,\varsigma)}(\text{Ж}_i^-)(\varrho) = rmin\ \{\varsigma_i, \text{Ж}_i^-(\varrho)\},$

$\forall\ \varrho \in \mathcal{W}$, $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in D[0,1]$ *and* $\varsigma_i \in D[-1,0]$.

(v) $\mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}) = \langle\, \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_1^+), \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_2^+), \dots, \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_n^+), \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_1^-), \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_2^-), \dots, \mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_n^-) \rangle$, where $\mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_i^+)(\varrho) = \varpi_i \text{Ж}_i^+(\varrho)$ and $\mathfrak{S}_{(\varpi,\varsigma)}(\text{Ж}_i^-)(\varrho) = -\varsigma_i \text{Ж}_i^-(\varrho)$, $\forall\ \varrho \in \mathcal{W}$, $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in [0,1]$ *and* $\varsigma_i \in [-1,0]$.

2 – THEOREMS.

Theorem 2.1. *If* $\text{Ѣ} = \langle\, \text{Ѣ}_1^+, \text{Ѣ}_2^+, \dots, \text{Ѣ}_n^+, \text{Ѣ}_1^-, \text{Ѣ}_2^-, \dots, \text{Ѣ}_n^- \rangle$ and $\mathfrak{F} = \langle\, \mathfrak{F}_1^+, \mathfrak{F}_2^+, \dots, \mathfrak{F}_n^+, \mathfrak{F}_1^-, \mathfrak{F}_2^-, \dots, \mathfrak{F}_n^- \rangle$ *are two* $B_V MIFNSRs$ of a ring Ĕ_1, then their intersection $\text{Ѣ} \cap \mathfrak{F}$ *is also a* $B_V MIFNSR$ of Ĕ_1.

Theorem 2.2. *If* $\text{Ѝ} = \langle\, \text{Ѝ}_1^+, \text{Ѝ}_2^+, \dots, \text{Ѝ}_n^+, \text{Ѝ}_1^-, \text{Ѝ}_2^-, \dots, \text{Ѝ}_n^- \rangle$ is *a* $B_V MIFSR$ of a ring ß_1, then

ȣ (Ѝ)*is also a*$B_V MIFSR$ of ß_1.

Theorem 2.3. *If* $\mathfrak{C} = \langle\, \mathfrak{C}_1^+, \mathfrak{C}_2^+, \dots, \mathfrak{C}_n^+, \mathfrak{C}_1^-, \mathfrak{C}_2^-, \dots, \mathfrak{C}_n^- \rangle$ is *a* $B_V MIFNSR$ of a ring Ʊ_1, then

ȣ $(\mathfrak{C})$*is also a* $B_V MIFNSR$ of the ring Ʊ_1.

Proof. Let ζ, υ be in Ʊ₁. For all i = 1, 2, …, n, by Theorem 2.2, ȣ$(\mathfrak{C})$ is a $B_V MIFSR$ of Ʊ_1, ȣ$(\mathfrak{C}_i^+)(\zeta\upsilon)$ = rmin{[½,½], $\mathfrak{C}_i^+(\zeta\upsilon)$}= rmin{[½,½], $\mathfrak{C}_i^+(\upsilon\zeta)$}= ȣ$(\mathfrak{C}_i^+)(\upsilon\zeta)$, for all ζ, υ in Ʊ₁. Also ȣ$(\mathfrak{C}_i^-)(\zeta\upsilon)$ = rmax{[–½,–½], $\mathfrak{C}_i^-(\zeta\upsilon)$} = rmax{[–½,–½], $\mathfrak{C}_i^-(\upsilon\zeta)$}= ȣ$(\mathfrak{C}_i^-)(\upsilon\zeta)$, for all ζ, υ in Ʊ₁. Hence ȣ$(\mathfrak{C})$ is a $B_V MIFNSR$ of Ʊ₁**.**

Corollary 2.4. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ ring$ Ʊ_1, then ȣ$(\mathfrak{P} \cap \mathfrak{W})$ is a $B_V MIFNSR$ of Ʊ_1.

Proof. From the above Theorems, it is trivial.

Corollary 2.5. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $\text{Ʊ}_1$and Ʊ_2, then ȣ $\mathfrak{P}$ ∩ȣ $\mathfrak{W}$ is a $B_V MIFNSR$ of $\text{Ʊ}_1 \cap \text{Ʊ}_2$.

Proof. From the above Theorems, it is trivial.

Corollary 2.6. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ Ʊ_1, then ȣ $\mathfrak{P}$ ∩ȣ $\mathfrak{W}$ is a $B_V MIFNSR$ of Ʊ_1.

Proof. From the above Theorems, it is trivial.

Theorem 2.7. *If* $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$*are* $B_V MIFNSR$s of the rings $\text{Ʊ}_1, \text{Ʊ}_2, \dots, \text{Ʊ}_\mathrm{m}$

respectively, then ȣ $(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR$ of the ring $\text{Ʊ}_1 \cap \text{Ʊ}_2 \cap \dots \cap \text{Ʊ}_\mathrm{m}$.

Proof. From the above Theorems, *the proof is trivial.*

Corollary 2.8. *If* $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ *are* $B_V MIFNSR$s of the rings $\text{Ʊ}_1, \text{Ʊ}_2, \dots, \text{Ʊ}_\mathrm{m}$ respectively, then ȣ $\mathfrak{P}_1$ ∩ȣ $\mathfrak{P}_2$ ∩ … ∩ȣ $\mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $\text{Ʊ}_1 \cap \text{Ʊ}_2 \cap \dots \cap \text{Ʊ}_\mathrm{m}$.

Proof. From the above Theorems, *the proof is trivial.*

Corollary 2.9. *If* $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring Ʊ_1, then ȣ $(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR\ of$ the ring Ʊ_1.

Proof. From the above Theorems, $the\ proof\ is\ trivial$.

Corollary 2.10. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\between \mathfrak{P}_1 \cap \between \mathfrak{P}_2 \cap \dots \cap \between \mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial$.

Theorem 2.11. If $Ѣ = \langle Ѣ_1^+, Ѣ_2^+, \dots, Ѣ_n^+, Ѣ_1^-, Ѣ_2^-, \dots, Ѣ_n^- \rangle$ is a $B_V MIFSR$ of a ring $Ѝ_1$,

then $\bowtie (Ѣ)$ $is\ also\ aB_V MIFNSR$ of $Ѝ_1$.

Theorem 2.12. If $\mathfrak{C} = \langle \mathfrak{C}_1^+, \mathfrak{C}_2^+, \dots, \mathfrak{C}_n^+, \mathfrak{C}_1^-, \mathfrak{C}_2^-, \dots, \mathfrak{C}_n^- \rangle$ is a $B_V MIFNSR$ of a ring $Ʊ_1$, then

$\bowtie (\mathfrak{C})$ $is\ also\ aB_V MIFNSR$ of the ring $Ʊ_1$.

Proof. Let ζ, υ be in $Ʊ_1$. For all i = 1, 2, …, n, by Theorem 2.11, $\bowtie(\mathfrak{C})$ is a $B_V MIFSR$ of $Ʊ_1$, $\bowtie(\mathfrak{C}_i^+)(\zeta\upsilon)$ = rmax{[½,½], $\mathfrak{C}_i^+(\zeta\upsilon)$}= rmax{[½,½], $\mathfrak{C}_i^+(\upsilon\zeta)$}= $\bowtie(\mathfrak{C}_i^+)(\upsilon\zeta)$, for all ζ, υ in $Ʊ_1$. Also $\bowtie(\mathfrak{C}_i^-)(\zeta\upsilon)$ = rmin{[–½,–½], $\mathfrak{C}_i^-(\zeta\upsilon)$} = rmin{[–½,–½], $\mathfrak{C}_i^-(\upsilon\zeta)$}= $\bowtie(\mathfrak{C}_i^-)(\upsilon\zeta)$, for all ζ, υ in $Ʊ_1$. Hence $\bowtie(\mathfrak{C})$ is a $B_V MIFNSR$ of $Ʊ_1$.

Corollary 2.13. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ ring$ $Ʊ_1$, then $\bowtie(\mathfrak{P} \cap \mathfrak{W})$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Corollary 2.14. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$and $Ʊ_2$, then $\bowtie \mathfrak{P} \cap \bowtie \mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1 \cap Ʊ_2$.

Proof. From the above Theorems, it is trivial.

Corollary 2.15. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$, then $\bowtie \mathfrak{P} \cap \bowtie \mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Theorem 2.16. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m are$ $B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_{\mathrm{m}}$

respectively, then $\bowtie (\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR$ of the ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_{\mathrm{m}}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial$.

Corollary 2.17. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_{\mathrm{m}}$ respectively, then $\bowtie \mathfrak{P}_1 \cap \bowtie \mathfrak{P}_2 \cap \dots \cap \bowtie \mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_{\mathrm{m}}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial$.

Corollary 2.18. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\bowtie (\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR\ of$ the ring $Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial$.

Corollary 2.19. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\bowtie \mathfrak{P}_1 \cap \bowtie \mathfrak{P}_2 \cap \dots \cap \bowtie \mathfrak{P}_m$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Theorem 2.20. $If\ Ⅎ = \langle Ⅎ_1^+, Ⅎ_2^+, \dots, Ⅎ_n^+, Ⅎ_1^-, Ⅎ_2^-, \dots, Ⅎ_n^- \rangle$ is $a\ B_V MIFSR$ of a ring $\aleph_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(Ⅎ)$ is a $B_V MIFSR$ of $\aleph_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in D[0,1]\ and$ $\varsigma_i \in D[-1,0]$.

Theorem 2.21. $If\ \mathfrak{P} = \langle \mathfrak{P}_1^+, \mathfrak{P}_2^+, \dots, \mathfrak{P}_n^+, \mathfrak{P}_1^-, \mathfrak{P}_2^-, \dots, \mathfrak{P}_n^- \rangle$ is $a\ B_V MIFNSR$ of a ring $\mathfrak{R}_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $B_V MIFNSR$ of $\mathfrak{R}_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in D[0,1]\ and\ \varsigma_i \in D[-1,0]$.

Proof. Let $\xi, \hbar$ be in $\mathfrak{R}_1, \varpi_i \in D[0,1] and\ \varsigma_i \in D[-1,0]$. For all i = 1, 2, …, n, by Theorem 2.20, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $B_V MIFSR$ of $\mathfrak{R}_1$, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_i^+)(\xi\hbar)$ = rmin$\{\varpi_i,\ \mathfrak{P}_i^+(\xi\hbar)\}$ = rmin$\{\varpi_i,\ \mathfrak{P}_i^+(\hbar\xi)\}$= $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_i^+)(\hbar\xi)$, for all ξ, $\hbar$ in $\mathfrak{R}_1$. And $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_i^-)(\xi\hbar)$ = rmax$\{\varsigma_i,\ \mathfrak{P}_i^-(\xi\hbar)\}$ = rmax$\{\varsigma_i,$ $\mathfrak{P}_i^-(\hbar\xi)\}$= $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_i^-)(\hbar\xi)$, for all ξ, $\hbar$ in $\mathfrak{R}_1$. Hence $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $B_V MIFNSR$ of $\mathfrak{R}_1$**.**

Corollary 2.22. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ ring$ $Ʊ_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Corollary 2.23. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$and $Ʊ_2$, then $\mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1 \cap Ʊ_2$.

Proof. From the above Theorems, it is trivial.

Corollary 2.24. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Theorem 2.25. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m are\ B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_\mathrm{m}$ respectively, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR$ of the ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_\mathrm{m}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.26. $If\ \mathfrak{P}_1,\ \mathfrak{P}_2,\ \dots,\ \mathfrak{P}_m\ are\ B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_\mathrm{m}$ respectively, then $\mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_\mathrm{m}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.27. $If\ \mathfrak{P}_1,\ \mathfrak{P}_2,\ \dots,\ \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR\ of\ Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.28. $If\ \mathfrak{P}_1,\ \mathfrak{P}_2,\ \dots,\ \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{Q}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Theorem 2.29. $If\ \text{ƀ} = \langle \text{ƀ}_1^+, \text{ƀ}_2^+, \dots, \text{ƀ}_n^+, \text{ƀ}_1^-, \text{ƀ}_2^-, \dots, \text{ƀ}_n^- \rangle$ is $a\ B_V MIFSR$ of a ring $\ddot{\breve{O}}_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}(\text{ƀ})$ is a $B_V MIFSR$ of $\ddot{\breve{O}}_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in D[0,1]\ and\ \varsigma_i \in D[-1,0]$.

Theorem 2.30. $If\ \mathfrak{C} = \langle \mathfrak{C}_1^+, \mathfrak{C}_2^+, \dots, \mathfrak{C}_n^+, \mathfrak{C}_1^-, \mathfrak{C}_2^-, \dots, \mathfrak{C}_n^- \rangle$ is $a\ B_V MIFNSR$ of a ring $\mathbb{R}_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C})$ is a $B_V MIFNSR$ of $\mathbb{R}_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in D[0,1]\ and\ \varsigma_i \in D[-1,0]$.

Proof. Let $\xi, \hbar$ be in $\mathbb{R}_1$, $\varpi_i \in D[0,1] and\ \varsigma_i \in D[-1,0]$. For all i = 1, 2, …, n, by Theorem 2.29, $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C})$ is a $B_V MIFSR$ of $\mathbb{R}_1$, $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C}_i^+)(\xi\hbar)$ = rmax{ϖ_i, $\mathfrak{C}_i^+(\xi\hbar)$} = rmax{$\varpi_i$, $\mathfrak{C}_i^+(\hbar\xi)$}= $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C}_i^+)(\hbar\xi)$, for all ξ, $\hbar$ in $\mathbb{R}_1$. And $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C}_i^-)(\xi\hbar)$ = rmin{ς_i, $\mathfrak{C}_i^-(\xi\hbar)$} = rmin{$\varsigma_i$, $\mathfrak{C}_i^-(\hbar\xi)$}= $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C}_i^-)(\hbar\xi)$, for all ξ, $\hbar$ in $\mathbb{R}_1$. Hence $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{C})$ is a $B_V MIFNSR$ of $\mathbb{R}_1$.

Corollary 2.31. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ ring\ \mho_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $B_V MIFNSR$ of $\mho_1$.

Proof. From the above Theorems, it is trivial.

Corollary 2.32. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings\ \mho_1$and $\mho_2$, then $\mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $\mho_1 \cap \mho_2$.

Proof. From the above Theorems, it is trivial.

Corollary 2.33. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings\ \mho_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $\mho_1$.

Proof. From the above Theorems, it is trivial.

Theorem 2.34. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m are\ B_V MIFNSR$s of the rings $\mho_1, \mho_2, \dots, \mho_\mathrm{m}$ respectively, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR$ of the ring $\mho_1 \cap \mho_2 \cap \dots \cap \mho_\mathrm{m}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.35. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m\ are\ B_V MIFNSR$s of the rings $\mho_1, \mho_2, \dots, \mho_\mathrm{m}$ respectively, then $\mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $\mho_1 \cap \mho_2 \cap \dots \cap \mho_\mathrm{m}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.36. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $\mho_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR\ of\ \mho_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.37. $If\ \mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $\mho_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{R}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of $\mho_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Theorem 2.38. If $Ж = \langle Ж_1^+, Ж_2^+, \dots, Ж_n^+, Ж_1^-, Ж_2^-, \dots, Ж_n^- \rangle$ is a $B_V MIFSR$ of a ring $ɮ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}(Ж)$ is a $B_V MIFSR$ of $ɮ_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in [0,1]$ and $\varsigma_i \in [-1,0]$.

Theorem 2.39. If $\mathcal{B} = \langle \mathcal{B}_1^+, \mathcal{B}_2^+, \dots, \mathcal{B}_n^+, \mathcal{B}_1^-, \mathcal{B}_2^-, \dots, \mathcal{B}_n^- \rangle$ is a $B_V MIFNSR$ of a ring $Ʊ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B})$ is a $B_V MIFNSR$ of $Ʊ_1$, where $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)$ and $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_n)$, $\varpi_i \in [0,1]$ and $\varsigma_i \in [-1,0]$.

Proof. Let σ, υ be in $Ʊ_1$, $\varpi_i \in [0,1] and\ \varsigma_i \in [-1,0]$. For all i = 1, 2, …, n, by Theorem 2.38, $\mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B})$ is a $B_V MIFSR$ of $Ʊ_1$, $\mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B}_i^+)(\sigma\upsilon) = \varpi_i\ \mathcal{B}_i^+(\sigma\upsilon) = \varpi_i\ \mathcal{B}_i^+(\upsilon\sigma) = \mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B}_i^+)(\upsilon\sigma)$, for all σ, υ in $Ʊ_1$. And $\mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B}_i^-)(\sigma\upsilon) = (-\varsigma_i)\ \mathcal{B}_i^-(\sigma\upsilon) = (-\varsigma_i)\ \mathcal{B}_i^-(\upsilon\sigma) = \mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B}_i^-)(\upsilon\sigma)$, for all σ, υ in $Ʊ_1$. Hence $\mathfrak{S}_{(\varpi,\varsigma)}(\mathcal{B})$ is a $B_V MIFNSR$ of $Ʊ_1$.

Corollary 2.40. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ ring$ $Ʊ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Corollary 2.41. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$and $Ʊ_2$, then $\mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1 \cap Ʊ_2$.

Proof. From the above Theorems, it is trivial.

Corollary 2.42. If $\mathfrak{P}$ and $\mathfrak{W}$ are $B_V MIFNSRs\ of\ the\ rings$ $Ʊ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P} \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{W}$ is a $B_V MIFNSR$ of $Ʊ_1$.

Proof. From the above Theorems, it is trivial.

Theorem 2.43. If $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_m are$ $B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_{\mathrm{m}}$ respectively, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR$ of the ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_{\mathrm{m}}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.44. If $\mathfrak{P}_1$, $\mathfrak{P}_2$, …, $\mathfrak{P}_m$ are $B_V MIFNSR$s of the rings $Ʊ_1, Ʊ_2, \dots, Ʊ_{\mathrm{m}}$ respectively, then $\mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of the

ring $Ʊ_1 \cap Ʊ_2 \cap \dots \cap Ʊ_{\mathrm{m}}$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.45. If $\mathfrak{P}_1$, $\mathfrak{P}_2$, …, $\mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $B_V MIFNSR\ of$ the ring $Ʊ_1$.

Proof. From the above Theorems, $the\ proof\ is\ trivial.$

Corollary 2.46. If $\mathfrak{P}_1$, $\mathfrak{P}_2$, …, $\mathfrak{P}_m$ are $B_V MIFNSR$s of the ring $Ʊ_1$, then $\mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_1 \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_2 \cap \dots \cap \mathfrak{S}_{(\varpi,\varsigma)}\mathfrak{P}_m$ is a $B_V MIFNSR$ of the ring $Ʊ_1$.

Proof. From the above Theorems, *the proof is trivial.*

CONCLUSION

In this paper, *various types of translation of* bipolar valued multi I- fuzzy normal subring of a ring have been introduced. Some useful theorems have been found and using these theorem we can find more results. It can be extended into different types of algebra.

REFERENCES

[1]. Anitha.M.S., Muruganantha Prasad & K.Arjunan, "Notes on Bipolar-valued fuzzy subgroups of a group", *Bulletin of Society for Mathematical Services and Standards,*Vol. 2 No. 3 (2013), pp. 52 – 59.

[2]. Arsham Borumand Saeid, "Bipolar-valued fuzzy BCK/BCI-algebras", *World Applied Sciences Journal,* 7 (11) (2009), 1404 – 1411.

[3]. Azriel Rosenfeld, "Fuzzy groups", *Journal of mathematical analysis and applications,* 35(1971), 512 – 517.

[4]. Balasubramanian.A, K.L.Muruganantha Prasad & K.Arjunan, "Properties of Bipolar interval valued fuzzy subgroups of a group", *International Journal of Scientific Research,* Vol. 4, Iss. 4 (2015), 262 - 268.

[5]. Grattan-Guiness, "Fuzzy membership mapped onto interval and many valued quantities", *Z.Math.Logik. Grundladen Math.* 22 (1975), 149 – 160.

[6]. Kyoung Ja Lee, "Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI- algebras", *Bull. Malays.Math. Sci. Soc.,* (2) 32(3) (2009), 361 – 373.

[7]. K.M.Lee, "Bipolar-valued fuzzy sets and their operations". *Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand,* (2000), 307 – 312.

[8]. K.M.Lee, "Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets". *J. fuzzy Logic Intelligent Systems,* 14 (2) (2004), 125 –129.

[9]. Murugalingam.K and K.Arjunan, "A study on interval valued fuzzy subsemirings of a semiring", *International Journal of Applied Mathematics and Modeling,* Vol. 1, No. 5 (2013), 1 – 6.

[10]. Sabu Sebastian, T.V.Ramakrishnan, "Multi fuzzy sets", *International Mathematical Forum,* 5, no.50 (2010), 2471 –2476.

[11]. Vairamuthu.K,S.Loganathan, "Product in bipolar valued multi I-fuzzy subrings of a ring", *Gradiva Review Journal,* Vol.8, Issue 11(2022).

[12]. Vairamuthu.K,S.Loganathan, "Bipolar valued multi I-fuzzy subrings of a ring", *Journal for Basic Sciences,* Vol.23, Issue 1(2023).

[13]. Yasodara.S, KE. Sathappan, "Bipolar-valued multi fuzzy subsemirings of a semiring", *International Journal of Mathematical Archive*, 6(9) (2015), 75 –80.

[14]. L.A.Zadeh, fuzzy sets, Inform. And Control, 8(1965), 338 –353.

[15]. W.R.Zhang, Bipolar Fuzzy sets and Relations, *a computational Frame work for cognitive modeling and multiple decision Analysis, proceedings of Fuzzy IEEE conferences,* (1994), 305–309.