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CERTAIN BOUNDS FOR A SUBCLASSES OF ANALYTIC FUNCTIONS OF RECIPROCAL ORDER

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ABSTRACT. In this paper, we introduce certain class of bi univalent functions related to shell like curves connected with Fibonacci numbers. Also we determine initial Taylor-Maclaurin coefficient inequalities and Fekete Szego problem for the belonging class.

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1. INTRODUCTION AND PRELIMINARIES

Let $\mathcal{A}$ denote the family of normalized analytic functions f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \qquad (z \in \mathbb{U}) \tag{1.1}$$

in the open disc $\mathbb{U} = \{z : z \in \mathbb{C} : |z| < 1\}$. Further, let $\mathcal{S}$ denote the class of functions in $\mathcal{A}$ which are also univalent in $\mathbb{U}$.

The well-known Koebe one-quarter theorem [2] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{A}$ contains a disk of radius 1/4. Hence every univalent function f has an inverse f^{-1} satisfying $f^{-1}(f(z)) = z, (z \in \mathbb{U})$ and

$$f^{-1}(f(w)) = w, (|w| < r_0(f), r_0(f) \geq 1/4)$$

where,

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). For example, functions in the class Σ are given below [13]:

$$\frac{z}{1-z}, \quad -log(1-z), \quad \frac{1}{2}log\left(\frac{1+z}{1-z}\right).$$

In 1967, Lewin [8] introduced the class Σ of bi-univalent functions and shown that $|a_2| < 1.51$. In 1969, Netanyahu [10] showed that $max_{f\in\Sigma}|a_2| = 4/3$ and Suffridge [14] have given an example of $f \in \Sigma$ for which $|a_2| = 4/3$. Later, in 1980, Brannan

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and Clunie [1] improved the result as $|a_2| \leq \sqrt{2}$. In 1985, Kedzier-awski [7] proved this conjecture for a special case when the function f and f^{-1} are starlike. In 1984, Tan [15] proved that $|a_2| \leq 1.485$ which is the best estimate for the function in the class of bi-univalent functions.

Let the functions f and g be analytic in $\mathbb{U}$. Then we say that f is subordinate to g, if there exits a Schwarz function $\omega(z)$, analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ in $\mathbb{U}$, such that $f(z) = g(\omega(z)), z \in \mathbb{U}$. We denote this subordination by $f \prec g$. Sokol [12] introduced shell like functions $\mathcal{SL}$ and convex shell like functions $\mathcal{KSL}$ as the set of functions $f \in \mathbb{U}$ which are defined as follows

Definition 1. *The function $f \in \mathcal{A}$ belongs to the class $\mathcal{SL}$ and $\mathcal{KSL}$ if it satisfies the following condition*

$$\frac{zf'(z)}{f(z)} \prec \tilde{p}$$

and

$$1 + \frac{zf''(z)}{f'(z)} \prec \tilde{p}$$

with

$$\frac{1+\tau^2 z^2}{1-\tau z-\tau^2 z^2}$$

where $\tau = \dfrac{1-\sqrt{5}}{2} \approx -0.618$.

The function $\tilde{p}$ is not univalent in $\mathbb{U}$, but it is univalent in the disk $|z| < (3-\sqrt{5})/2 \approx 0.38$. For example, $\tilde{p}(0) = \tilde{p}(-1/2\tau) = 1$ and $\tilde{p} = \sqrt{5}/5$, and it may also be noticed that $\dfrac{1}{|\tau|} = \dfrac{|\tau|}{1-|\tau|}$, which shows that the number $|\tau|$ divides $[0,1]$ such that it fulfils the golden section. The image of the unit circle $|z| = 1$ under $\tilde{p}$ is a curve described by the equation given by

$$(10x-\sqrt{5})y^2 = (\sqrt{5}-2x)(\sqrt{5}x-1)^2,$$

which is translated and revolved trisectrix of Maclaurin. The curve $\tilde{p}(re^{it})$ is a closed curve without any loops for $0 < r \leq r_0 = (3-\sqrt{5})/2 \approx 0.38$. For $r_0 < r < 1$, it has a loop, and for r=1, it has a vertical asymptote. Since τ satisfies the equation $\tau^2 = 1+\tau$, this expression can be used to obtain higher powers τ^n as a linear function of lower powers, which in turn can be decomposed all the way down to a linear combination of τ and 1. The resulting recurrence relationships yield Fibonacci

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numbers u_n: $\tau^n = u_n\tau + u_{n-1}$. In [6], taking τz=t, Raina and Sokol showed that

$$\begin{aligned}\tilde{p}(z) &= \frac{1+\tau^2z^2}{1-\tau z-\tau^2z^2}\\ &= (t+\frac{1}{t})\frac{t}{1-t-t^2}\\ &= \frac{1}{\sqrt{5}}(t+\frac{1}{t})(\frac{1}{1-(1-\tau)t}-\frac{1}{1-\tau t})\\ &= (t+\frac{1}{t})\sum_{n=1}^{\infty}\frac{(1-\tau)^n-\tau^n}{\sqrt{5}}\tau^n\\ &= (t+\frac{1}{t})\sum_{n=1}^{\infty}u_nt^n = 1+\sum_{n=1}^{\infty}(u_{n-1}+u_{n+1})\tau^nz^n,\end{aligned}$$

where

$$u_n = \frac{(1-\tau)^n-\tau^n}{\sqrt{5}}, \tau = \frac{1-\sqrt{5}}{2} \qquad (n = 1, 2, ...).$$

This shows that the relevant connection of $\tilde{p}$ with the sequence of Fibonacci numbers u_n, such that $u_o = 0$, $u_1 = 1, u_{n+2} = u_n + u_{n+1}$ for n=0,1,2,... and they got

$$\begin{aligned}\tilde{p}(z) &= 1+\sum_{n=1}^{\infty}\tilde{p}_nz^n\\ &= 1+(u_o+u_z)\tau z+(u_1+u_3)\tau^2z^2\\ &+\sum_{n=3}^{\infty}(u_n-3+u_n-2+u_n-1+u_n)\tau^nz^n\\ &= 1+\tau z+3\tau^2z^2+4\tau^3z^3+7\tau^4z^4+11\tau^5z^5+...\end{aligned}$$

Let $\mathcal{P}(\beta)$,$0 \leq \beta < 1$, denote the class of analytic functions p in $\mathbb{U}$ with $p(0) = 1$ and $Re\{P(z)\} > \beta$. Especially, we will use $\mathcal{P}$ instead of $\mathcal{P}(0)$.

Firstly, let $p(z) = 1 + p_1z + p_2z^2 + ...$ and p $\prec \tilde{p}$. Then there exists an analytic function $u$such that $|u(z)| < 1$ in $\mathbb{U}$ and $p(z) = \tilde{p}(u(z))$. Therefore, the function

$$h(z) = \frac{1+u(z)}{1-u(z)} = 1 + c_1z + c_2z^2 + ...$$

is in the class $P(0)$. It follows that

$$u(z) = \frac{c_1z}{2} + \left(c_2 - \frac{c_1^2}{2}\right)\frac{z^2}{2} + \left(c_3 - c_1c_2 + \frac{c_1^3}{4}\right)\frac{z^3}{2} + ...$$

and

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$$\begin{aligned}\tilde{p}(u(z)) =& 1+\tilde{p_1}\left\{\frac{c_1 z}{2}+\left(c_2-\frac{c_1^2}{2}\right)\frac{z^2}{2}+\left(c_3-c_1c_2+\frac{c_1^3}{4}\right)\frac{z^3}{2}+...\right\}\\ &+\tilde{p_2}\left\{\frac{c_1 z}{2}+\left(c_2-\frac{c_1^2}{2}\right)\frac{z^2}{2}+(c_3-c_1c_2\ +\ \frac{c_1^3}{4}\right)\frac{z^3}{2}+...\right\}^2\\ &+\tilde{p_3}\left\{\frac{c_1 z}{2}+\left(c_2-\frac{c_1^2}{2}\right)\frac{z^2}{2}+(c_3-c_1c_2\ +\ \frac{c_1^3}{4}\right)\frac{z^3}{2}+...\right\}^3+...\end{aligned}$$

$$\begin{aligned}\tilde{p}(u(z)) =& 1+\frac{\tilde{p_1}c_1 z}{2}+\left\{\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)\tilde{p_1}+\frac{c_1^2}{4}\tilde{p_2}\right\}z^2\\ &+\left\{\frac{1}{2}\left(c_3-c_1c_2+\frac{c_1^3}{4}\right)\tilde{P_1}+\frac{1}{2}c_1\left(c_2-\frac{c_1^2}{2}\right)\tilde{p_2}+\frac{c_1^3}{8}\tilde{p_3}\right\}z^3+...\end{aligned} \tag{1.3}$$

and similarly, there exists an analytic function v such that $|v(w)|<1$ in $\mathbb{U}$ and $p(w)=\tilde{p}(v(w))$. Therefore, the function

$$k(w)=\frac{1+v(w)}{1-v(w)}=1+d_1w+d_2w^2+...$$

is in the class P(0). It follows that

$$v(w)=\frac{d_1 w}{2}+\left(d_2-\frac{d_1^2}{2}\right)\frac{w^2}{2}+\left(d_3-d_1d_2+\frac{d_1^3}{4}\right)\frac{w^3}{2}+...$$

and

$$\begin{aligned}\tilde{p}(v(w)) =& 1+\frac{\tilde{p_1}d_1 w}{2}+\left\{\frac{1}{2}\left(d_2-\frac{d_1^2}{2}\right)\tilde{p_1}+\frac{d_1^2}{4}\tilde{p_2}\right\}w^2\\ &+\left\{\frac{1}{2}\left(d_3-d_1d_2+\frac{d_1^3}{4}\right)\tilde{p_1}+\frac{1}{2}d_1\left(d_2-\frac{d_1^2}{2}\right)\tilde{p_2}+\frac{d_1^3}{8}\tilde{p_3}\right\}w^3+...\end{aligned} \tag{1.4}$$

Recently many authors introduced and investigated subclasses of bi-univalent functions related to shell - like curves([5],[9]).

Definition 2. *The function $f(z)$ in (1.1) belongs to the class $\mathcal{M}_{\tau,\lambda}^{b,q}$ if it satisfies the following condition*

$$(\Delta_q f(z))\left(\frac{(1-b)z}{f(z)-F(bz)}\right)^{\lambda}\prec\frac{1+\tau^2z^2}{1-\tau z-\tau^2z^2}\qquad(\lambda\geq 1) \tag{1.5}$$

where $\tau=\dfrac{1-\sqrt{5}}{2}\approx -0.618.$

Theorem 1. [3] *The function* $\tilde{p}(z)=\dfrac{1+\tau^2z^2}{1-\tau z-\tau^2z^2}$ *belongs to the class* $\mathcal{P}(\beta)$ *with* $\beta=\sqrt{5}/10\approx 0.2236$. *The following lemma will be useful to prove our result.*

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Lemma 1. [11] *Let* $p \epsilon \mathcal{P}$ *with* $p(z) = 1 + c_1 z + c_2 z^2 + \ldots$, *then*

$$|c_n| \leq 2, \quad for \quad n \geq 1.$$

2. Initial Coefficients

In this section, we introduce a new subclass of Σ associated with λ-pseudo bistar-like functions with respect to symmetric points related to shell-like curves connected with Fibonacci numbers. Motivated by the works of Dziok et al. in [3] on the class of convex shell like functions connected with Fibonacci numbers we obtain the initial Taylor coefficients $|a_2|$ and $|a_3|$ for the function class by subordination.

Theorem 2. *If f is given by* (1.1) *is in the class* $\mathcal{M}_{\tau,\lambda}^{b,q}$ *then*

$$|a_2| \leq \frac{|\tau|}{\sqrt{\left|\lambda\tau\left[\frac{(1+\lambda)}{2}(1+b)^2 - [2]_q(1+b)\right] + \tau\left[[3]_q - \lambda(1+b+b^2)\right] + (\lambda(1+b) - [2]_q)^2(1+3\tau)\right|}}$$

and

$$|a_3| \leq \frac{|\tau|^2}{|(\lambda(1+b) - [2]_q)^2|} + \frac{|\tau|}{|([3]_q - \lambda(1+b+b^2))|}.$$

Proof.

$$\begin{aligned}(\Delta_q f(z))\left(\frac{(1-b)z}{f(z)-F(bz)}\right)^{\lambda} &= 1 + \left[[2]_q - \lambda(1+b)\right] a_2 z \\ &+ \left[([3]_q - \lambda(1+b+b^2))a_3 + \lambda\left(\frac{(\lambda-1)}{2}(1+b)^2 - (1+b)[2]_q\right) a_2^2\right] z^2 \\ &+ \left[([4]_q - \lambda(1+b)(1+b)^2)a_4 + \lambda(\lambda-1)(1+b)(1+b+b^2)\right. \\ &- \lambda[3]_q(1+b)a_2 a_3 + \left(\frac{[2]_q\lambda(\lambda-1)(1+b)^2}{2} - \frac{\lambda(\lambda+1)(\lambda+2)(1+b)^2}{6}\right. \\ &- \left.\left.\lambda[3]_q(1+b+b^2)\right) a_2^3\right] z^3 + \ldots\end{aligned} \tag{2.1}$$

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and

$$(\Delta_q g(w))\left(\frac{(1-b)z}{g(w)-g(bw)}\right)^{\lambda} = 1+\left[\left(\lambda(1+b)-[2]_q\right)a_2\right]w$$
$$\left\{\left[\lambda\left(\frac{(\lambda+1)(1+b)^2}{2}-[2]_q(1+b)\right)+2\left[[3]_q-\lambda(1+b+b^2)\right]\right]a_2^2\right.$$
$$\left.-\left[[3]_q-\lambda(1+b+b^2)a_3\right]\right\}w^2+\left[\left\{5[4]_q-\lambda[2]_q(1+b+b^2)\right.\right.$$
$$\left.-\lambda[3]_q(1+b)-5\lambda(1+b)(1+b^2)\right\}a_2a_3+\left\{5\lambda(1+b)(1+b^2)\right.$$
$$-2\lambda(\lambda+1)(1+b)(1+b+b^2)+\frac{\lambda(\lambda+1)(\lambda+2)(1+b)^3}{6}$$
$$+2\lambda[2]_q(1+b+b^2)-\frac{[2]_q\lambda(\lambda+1)(1+b)^2}{2}$$
$$\left.+2\lambda[3]_q(1+b)-5[4]_q\right\}a_2^3-\left\{[4]_q-\lambda(1+b)(1+b^2)\right\}a_4\Big]w^3+... \tag{2.2}$$

By Comparing (2.1) and (1.3) gives,

$$\left[[2]_q-\lambda(1+b)\right]a_2=\frac{c_1\tau}{2} \tag{2.3}$$

$$\lambda\left[\frac{(1+\lambda)}{2}(1+b)^2-[2]_q(1+b)\right]a_2^2+\left([3]_q-\lambda(1+b+b^2)\right)a_3=\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)\tau+\frac{c_1^2}{4}3\tau^2 \tag{2.4}$$

$$\left[\lambda(1+b)-[2]_q\right]a_2=\frac{d_1}{2}\tau \tag{2.5}$$

$$\left(\lambda\left[\frac{(1+\lambda)}{2}(1+b)^2-[2]_q(1+b)\right]+2\left[[3]_q-\lambda(1+b+b^2)\right]\right)a_2^2-\left([3]_q-\lambda(1+b+b^2)\right)a_3$$
$$=\frac{1}{2}\left(d_2-\frac{d_1^2}{2}\right)\tau+\frac{d_1^2}{4}3\tau^2 \tag{2.6}$$

Adding (2.3) and (2.5),

$$c_1=-d_1\Rightarrow c_1^2=d_1^2 \tag{2.7}$$

Squaring (2.3) and (2.5) and adding,we have

$$2(\lambda(1+b)-[2]_q)^2a_2^2=\frac{1}{4}(c_1^2+d_1^2)\tau^2 \tag{2.8}$$

$$a_2^2=\frac{(c_1^2+d_1^2)\tau^2}{\gamma(\lambda(1+b)-[2]_q)^2} \tag{2.9}$$

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Adding (2.4) and (2.6) and using (2.4) implies

$$2\lambda\left[\frac{(1+\lambda)}{2}(1+b)^2-[2]_q(1+b)\right]a_2^2+2\left[[3]_q-\lambda(1+b+b^2)\right]a_2^2=\frac{\tau}{2}(c_2+d_2)-c_1^2\left(\frac{\tau-3\tau^3}{2}\right) \tag{2.10}$$

Using (2.7) in (2.8), we have

$$c_1^2=\frac{4(\lambda(1+b)-[2]_q)^2}{\tau^2}$$

Substitute for c_1^2 in (2.10), we have

$$|a_2|^2\leq\frac{|\tau|^2|c_2+d_2|}{|\lambda\tau\left[\frac{(1+\lambda)}{2}(1+b)^2-[2]_q(1+b)\right]+\tau\left[[3]_q-\lambda(1+b+b^2)\right]+(\lambda(1+b)-[2]_q)^2(1+3\tau)|}$$

Using lemma $|c_n|\leq 2,|d_n|\leq 2$

$$|a_2|\leq\frac{|\tau|}{\sqrt{|\lambda\tau\left[\frac{(1+\lambda)}{2}(1+b)^2-[2]_q(1+b)\right]+\tau\left[[3]_q-\lambda(1+b+b^2)\right]+(\lambda(1+b)-[2]_q)^2(1+3\tau)|}}$$

By subtracting (2.6) from (2.4) and simplifying,

$$2([3]_q-\lambda(1+b+b^2))a_3-2([3]_q-\lambda(1+b+b^2))a_2^2=\frac{\tau}{2}(c_2-d_2)-\left(\frac{\tau}{4}-\frac{3\tau^2}{4}\right)(c_1^2-d_1^2)$$

Applying equation (2.7) and $|c_n|\leq 2,|d_n|\leq 2$, we have

$$a_3=a_2^2+\frac{\tau(c_2-d_2)}{4([3]_q-\lambda(1+b+b^2))} \tag{2.11}$$

$$|a_3|\leq\frac{|\tau|^2}{|(\lambda(1+b)-[2]_q)^2|}+\frac{|\tau|}{|([3]_q-\lambda(1+b+b^2))|}$$

□

3. Fekete-Szegö Inequality

Fekete and Szegö [4] introduced the generalized functional $|a_3-\mu a_2^2|$, where μ is some real number. Due to Zaprawa [16], in the following theorem we determine the Fekete-Szegö functional for $f\in\mathcal{M}_{\tau,\lambda}^{b,q}$.

Theorem 3. *If f is given by* (1.1) *is in the class* $\mathcal{M}_{\tau,\lambda}^{b,q}$ *then,*

$$|a_3-\nu a_2^2|\leq\begin{cases}2|1-\mu|\Omega_1(\tau,\lambda,b,q) & \textit{if } |(1-\mu)|\Omega_1(\tau,\lambda,b,q)\geq\dfrac{\tau}{([3]_q-\lambda(1+b+b^2))}\\ \dfrac{2\tau}{([3]_q-\lambda(1+b+b^2))} & \textit{if } |(1-\mu)|\Omega_1(\tau,\lambda,b,q)\leq\dfrac{\tau}{([3]_q-\lambda(1+b+b^2))}\end{cases}$$

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Proof. By using the values of a_2 and a_3,

$$a_3 - \mu a_2^2 = a_2^2 + \frac{\tau(c_2 - d_2)}{4([3]_q - \lambda(1+b+b^2))} - \mu a_2^2$$

$$|a_3 - \mu a_2^2| \leq \frac{\tau}{([3]_q - \lambda(1+b+b^2))} + (1-\mu)\Omega_1(\tau, \lambda, b, q) \tag{3.1}$$

Where

$$\Omega_1(\tau, \lambda, b, q) = \frac{\tau^2}{\lambda\tau\left[\frac{(1+\lambda)}{2}(1+b)^2 - [2]_q(1+b)\right] + \tau\left[[3]_q - \lambda(1+b+b^2)\right] + (\lambda(1+b) - [2]_q)^2(1+3\tau)}$$

If

$$(1-\mu)\Omega_1(\tau, \lambda, b, q) \geq \frac{\tau}{([3]_q - \lambda(1+b+b^2))}$$

then,

$$|a_3 - \mu a_2^2| \leq 2|1-\mu|\Omega_1(\tau, \lambda, b, q)$$

where

$$|(1-\mu)| \geq \frac{\tau}{\Omega_1([3]_q - \lambda(1+b+b^2))}$$

and suppose

$$(1-\mu)\Omega_1(\tau, \lambda, b, q) \leq \frac{\tau}{([3]_q - \lambda(1+b+b^2))}$$

then

$$|a_3 - \mu a_2^2| \leq \frac{2\tau}{([3]_q - \lambda(1+b+b^2))}$$

where

$$|1-\mu| \leq \frac{\tau}{\Omega_1(\tau, \lambda, b, q)([3]_q - \lambda(1+b+b^2))}.$$

□

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