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Series Solution and Wave Solution of Time Fractional Generalized Korteweg-de Vries Equation

Mitu Nagpal[1] and Rajeev Kumar[2]

[1,2]Department of Mathematics, MMEC, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala-133207(Haryana), India

Corresponding should be addressed to Mitu Nagpal; gakharmitu@gmail.com

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Abstract: In the present article, exact solutions of nonlinear fractional p^{th} order Korteweg-de Vries equation time fractional derivatives are deduced and analyzed. The Lie symmetry approach has been used to identify the fractional KdV equation's infinitesimal generators and symmetry reductions. Some new exact solutions are obtained with using $\left(\frac{G'}{G}\right)$–expansion method. The wave solutions and series solutions of KdV nonlinear fractional partial differential equation has been evaluated and in the form of rational and exponential. All the calculations have done in Maple.

Keywords: Korteweg-de Vries Equation, Lie Symmetry, Conformable Derivative, Series Solution, Wave Solution, $\left(\frac{G'}{G}\right)$– Expansion Method.

1. Introduction

1.1 Scope

In applied mathematics and physics, nonlinear phenomena are commonly present. In applied mathematics and mathematical physics Non-Linear Partial Differential Equations (NLPDEs) are applied for modeling several scientific processes and issues. In obtaining the Non-Linear Fractional Partial Differential Equations NLFPDEs [1] approximate solutions are vital in all of these fields of study. We mainly concentrate on estimating the precise solutions because we don't have a mechanism for finding the precise solutions of these kinds of Fractional Partial Differential Equations (FPDEs).

To solve FPDEs, a number of techniques have been used, e.g., Elzaki transform decomposition method [2], Laplace transforms approach [3], and Adomian's decomposition technique [4] furthermore. There are various techniques to analyze the NLPDEs but lie group of symmetry is one of the approaches for deducing the solution of NLPDEs. Lie Symmetries studies differential equation invariance under a one-parameter group transformations that turn into a new solution and minimize the order of differential equation. Lie developed the theory of one parameter group of transformation in 17th century which applied for findings the solution of Partial Differential Equations (PDEs). Later on, many contributors like Bluman [6], Birkoff [5] and Olver [7] et. al. Many researchers developed different methods [8] for finding exact solution of NLPDEs, such as tanh method [9], the hyperbolic B-spline differential quadrature method [10] and new extended $\left(\frac{G'}{G}\right)$ method [11], etc.

1.2 Review of Related Work:

Korteweg-de Vries (KdV) equation is the generalized form of fractional PDEs. KdV equations can be utilized for predicting solitary waves, long waves, tides and wave propagation in a shallow canal [12-16]. Fluid mechanics [17], viscoelasticity, signal processing, fractional kinetics and hydrology are among some of the disciplines that use the KdV equations.

J. S. Russell's (1834) provided the concept of KdV equations. However, the equation is developed by Lord Rayleigh, Joseph Boussinesq (1870) and KdV (1895). The numerical and exact solitary wave solutions of the generalized long wave and KdV equation has obtained by D. Kaya and S. M. EL Sayad [18] for the initial conditions. D. D. Bhatta and M. I. Bhatti [19] presented an algorithm for numerical solutions of KdV equation on reformed Bernstein polynomials.

A. R. Seadawy [20] studied the exact solution for the KdV equation of higher order non-linearity by using the variational technique. Exact solutions of KdV equation of higher order non-linearity are determined for utilizing the variational principle method without requiring for significant calculations. A. R. Seadawy, D. Lu and C. Yue studied fifth-order generalized KdV equations [21] on water wave equations.

G. Wang obtained series solution and invariant solutions of KdV–Burgers–Kuramoto generalized equation and deduced invariants and invariant solutions based on the Lie point symmetries [22]. A. A. Alderremy, A. Shaban, et.al. [23] investigated third order KdV equations and coupled Burgers equations with the help of two methods. H. Rezazadeh, A. G. Davodi, et.al. [24] applied the $\left(\frac{G'}{G}\right)$ expansion technique for traveling wave solutions of the Schrödinger-KdV equations by the conformable derivative.

1.3 Motivation of study

In the above related work we observed that the solution of KdV equation solved by the theory of Riemann – Liouville. This theory has limitation that cannot obtain the traveling wave solutions. Fractional derivatives have changed as a result of the use of conformable fractional derivatives [26] by Khalil et.al. to identify Lie symmetries in differential equations. In this paper, by conformable fractional derivatives [32-37] we will find Lie symmetry of KdV equation.

The generalized pth order KdV nonlinear fractional partial differential equation:

$$u_t^{\alpha} - u_{xx} - Au^{p}u_x = 0 \ , 0 < \alpha \leq 1, p > 0 \ \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and u_t^{α} is the conformable fractional derivative [38-39].

Objective of the study:

a. To find the lie symmetry of KdV NLFPDEs.
b. To find series solution of the problem.
c. To apply the $\left(\frac{G'}{G}\right)$ - expansion approach to determine the wave solution.

This paper divides in following sections:

Section 2, Preliminaries are described.

Section3, Analysis of NLFPDEs by Lie Symmetry.

Section 4, Includes the description of $\left(\frac{G'}{G}\right)$- Expansion Method.

Section 5, Symmetry analysis of KdV NLFPDEs.

Section 6, Findings of series solution of KdV equation

Section 7, Exact traveling wave solution of fractional KdV equation is provided in trigonometric form.

At last, the brief of the research work is recorded in the section 8.

2. Preliminaries:

2.1 Conformable Derivative and Conformable Fractional Derivatives:

If function f: $[0,\infty) \rightarrow R$ then conformable fractional derivative [7] is defined by:

$$T_\alpha\,(\text{f})(\text{ŧ}) = \lim_{\epsilon \to 0} \frac{\text{f}(\text{ŧ} + \epsilon \text{ŧ}^{1-\alpha}) - \text{f}(\text{ŧ})}{\epsilon}$$

For all ŧ > 0,$\in (0,1)$, if f is α -differentiable in some (0, α), a >0 and $\lim_{t\to\infty} \text{f}^{\alpha}(t)$ occurs, then define $\text{f}^{\alpha}(0)$ =$\lim_{t\to 0} \text{f}^{\alpha}(\text{ŧ})$. We generally use $\text{f}^{\alpha}(\text{ŧ})$ for $T_\alpha\,(\text{f})(\text{ŧ})$ to represent conformable fractional derivatives and consideration that $T_\alpha\,(\text{ŧ}^p) = p\text{ŧ}^{p-\alpha}$.

2.2 Conformable Fractional Derivative:

A conformable fractional derivative, $\mathfrak{f}$ of order α with respect to function $\mathfrak{f}$: $[0,\infty) \to R$ defined as $D^{\alpha}\mathfrak{f}(x) = \lim_{h \to 0} \frac{\mathfrak{f}(x+he^{(\alpha-1)x})-\mathfrak{f}(x)}{h}$ Where $x > 0$, $\alpha \in (0,1)$, f is α – differentiable in (0, α) , and $\lim_{x \to 0^{+}} D^{\alpha}\mathfrak{f}\,(x)$ exists and $(D^{\alpha}\mathfrak{f})(0) = \lim_{x \to 0^{+}} (D^{\alpha}\mathfrak{f})(x)$. Based on the product rule and quotient rule, conformable derivative [7] yields conclusion which is similar to the Mean Value Theorem and Rolle's Theorem.

3. Analysis of NLFPDEs by Lie Symmetry:

The time-fractional partial differential equation is

$$\frac{\partial^{\alpha} w}{\partial t^{\alpha}} = \mathrm{H}\,[w], \quad 0 < \alpha \leq 1 \quad \ldots\ldots\ldots\ldots, (2)$$

where $w = w$ (x, t), non-linear differential operator is H $[w]$ and conformable fractional derivative is $\left(\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)$. Now, we analyze symmetry transformation of equation (2), take invertible point transformations

$$\breve{x} = \mathrm{X}\,(\mathrm{x}, \mathrm{t}, w, \varepsilon)\ ,\ \breve{t} = \mathrm{T}\,(\mathrm{x}, \mathrm{t}, w, \varepsilon),\ \ \breve{w} = \mathrm{W}\,(\mathrm{x}, \mathrm{t}, w, \varepsilon)\ \ldots\ldots\ldots\ ,\qquad (3)$$

based on a continuous parameter ε and Eq. (1) that have the same form in the new variable $\breve{x}, \breve{t}, \breve{w}$ are said to be symmetry transformation. The symmetry group is a continuous group made up of H* transformations. The Lie group is another name of the symmetry group H*. The key point in Lie group of transformation [8] is to deduced infinitesimal generator [9] and determining equations.

Infinitesimal transformation of (3) be

$$\begin{aligned}
\breve{x} &= x + \varepsilon\xi(x,t,w) + o(\varepsilon^2), \\
\breve{t} &= t + \varepsilon\tau(x,t,w) + o(\varepsilon^2)\ , \qquad (4) \\
\breve{w} &= w + \varepsilon\eta(x,t,w) + o\,(\varepsilon^2),
\end{aligned}$$

$$\mathrm{W} = \xi(x,t,w)\frac{\partial}{\partial x} + \tau(x,t,w)\frac{\partial}{\partial t} + \eta(x,t,w)\frac{\partial}{\partial w}\ \ldots\ldots\ldots, \qquad (5)$$

Here, W is the infinitesimal operator.

$\frac{d\breve{x}}{d\varepsilon} = \xi(\breve{x},\breve{t},\breve{w})$, $\frac{d\breve{t},}{d\varepsilon} = \tau(\breve{x},\breve{t},\breve{w})$, $\frac{d\breve{w}}{d\varepsilon} = \eta(\breve{x},\breve{t},\breve{w})$

can be computed to obtain the group transformation (3) relating to operator (5) and subject to initial conditions

$$\breve{x}|_{\varepsilon=0} = x \quad , \quad \breve{t}\,|_{\varepsilon=0} = t \quad , \quad \breve{w}|_{\varepsilon=0} = w \,\ldots\ldots., \qquad (6)$$

A surface $w = w$ (x, t) is mapped as the group transformation, generated by W if

W (w - w (x, t)) = 0 when w = w (x, t) …………, (7)

Here, $\breve{w}(\breve{x}\,,\breve{t}\,)$ satisfies to $\frac{\partial^{\alpha}\breve{w}}{\partial\tilde{t}^{\alpha}}$ = H [$\breve{w}$] , 0<$\alpha \leq 1$………., (8)

As the function $w = w\,(x,\mathrm{t})$ satisfies Eq. (2), then the transformation (3) forms a symmetry group H of Eq. (2). Extended transformation (4) of fractional differentiation $\frac{\partial^{\alpha}w}{\partial t^{\alpha}}$ and operator of x differentiation of several order $\frac{\partial^{\alpha}w}{\partial x^{k}}$, k=1, 2, 3, …. , we can find

$$\frac{\partial^{\alpha}\breve{w}}{\partial\tilde{t}^{\alpha}} = \frac{\partial^{\alpha}w}{\partial t^{+\alpha}} + \varepsilon\eta_{\alpha}^{t}(x,t,w) + \mathrm{o}\,(\varepsilon^{2}),$$

$$\frac{\partial\breve{w}}{\partial\tilde{x}} = \frac{\partial w}{\partial x} + \varepsilon\eta^{x}(x,\mathrm{t},w) + \mathrm{o}\,(\varepsilon^{2}),$$

$$\frac{\partial^{2}\breve{w}}{\partial\tilde{x}^{2}} = \frac{\partial^{2}w}{\partial x^{2}} + \varepsilon\eta^{xx}(x,\mathrm{t},w) + \mathrm{o}\,(\varepsilon^{2}),$$

$$\frac{\partial^{3}\breve{w}}{\partial\tilde{x}^{3}} = \frac{\partial^{3}w}{\partial x^{3}} + \varepsilon\eta^{xxx}(x,\mathrm{t},w) + \mathrm{o}\,(\varepsilon^{2}),$$

$$\vdots$$

$$\vdots$$

where

$$\begin{aligned} \eta^{x} &= D_{x}(\eta) - w_{t}D_{x}(\tau) - w_{x}D_{x}(\xi), \\ \eta^{xx} &= D_{x}(\eta^{x}) - w_{xt}D_{x}(\tau) - w_{xx}D_{x}(\xi), \qquad (9) \\ \eta^{xx} &= D_{x}(\eta^{xx}) - w_{xxt}D_{x}(\tau) - w_{xxx}D_{x}(\xi). \\ &\vdots \end{aligned}$$

Here, derivative operator D_{x} is defined as

$$D_x = \frac{\partial}{\partial x} + w_x \frac{\partial}{\partial w} + w_{xx} \frac{\partial}{\partial w_x} + w_{tx} \frac{\partial}{\partial w_t} + \ldots\ldots\ldots, \qquad (10)$$

and $\frac{\partial^{\alpha} \tilde{w}}{\partial \tilde{t}^{\alpha}} = \frac{\partial^{\alpha} w}{\partial t^{\alpha}} + \varepsilon \eta_{\alpha}^{t} + o\ (\varepsilon^{2})$, where η_{α}^{t} extended infinitesimal associated to conformable fractional time derivative of $v \; and \; \tilde{v}$ differentiable functions. The prolongation of the point transformation (3) to the α^{th} derivative for some $\alpha \in (0, 1]$.

4. Description of $\left(\frac{G'}{G}\right)$-Expansion Method [40]:

$\left(\frac{G'}{G}\right)$- Expansion Method (Zayed, 2011) has been discussed in this section as a way to obtain traveling wave solutions of NLPDEs. Assume that non-linear equation,

F $(u, u_x, u_{xx}, u_t, u_{tt} \ldots) = 0 \ldots\ldots\ldots\ldots,$ (11)

Where F is higher order derivatives and non-linear terms of polynomial with unknown variable $u = u(x, t)$ and its derivatives $u, u_x, u_{xx}, u_t, u_{tt}$.

A transformation $u = u(x, t) = Q\ (\xi)$ and $\xi = gx + hy - ct$, $Q\ (\xi)$ represents the traveling wave solutions at speed c and g and h define the wave numbers.

The method can be applied utilizing the following methods to find the wave solutions:

Step 1: Firstly, change NLPDEs equation into nonlinear ordinary differential equation (ODE) using transformation (Guner and Ozkan 2016) and the system reduce into ODE.

$F\ (Q, Q', Q'' \ldots\ldots) = 0 \ldots\ldots\ldots\ldots\ldots\ldots,$ (12)

Step 2: Assume that the solution to equation (12) may be defined as :

$Q\ (\xi) = a_m \left(\frac{G'}{G}\right)^{m} + a_{m-1} \left(\frac{G'}{G}\right)^{m-1} + \ldots\ldots\ldots\ldots,$ (13)

the following form, $G = G\ (\xi)$ of second order linear differential equation is satisfied:

$G'' + \lambda G' + \mu G = 0 \ldots\ldots\ldots\ldots,$ (14)

Whereas $a_m, a_{m-1}, \ldots\ldots a_0, \lambda \; and \; \mu$ are constants, a_m ǂ 0. The non-linear terms and highest order derivatives in Eq. (11) are balanced by using the homogeneous balance method for find the positive integer m.

Step 3: Using Eq. (14) and putting in Eq. (13) into Eq. (12). Collecting the same order terms of $\left(\frac{G'}{G}\right)$, and then equating each coefficient of polynomial to zero. A set of equations for a_m, a_{m-1}, ... a_0 are obtained.

Step 4: As a result of Eq. (14) we able to obtain travelling wave solutions for NLPDEs (2). The general solutions to Eq. (14) are given as

$$\frac{G\prime}{G}=\begin{cases} -\frac{\lambda}{2}+\sqrt{\frac{\lambda^2-4\mu}{2}}\left(\frac{c_1\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+c_2\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{c_1\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+c_2\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}\right), \lambda^2-4\mu>0 \\ -\frac{\lambda}{2}+\sqrt{\frac{4\mu-\lambda^2}{2}}\left(\frac{-c_1\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+c_2\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{c_1\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+c_2\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}\right), \lambda^2-4\mu<0 \\ -\frac{\lambda}{2}+\frac{c_1}{c_1+c_1\xi}\ , \lambda^2-4\mu=0 \end{cases}$$

5. Symmetry Analysis of KdV NLFPDEs:

The following p th order KdV equation in fractional form is

$\tilde{u}_{ţ}^{\bar{\alpha}}-\tilde{u}_{\varkappa\varkappa}-A\tilde{u}^{p}\tilde{u}_{\varkappa}=0\ ,\ 0\leq\alpha\leq1$

and α (parameter) expresses the order of conformal fractional derivative.

Here $\tilde{u}(x,ţ)=\tilde{u}(\xi),\ \xi=k\varkappa-\frac{\lambda t^{\bar{\alpha}}}{\tau(1+\bar{\alpha})}$.

Construct an ODE for the expression $\tilde{u}(x,ţ)$= $Q\ (\xi)$ using the Eq. (1),

$c^2Q'(\xi)-Q''(\xi)\ g^2-a^2Q(\xi)+b^2Q^3(\xi)=0$……….., (15)

Applying lie group of transformation in Eq. (1), we get Infinitesimal equation

$$\eta_{\text{ţ}}^{\bar{\alpha}} - \eta^{\varkappa\varkappa} - A(\tilde{u}^{p}\eta^{\varkappa} + p\tilde{u}^{p-1}\eta\tilde{u}_{\varkappa})=0 \qquad (16)$$

Substituting the values of $\eta^{\varkappa\varkappa}$ and $\eta_{\text{ţ}}^{\bar{\alpha}}$ into (16). In partial derivatives of where $0 < \bar{\alpha} \leq 1$ and the parameter $\bar{\alpha}$ represents order of conformable fractional derivatives, equate the coefficients of the various equations. The infinitesimal equation is given by $\tilde{u}$ and determining equations are obtained for the symmetry of Eq. (16) by the Lie theory.

$$\begin{gathered}
D_1(\tau) = 0 \\
D_{3,3}(\xi) = 0, D_3(\xi) = 0, D_{3,3}(\eta) = 0 \\
D_{1,1}(\xi) - \text{ţ}^{1-\bar{\alpha}}D_2(\xi) - A\tilde{u}^{p}D_1(\xi) - 2D_{1,3}(\eta) - Ap\tilde{u}^{p-1}\eta = 0 \\
-\tau\bar{\alpha}\text{ţ}^{-\bar{\alpha}} + \text{ţ}^{1-\bar{\alpha}}\eta_{\tilde{u}} - \text{ţ}^{1-\bar{\alpha}}\tau_{\text{ţ}} + \tau\text{ţ}^{-\bar{\alpha}} = 0 \qquad (17) \\
\text{ţ}^{1-\bar{\alpha}}\eta_{\text{ţ}} - a\tilde{u}^{p}\eta_{\varkappa} - \eta_{\varkappa\varkappa} = 0 \\
2\xi_{\varkappa} - \eta_{\tilde{u}} = 0
\end{gathered}$$

These determining equations can be solved to obtain

$$\begin{gathered}
\eta = \tilde{u}c_1 \\
\xi = -pc_1\varkappa + c_2 \qquad (18) \\
\tau = \frac{-2pc_1\text{ţ}}{\bar{\alpha}} + c_3\text{ţ}^{1-\bar{\alpha}}
\end{gathered}$$

Vector fields span the associated symmetry group are

$$\begin{gathered}
V_1 = \tilde{u}\frac{\partial}{\partial\tilde{u}} - p\varkappa\frac{\partial}{\partial\varkappa} - 2p\frac{\text{ţ}}{\bar{\alpha}}\frac{\partial}{\partial\text{ţ}} \\
V_2 = \frac{\partial}{\partial\varkappa} \qquad (19) \\
V_3 = \text{ţ}^{1-\bar{\alpha}}\frac{\partial}{\partial\text{ţ}}
\end{gathered}$$

The equivalent solution is $\xi = x - c\frac{\text{ţ}^{\bar{\alpha}}}{\bar{\alpha}}$ for the symmetry $V_2 + V_3$.

6. Series solution of KdV NLFPDEs:

$$V_1 = \tilde{u}\frac{\partial}{\partial \tilde{u}} - p\varkappa\frac{\partial}{\partial \varkappa} - 2p\frac{\text{ţ}}{\bar{\alpha}}\frac{\partial}{\partial \text{ţ}}$$

$$\frac{dx}{-p\varkappa} = \frac{\bar{\alpha}d\text{ţ}}{-2p\text{ţ}} = \frac{d\tilde{u}}{\tilde{u}} \quad \ldots\ldots\ldots\ldots, \qquad (20)$$

Now solving Eq. (20), we get the similarity variable

$$\tilde{u} = \text{ţ}^{-\frac{\bar{\alpha}}{2p}}F(\xi), \quad \xi = x\text{ţ}^{\frac{\bar{\alpha}}{2}} \quad \ldots\ldots\ldots\ldots, \qquad (21)$$

Inserting these values in Eq. (1), which yields

$$-\frac{\bar{\alpha}}{2p}\text{ţ}^{-\frac{\bar{\alpha}}{2p}-\bar{\alpha}}F(\xi) - \frac{\bar{\alpha}}{2}\text{ţ}^{-\frac{\bar{\alpha}}{2p}-\bar{\alpha}}\xi F'(\xi) - \text{ţ}^{-\frac{\bar{\alpha}}{2p}}F''(\xi)\text{ţ}^{-\bar{\alpha}} - A\text{ţ}^{-\frac{\bar{\alpha}}{2}}F^p(\xi)\text{ţ}^{-\frac{\bar{\alpha}}{2p}}F'(\xi)\text{ţ}^{-\frac{\bar{\alpha}}{2}} = 0 \quad \ldots\ldots\ldots.., \qquad (22)$$

To obtain a solution of Eq. (22)

We take $F(\xi) = A(\xi)^p$ ……., (23)

Where A and p are constant. Equating the similar exponent of ξ, we get $p = -\frac{1}{q}$.

Substituting (23) in to (22), the following solution is

$$\tilde{u} = -x^p\text{ţ}^{\frac{-\bar{\alpha}(1+p^2)}{2p}}\left(\frac{p+1}{p}\right)^{\frac{1}{p+1}}$$

7. Wave Solution of KdV Fractional Equation:

The $\left(\frac{G'}{G}\right)$ - expansion method [23] is used to solve the NLFPDEs defined as the KdV as given

$$\tilde{u}_{\text{ţ}}^{\bar{\alpha}} - \tilde{u}_{\varkappa\varkappa} - A\tilde{u}^p\tilde{u}_{\varkappa} = 0 \quad \ldots\ldots\ldots\ldots.., \ (24)$$

And wave transformation is

$$\tilde{u} = F(\xi), \xi = x - c\frac{\text{ţ}^{\bar{\alpha}}}{\bar{\alpha}} \ldots\ldots\ldots\ldots\ldots\ldots., \quad (25)$$

Now modify the Eq. (1) into ODE as

$$-\text{ƈ}F'(\xi)-F''(\xi)-AF'(\xi)F^{p}(\xi)=0\ \ldots\ldots\ldots.., \qquad (26)$$

integrate Eq. (27), we get

$$-\text{ƈ}F(\xi)-F'(\xi)-A\frac{F^{p+1}(\xi)}{p+1}=0\ \ldots\ldots.., \qquad (27)$$

The non-linear terms and highest order derivatives identified in Eq. (26) that is not positive integers then using the homogeneous balance approach, we obtain the positive integer $m=\frac{1}{p}$. Considering the solution in the form as

$$F(\xi)=A\left(\frac{G'}{G}\right)^{\frac{1}{p}},p>0\ \ldots\ldots\ldots, \qquad (28)$$

Where A is constant to be find out.

Now Eq. (28) becomes

$$-\text{ƈ}AX(\xi)^{\frac{1}{p}}+\frac{AuX(\xi)^{\frac{1}{p}}}{PX(\xi)}+\frac{A\lambda}{p}X(\xi)^{\frac{1}{p}}+\frac{A\lambda}{p}X(\xi)^{\frac{1}{p}+1}-\frac{A^{p+2}}{p+1}X(\xi)^{\frac{1}{p}+1}=0\ldots\ldots\ldots, \qquad (29)$$

Where $X(\xi)=\frac{G'}{G}$, After collecting the coefficient of $X(\xi)^{\frac{1}{p}}, X(\xi)^{\frac{1}{p}+1}$and $X(\xi)^{\frac{1}{p}-1}$,we get $A=\left(\frac{p+1}{p}\right)^{\frac{1}{p+1}}, u=0\ and\ \lambda=p\text{ƈ}\ \ldots\ldots\ldots.,$ (30)

Determined wave solutions of Eq. (30) as

$$F(\xi)=\left(\frac{p+1}{p}\right)^{\frac{1}{p+1}}\left(\frac{\lambda e^{-\lambda\xi}}{c_1e^{-\lambda\xi}+c_2}\right)^{\frac{1}{p+1}}\text{where } \xi=x-\ \text{ƈ}\frac{\text{ƫ}^{\bar{\alpha}}}{\bar{\alpha}}$$

The exact solution of Eq. (1) is determined as

$$F(\xi)=\left(\frac{p+1}{p}\right)^{\frac{1}{p+1}}\left(\frac{\lambda e^{-\lambda\left(x-\text{ƈ}\frac{\text{ƫ}^{\bar{\alpha}}}{\bar{\alpha}}\right)}}{c_1e^{-\lambda\left(x-\text{ƈ}\frac{\text{ƫ}^{\bar{\alpha}}}{\bar{\alpha}}\right)}+c_2}\right)^{\frac{1}{p+1}}$$

8. Conclusion:

The conformable derivatives have been utilized to interpret the NLFPDEs generalized KdV equation. The symmetry aspects of KdV equation are studied by Lie group analysis methodology. Afterwards, vector fields of KdV equation are discussed on the basis of the point symmetry. Also, the symmetry reductions are constructed. Additionally, the paper shows exact traveling wave solutions of nonlinear fractional p^{th} order Korteweg-de Vries equation that have been obtained by using the $\left(\frac{G'}{G}\right)$-expansion method. At last, the generalized fractional KdV equation series solution and several explicit and exact solutions are established. The results of this paper provide practical and significant insights for future research. Additionally, the exact solutions we have derived may be useful across different areas of applied mathematics, particularly for analyzing specific physical phenomena.

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