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Subclasses on Negative Coefficints by Linear Differential Operator

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Abstract: In this paper, we use the linear operator $AS_{\lambda,q}^{\delta,n}$ to define the class $T_n(\alpha,\beta,\delta,\lambda;q)$. We derive coefficient estimates and numerous other features for functions that fall under this class. We identify the extreme points and integral means as well.

Keywords: Analytic Function, Linear Differential Operator, Coefficient inequalities, Extreme Points and Integral means.

AMS Classification: Primary 30C45; Secondary 30C50;30C80

1. Introduction

Linear differential operators are crucial in geometric function theory, a branch of mathematics that studies the properties of functions and their translations in geometric contexts. In particular, linear differential operators are used to study the characteristics of conformal mappings, quasi-conformal mappings and other types of mappings between Riemann surfaces and other geometric objects.

Numerous aspects of functions and mappings, including their regularity, smoothness, and geometric features like curvature and conformality, are explored using linear operators.

Let A be the class of functions f of the form

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j, \tag{1.1}$$

which are analytic in the open unit disc $U = \{z \in C; |z| < 1\}$.

Let T denote the subclass of A in U, consisting of analytic functions whose non-zero coefficients from the second terms onwards are negative. That is, an analytic function $f \in T$ if it has Taylor series expansion of the form

$$f(z) = z - \sum_{j=2}^{\infty} a_j z^j, \qquad (a_j \geq 0) \tag{1.2}$$

which are univalent in the open unit disc U.

Using the idea of convolution, Annapoorna S and Dileep L [4], introduced a Linear differential operator $\boldsymbol{AS_{\lambda,q}^{\delta,n}} : \boldsymbol{A} \to \boldsymbol{A}$ defined by

$$\boldsymbol{AS_{\lambda,q}^{\delta,n} f(z)} = [(\mathbf{1}-\boldsymbol{\lambda})[\mathbf{1}+(\boldsymbol{j}-\mathbf{1})\boldsymbol{\delta}]^{\boldsymbol{n}} + \boldsymbol{\lambda\Phi(a,c)}] * \boldsymbol{f(z)} \ .$$

For Functions $f \in A$ of the form (1.1), we have

$$\boldsymbol{AS_{\lambda,q}^{\delta,n} f(z)} = z + \sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q) a_j z^j \tag{1.3}$$

where

$$B_\lambda^\delta(a,c,j,n;q) = \left[(\mathbf{1}-\boldsymbol{\lambda})[\mathbf{1}+(\boldsymbol{j}-\mathbf{1})\boldsymbol{\delta}]^{\boldsymbol{n}} + \boldsymbol{\lambda}\ \frac{(\boldsymbol{a})_{j-1}}{(\boldsymbol{c})_{j-1}}\right]_q \tag{1.4}$$

$n \in \mathbb{N}_0,\ \ \lambda \geq 0,\ \ \delta \geq 0\ \ and\ a,c \in \mathbb{R}\setminus\mathbb{Z}.$

Here $(a)_j$ is the Pochhammer symbol defined interms of the Gamma function by,

$$(a)_j = \frac{\Gamma(a+j)}{\Gamma(a)} = \begin{cases} 1, & for\ j=0 \\ a(a+1)(a+2)\cdots(a+j-1), & for\ j \in \mathbb{N} \end{cases}.$$

We Obtain the AL-Oboudi differential operator [2], for a range of parametric values of $q \to 1^-, \lambda = 0$.

The Carlson-Shaffer operator [5], is obtained for a range of parametric values of $q \to 1^-, \lambda = 1$.

For a varied parametric values of $\lambda = 0$, we get the differential operator investigated by Dileep L and Mallige Rajeev [9].

We obtain the operator studied by Dileep L and S Latha [8], for $q \to 1^-, \delta = 1$.

Now using linear differential operator $\boldsymbol{AS_{\lambda,q}^{\delta,n}}$, we define the following subclass of T. Let $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ be the subclass of T consisting of functions which satisfy the conditions

$$R\left\{\frac{z\left(AS_{\lambda,q}^{\delta,n}f\right)'}{\beta z\left(AS_{\lambda,q}^{\delta,n}f\right)' + (1-\beta)AS_{\lambda,q}^{\delta,n}f}\right\} > \alpha, \tag{1.5}$$

for some $\alpha, \beta\ (\ 0 \leq \alpha,\ \beta < 1)\ and\ \ n \in \mathbb{N}_0$.

For a different parametric values of $q \to 1^-, \delta = 1\ and\ \lambda = 0$ the above class reduces to the class defined by Dileep L and S Latha [8].

2. Prime Results:

Theorem 2.1: A function f defined by (1.2) is in the class $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ if and only if

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)a_j\,[j-\alpha+\alpha\beta-\alpha\beta j] < 1-\alpha, \tag{2.1}$$

where, $\alpha, \beta\ (\ 0 \leq \alpha,\ \beta < 1)\ and\ \ n \in \mathbb{N}_0$.

Proof: Suppose $f \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$. Then

$$R\left\{\frac{z(AS_{\lambda,q}^{\delta,n}f)'}{\beta z\left(AS_{\lambda,q}^{\delta,n}f\right)' + (1-\beta)AS_{\lambda,q}^{\delta,n}f}\right\} > \alpha$$

$$R\left\{\frac{z-\sum_{j=2}^{\infty} jB_\lambda^\delta(a,c,j,n;q)a_j\,z^j}{\beta\left[z-\sum_{j=2}^{\infty} jB_\lambda^\delta(a,c,j,n;q)a_j\,z^j\right] + (1-\beta)\left[z-\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)a_j\,z^j\right]}\right\} > \alpha$$

$R\left\{\frac{z-\sum_{j=2}^{\infty} jB_{\lambda}^{\delta}(a,c,j,n;q)a_j z^j}{\boldsymbol{z}-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j z^j}\right\} > \alpha.$

Let $z \to 1$, then we get

$$1-\sum_{j=2}^{\infty} jB_{\lambda}^{\delta}(a,c,j,n;q)a_j > \alpha\left\{1-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j\right\}$$

$$\sum_{j=2}^{\infty} jB_{\lambda}^{\delta}(a,c,j,n;q)a_j - \alpha\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j < 1-\alpha$$

$$\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,[j-\alpha+\alpha\beta-\alpha\beta j] < 1-\alpha.$$

Conversely, assume that (2.1) be true. We have to show that (1.5) is satisfied or equivalently

$$\left|\frac{z(\boldsymbol{AS_{\lambda,q}^{\delta,n}}f)'}{\beta z\left(\boldsymbol{AS_{\lambda,q}^{\delta,n}f}\right)'+(\boldsymbol{1-\beta})\boldsymbol{AS_{\lambda,q}^{\delta,n}f}}-1\right| < 1-\alpha.$$

But

$$\left|\frac{z-\sum_{j=2}^{\infty} jB_{\lambda}^{\delta}(a,c,j,n;q)a_j z^j}{\boldsymbol{z}-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j z^j}-1\right|$$

$$= \left|\frac{\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,(j-1)(\beta-1)z^j}{\boldsymbol{z}-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j z^j}\right|$$

$$\leq \frac{\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,(j-1)(\beta-1)|z^j|}{|\boldsymbol{z}|-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j\,|z^j|}$$

$$\leq \frac{\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,(j-1)(\beta-1)}{1-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j}.$$

The last expression is bounded by $1-\alpha$ if

$$\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,(j-1)(\beta-1) \leq (1-\alpha)\left(1-\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)[\beta(j-1)+1]a_j\right)$$

$$\sum_{j=2}^{\infty} B_{\lambda}^{\delta}(a,c,j,n;q)a_j\,[j-\alpha+\alpha\beta-\alpha\beta j] < 1-\alpha,$$

which is true by hypothesis. This completes the assertion of Theorem 2.1

For parametric values of $q \to 1^-, \delta = 1, \lambda = 0$ and different values of $n\,(n = 0, 1)$ in the above theorem, we have the following results of A O Mostafa [15].

Corollary 2.2: ($\boldsymbol{i}$) A function f defined by (1.2) is in the class $\boldsymbol{T(\alpha,\beta)}$ if and only if

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$$\sum_{j=2}^{\infty} [j-\alpha+\alpha\beta-\alpha\beta j]a_j \le 1-\alpha.$$

(ii) A function f defined by (1.2) is in the class $\boldsymbol{C}(\boldsymbol{\alpha},\boldsymbol{\beta})$ if and only if

$$\sum_{j=2}^{\infty} j[j-\alpha+\alpha\beta-\alpha\beta j]a_j \le 1-\alpha.$$

Corollary 2.3: If $f \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$**,** then

$|a_j| \le \frac{1-\alpha}{B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]}$.

Theorem 2.4: Let $0 \le \alpha < 1,\ 0 \le \beta_1 \le \beta_2 < 1, n \in N_0,$ then $\boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta_2},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q}) \subset \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta_1},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$**.**

Proof: From the Theorem 2.1,

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)a_j\,[j-\alpha+\alpha\beta_2-\alpha\beta_2 j]$$

$$\le \sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)a_j\,[j-\alpha+\alpha\beta_1-\alpha\beta_1 j] \le 1-\alpha.$$

For $f(z) \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta_2},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$. Hence $f(z) \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta_1},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$.

Theorem 2.5: Let $f(z) \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$. Define $f_1(z) = z$ and

$$\boldsymbol{f_j}(\boldsymbol{z}) = \boldsymbol{z} + \frac{1-\alpha}{B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]}\, z^j, \quad j = 2,3,\cdots,$$

for some $\alpha,\beta\ (0 \le \alpha,\ \beta < 1), n \in \mathbb{N}_0$ *and* $z \in U$. $f \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$ if and only if f can be expressed as $f(z) = \sum_{j=1}^{\infty}\mu_j\, f_j(z)$ where $\mu_j \ge 0$ and $\sum_{j=1}^{\infty}\mu_j = 1$.

Proof: If $f(z) = \sum_{j=1}^{\infty}\mu_j\, f_j(z)$ with $\sum_{j=1}^{\infty}\mu_j = 1,\ \mu_j \ge 0,$ then

$$\sum_{j=2}^{\infty}\frac{B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\,\mu_j}{B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]}\,(1-\alpha)$$

$$= \sum_{j=2}^{\infty}\mu_j\,(1-\alpha) = (1-\mu_1)(1-\alpha) \le (1-\alpha).$$

Hence $f(z) \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$.
Conversely, let $f(z) = z - \sum_{j=2}^{\infty} a_j z^j \in \boldsymbol{T_n}(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\delta},\boldsymbol{\lambda};\boldsymbol{q})$, define

$\boldsymbol{\mu_j} = \frac{B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]|a_j|,}{(\boldsymbol{1-\alpha})}\quad \boldsymbol{j = 2,3\cdots,}$

and define $\mu_1 = 1 - \sum_{j=2}^{\infty}\mu_j$. From Theorem 2.1, $\sum_{j=2}^{\infty}\mu_j \le 1$ *and so* $\mu_1 \ge 0$.
Since $\mu_j f_j(z) = \mu_j f + a_j\, z^j,\quad \sum_{j=1}^{\infty}\mu_j\, f_j(z) = z - \sum_{j=2}^{\infty} a_j z^j = f(z)$.

Theorem 2.6: The class $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ is closed under convex linear combination.
Proof: Let $f, g \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ and let

$$f(z) = z - \sum_{j=2}^{\infty} a_j z^j, \qquad g(z) = z - \sum_{j=2}^{\infty} b_j z^j.$$

For η such that $0 \le \eta \le 1$, it suffices to show that the function defined by

$$h(z) = (1-\eta)f(z) + \eta g(z), \ \ z \in U$$

belongs to $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$. Now

$$h(z) = z - \sum_{j=2}^{\infty} \left[(1-\eta)a_j + \eta b_j\right] z^j,$$

Applying Theorem 2.1, to $f, g \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ we have

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\left[(1-\eta)a_j + \eta b_j\right]$$

$$= (1-\eta)\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\, a_j + \eta \sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\, b_j$$

$$\le (1-\eta)(1-\alpha) + \eta\,(1-\alpha) = 1-\alpha.$$

This implies that $h \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$.

Corollary 2.7: If $f_1(z),\ f_2(z) \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ then the function defined by $\boldsymbol{g(z) = \frac{1}{2}[f_1(z) + f_2(z)]}$ is also in $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$.

Theorem 2.8: Let for $m = 1,2,\cdots,j$ $f_m(z) = z - \sum_{j=2}^{\infty} a_{j,m} z^j \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$ and $0 < \beta_m < 1$ such that $\sum_{m=2}^{\infty} \beta_m = 1$, then the function $F(z)$ defined by $F(z) = \sum_{m=2}^{j} \beta_m f_m(z)$ *is also in* $\boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$.
***Proof*:** For each $m \in \{1,2,\cdots,j\}$ we obtain

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\,|a_j| < 1-\alpha.$$

$$F(z) = \sum_{m=1}^{j} \beta_m \left(z - \sum_{j=2}^{\infty} a_{j,m} z^j\right)$$

$$= z - \sum_{j=2}^{\infty}\left(\sum_{m=1}^{j} \beta_m\, a_{j,m}\, z^j\right)$$

Since,

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a,c,j,n;q)[j-\alpha+\alpha\beta-\alpha\beta j]\left[\sum_{m=1}^{j} \beta_m\, a_{j,m}\right] < \sum_{m=1}^{j} \beta_j(1-\alpha) < 1-\alpha.$$

Therefore, $F(z) \in \boldsymbol{T_n(\alpha,\beta,\delta,\lambda;q)}$.

Theorem 2.9: Let $f(z) \in \boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$. Komato operator of f is defined by

$$k(z) = \int_0^1 \frac{(c+1)^\gamma}{\Gamma(\gamma)}\, t^c \left(\log\left(\frac{1}{t}\right)\right)^{\gamma-1} \frac{f(tz)}{t}\, dt,$$

$c > -1,\ \gamma \geq 0$ then $k(z) \in \boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$.

$\boldsymbol{Proof}$: We have

$$\int_0^1 t^c \left(\log\left(\frac{1}{t}\right)\right)^{\gamma-1} dt = \frac{\boldsymbol{\Gamma(\gamma)}}{\boldsymbol{(c+1)^\gamma}}$$

$$\int_0^1 t^{j+c-1} \left(\log\left(\frac{1}{t}\right)\right)^{\gamma-1} dt = \frac{\boldsymbol{\Gamma(\gamma)}}{\boldsymbol{(c+1)^\gamma}}, \quad \boldsymbol{j = 2, 3, \cdots},$$

$$k(z) = \frac{(c+1)^\gamma}{\Gamma(\gamma)}\left[\int_0^1 t^c \left(\log\left(\frac{1}{t}\right)\right)^{\gamma-1} z\, dt - \sum_{j=2}^{\infty} z^j \int_0^1 a_j\, t^{j+c-1} \left(\log\left(\frac{1}{t}\right)\right)^{\gamma-1} dt\right]$$

$$= \boldsymbol{z} - \sum_{\boldsymbol{j=2}}^{\infty} \left(\frac{\boldsymbol{c+1}}{\boldsymbol{c+j}}\right)^{\boldsymbol{\gamma}} \boldsymbol{a_j z^j}.$$

Since $f(z) \in \boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$ and $\left(\frac{\boldsymbol{c+1}}{\boldsymbol{c+j}}\right)^{\boldsymbol{\gamma}} < \boldsymbol{1}$, we have

$$\sum_{j=2}^{\infty} B_\lambda^\delta(a, c, j, n; q)[j - \alpha + \alpha\beta - \alpha\beta j] \left(\frac{\boldsymbol{c+1}}{\boldsymbol{c+j}}\right)^{\boldsymbol{\gamma}} a_j < (1-\alpha).$$

Theorem 2.10: Let $f(z) \in \boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$, then for every $0 \leq \zeta < 1$ the function

$$\boldsymbol{H_\zeta(z) = (1-\zeta) f(z) + \zeta \int_0^z \frac{f(t)}{t}\, dt}.$$

$\boldsymbol{Proof}$: We have $H_\zeta(z) = z - \sum_{j=2}^{\infty}\left(1 + \frac{\zeta}{j} - \zeta\right) a_j z^j$.

Since $\left(1 + \frac{\zeta}{j} - \zeta\right) < 1,\ j \geq 2, so\ by$ Theorem 2.1,

$$\sum_{j=2}^{\infty}\left(1 + \frac{\zeta}{j} - \zeta\right) B_\lambda^\delta(a, c, j, n; q)[j - \alpha + \alpha\beta - \alpha\beta j] a_j$$

$$< \sum_{j=2}^{\infty} B_\lambda^\delta(a, c, j, n; q)[j - \alpha + \alpha\beta - \alpha\beta j] a_j < 1 - \alpha.$$

Therefore, $H_\zeta(z) \in \boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$.

3. Conclusion:

Conclusion: Here, in our present investigation, we have successfully introduced a new subclass of analytic functions $\boldsymbol{T_n(\alpha, \beta, \delta, \lambda; q)}$ using the Linear differential operator. Many properties and characteristics of this newly defined function class such as coefficient estimates, distortion theorem, extreme points, integral theorem have been studied.

4. References:

[1] H S Almiri, On Ruscheweyh derivatives, Ann., Polon Math., 38 (1980), 87-94.

[2] F M Al-Oboudi, On univalent functions defined by a generalized Salagean operator, Ind., J. Math. Math. Sci., 2004, No. 25-28, 1429-1436.

[3] O Altintas and S Owa, On Subclasses of univalent functions with negative coefficients, Pusam Kyonganma Math., J., 4 (1988), 41-56.

[4] Annapoorna S and Dileep L, Applications of Linear differential operator on varying arguments, Communications in Mathematics and Applications, Vol.15, No. 2, pp 791-799, 2024, DOI:10.26713/cam.v15i2.2735.

[5] B C Carlson and Shaffer, Starlike and pre-starlike hypergeometric functions, SIAM J., on Math. Analysis, 15(1984), 737-745.

[6] Dileep L and S Latha, A note on Salagean type functions, Global Journal of Mathematical Sciences, Theory and Practical, Vol.2, No.1, 2010, pp 29-35.

[7] Dileep L and S Latha, Certain subclasses of analytic functions involving Salagean-Rushceweyh operator, Vietnam Journal of Mathematics, Vol. 38(4), 403-412, 2010.

[8] Dileep L and S Latha, A note on Salagean -Carlson Shaffer operator, Int. J. Math. Archive, -2(2), 272-279, 2011.

[9] Dileep L and Mallige Rajeev, Convolution operators in Geometric Function Theory, GIS Science Journal, Vol. 7, Issue 7, 2020, 472-483.

[10] P L Duren, Univalent functions, Springer-Verlag, 1983.

[11] S S Eker and S Owa, Certain classes of analytic functions involving Salagean operator, J. Inequal. Pure Appl. Math.,

[12] S S Eker and S Owa, New applications of classes of analytic functions involving Salagean operator, Int. Symposium on Complex Function Theory and Applications, Brasov, Romania, 1-5, (2006).

[13] S S Eker and B Seker, On a class of multivalent functions defined by Salagean operator, General Mathematics, Vol. 15, N-.2-3(2007), 154-163.

[14] J E Littlewood, On inequalities in the theory of functions, Proc., London Math. Soc., 23(1925), 481-519.

[15] A O Mostafa, A study on starlike and convex properties for hypergeometric functions, JIPAM, Vol. 10(2009), Issue 3, 87.

[16] M S Robertson, On the theory of univalent functions, Annals of Math., 37 (1936), pp 374- 406.

[17] G S Salagean, On Some classes of univalent functions, Seminar of Geometric Function theory, Cluj-Napoca, 1983.

[18] G S Salagean, Subclasses of univalent functions, Lecture Notes in Mathe. Springer- Verlag, 2013 (1983), 362-372.

[19] S Shams, S R Kulakukarni and J M Jahangiri, Classes of uniformly starlike and functions, Int. J. Math. And Math. Si., 55 (2004), 2959-2961.

[20] H Silverman, Univalent functions with varying arguments, Huston Journal of Math., Vol.7, No.2 (1981).