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Some Fixed Point Results for Expansive Mappings in Dislocated Quasi-*B*-Metric Spaces with an Application to Integral Equations

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Abstract: In this paper, we prove some new fixed point results for expansive type mappings in complete dislocated quasi b-metric space. A common fixed point result is also established considering such mappings. Our results extend and generalize the results of Das et al. [9] from the dislocated quasi metric space setting to dislocated quasi-b-metric spaces. Suitable examples are provided to demonstrate our results. The solution to a system of Fredholm integral equations is also established to show the applicability of our results.

Keywords: Fixed points; dislocated qusi-b-metricspace; expansive map; integral equation.

AMS Subject Classification (2020): 47H10,54H25..

1. Introduction

The development of fixed point theory is based on the generalization of contraction conditions in one direction or/and generalization of ambient spaces of the operator under consideration on the other. Using the Picard iteration approach, Polish mathematician Banach developed the Banach contraction mapping concept in 1922. The existence of a solution for a differential equation with initial value condition, the implicit function existence theorem, and fixed point theory's elegant assertion and effective method of solving it have drawn the attention of academics and inspired people to conduct in-depth, comprehensive research. With the advent of the computer, particularly in the last few decades, many individuals have dealt with a large number of applications by using a range of iteration techniques to approach the fixed point. As a result, they made a breakthrough and gradually improved this subject. These days, nonlinear functional analysis relies heavily on fixed point theory.

The notion of dislocated metric space initially surfaced in domain theory, which was proposed by Matthews [15] in 1986 along with various concepts of metric domains. Hitzler et al. [13] presented the idea of dislocated metric space later in 2000, where a point's self-distance is not always zero. In this area, they also extended the Banach contraction concept. Topology, logical programming, computer science, electronic engineering, and other fields all heavily rely on dislocated metric space. Zeyadaet al. [26] expanded Hitzler's [13] result in dislocated quasi-metric space and introduced the entire dislocated quasi-metric space. For more details see [1, 11, 14, 16, 17, 21, 23]

The innovative idea of expansive mapping was first presented by Wang et al. [23] in 1984. They carried out a comprehensive investigation and revealed complex fixed point out comes in the domain of entire metric space. Applications for expansive mappings can be found in nonlinear analysis, dynamical system theory, and chaos theory. Since then, other academics have conducted thorough studies, methodically developing and extending fixed point theoretical results in this specific field [3,10,12,18– 20, 22, 25].

Definition 1.1. [8] Let X be a non-empty set and $s \geq 1$ be a given real number. Let $d: X \times X \rightarrow [0, \infty)$ be a mapping and for any $a, b, c \epsilon X$:

(i). $0 \leq d(a,b)$ and $d(a,b) = 0$ if and only if $a = b$;

(ii). $d(a,b) = 0$ implies $a = b$;

(iii). $d(a,b) = 0 = d(b,a)$ implies $a = b$;

(iv). $d(a,b) = d(b,a)$;

(v). $d(a,c) \leq d(a,b) + d(b,c)$;

(vi). $d(a,c) \leq s[d(a,b) + d(b,c)]$.

Then

(1) (X,d) is called a metric space if (i), (iv), and (v) hold;

(2) (X,d) is called a b-metric space if (i), (iv), and (vi) hold;

(3) (X,d) is called a quasi-metric space if (i), and (v) hold;

(4) (X,d) is called a quasi-b-metric space if (i) , and (vi) hold;

(5) (X,d) is called a dislocated metric space (d-metric space) if (ii) , (iv), and (v) hold;

(6) (X,d) is called a dislocated b-metric space ($d\ b$-metric space) if (ii), (iv) , and (vi) hold;

(7) (X,d) is called a dislocated quasi-metric space (dq-metric space) if (iii) and (v) hold;

(8) (X,d) is called a dislocated quasi-b-metric space ($dq\ b$-metric space) if (iii) and (vi) hold.

Even though the examples provided were well-known, we felt that providing a thorough review would be helpful for convenient reference.

Example 1.2. (a) Let X=$\mathbb{R}$ and $d: X \times X \rightarrow \mathbb{R}^+$ defined as $d(a,b) = \begin{cases} a - b, & a \geq b \\ 1, & otherise. \end{cases}$

Then (X,d) is a quasi-metric space, but it is not a metric space.

(b) Let X=$\mathbb{R}^+$ and $d: X \times X \rightarrow \mathbb{R}^+$ defined as $d(a,b) = \begin{cases} 0, & a = b \\ (a+b)^2, & otherise. \end{cases}$

Then (X,d) is a b-metric space, but it is not a metric space.

(c) Let $X = C([0,1],\mathbb{R})$ with the usual partial ordering, and let $d: X \times X \rightarrow \mathbb{R}^+$ be defined as

$$d(f,g)=\begin{cases}\int_0^1 (g(t)-f(t))^3\,dt, & f\le g,\\ \int_0^1 (f(t)-g(t))^3\,dt, & f\ge g.\end{cases}$$

Then (X,d) is a quasi-b-metric space, but it is not a quasi-metric space and b-metric space.

(d) Let $X=\mathbb{R}^+$ and $d: X\times X \to \mathbb{R}^+$ defined as $d(a,b)=\max\{a,b\}$.Then (X,d) is a dislocated metric space, but it is not a metric space.

(e) Let $X=[0,1]$ and $d: X\times X \to \mathbb{R}^+$ be defined as $d(a,b)=|a-b|+a$. Then (X,d) is a dislocated quasi-metric space, but it is not a dislocated metric space, and it is not a quasi-metric space.

(f) Let $X=[0,\infty)$ and $d: X\times X \to [0,\infty)$ be defined as $d(a,b)=(a+b)^2$. Then (X,d) is a dislocated b-metric space, but it is not a b-metric space.

(g) Let $X=\mathbb{R}$ and $d: X\times X \to \mathbb{R}^+$ be defined as $d(a,b)=|a-b|^2+\frac{|a|}{n}+\frac{|b|}{m}$, where $n,m\in\mathbb{N}\backslash\{1\}$, $n\neq m$.Then (X,d) is a dislocated quasi-metric space, but it is not a quasi b- metric space, dislocated b-metric space and dislocated quasi-metric space.

Thus, we get the process diagram (refer to Figure 1), in which generalization relationships are represented by arrows.

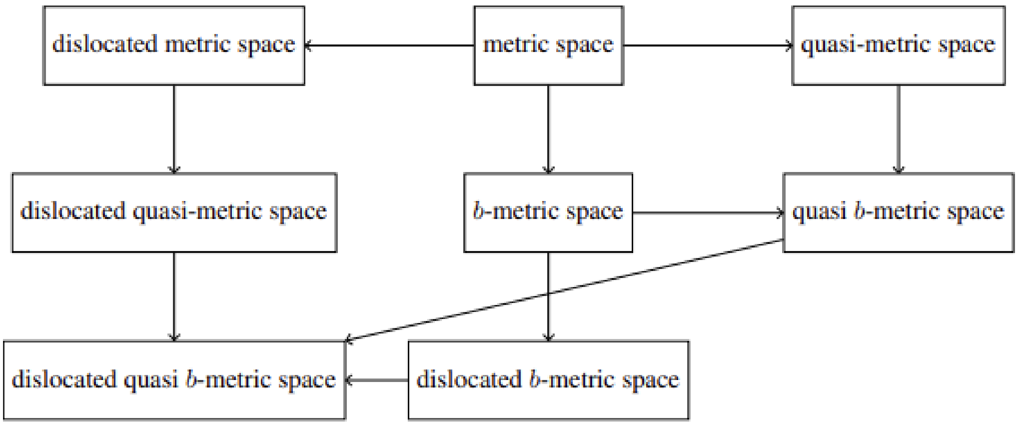

Figure 1: Process diagram.

The following lemmas are useful in proving our main results.

Lemma 1.3. [2] *Let (X,d) be a b-metric space with coefficients $s\geq 1$.*

Suppose that $\{a_n\}$ and $\{b_n\}$ are b-convergent to x and y respectively. Then we have

$\frac{1}{s^2}d(x,y)\le \liminf_{n\to\infty} d(a_n,b_n)\le \limsup_{n\to\infty} d(a_n,b_n)\le s^2 d(x,y)$.

In particular, if $x=y$, then we have $\lim_{n\to\infty} d(a_n,b_n)=0$.*Moreover for each $z\in X$ we have*

$\frac{1}{s}\, d(x,z) \leq \lim_{n\to\infty} \inf d(a_n, z) \leq \lim_{n\to\infty} \sup d(a_n, z) \leq s\, d(x,z)$.

Lemma 1.4. *Let a be a limit of some sequence $\{a_n\}$ in a dq b-metric space(X,d), then $d(a,a) = 0$.*

Proof. Let $a \in X$, $\{a_n\} \subseteq X$ and a sequence which converges to a. Then

$$d(a,b) \leq s[d(a,a_n) + d(a_n,a)], \forall\, n \in N.$$

By taking limit superior as $n \to \infty$ and using Lemma 1.3, we get $d(a,a) = 0$.

Recently, Das et al. [9] established the following theorems in dq-metric spaces.

Theorem 1.5. [9] *Let (X,d) be a complete dq-metric space and T be an onto self-mapping on X such that*

$$d(Ta,Tb) \geq k \min\left\{\alpha\, d(a,b); \beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Ta,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)\right\}$$

for all $a,b \in X$ with $d(a,b) \neq 0, k > 1$, nonnegative real numbers $\alpha, \beta_i, \gamma_j, \delta_j$ for

$i = 1,2;\ j = 1,2,3$ and $\frac{1}{k} = \min\{\alpha, \beta_2, \gamma_2 + \gamma_3, k^2\delta_2(\delta_1 + \delta_2) + k\delta_3\}$.

Then T has a unique fixed point.

Theorem 1.6. [9] *Let (X,d) be a complete dq -metric space and S, T be two onto self-mapping on X such that*

$$d(Sa,Tb) \geq k \min\left\{\alpha\, d(a,b); \beta_1 \frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)\right\}$$

for all $a,b \in X$ with $d(a,b) \neq 0, k > 1$, nonnegative real numbers $\alpha, \beta_i, \gamma_j, \delta_j$ for

$i = 1,2;\ j = 1,2,3$ *and* $\frac{1}{k} = \min\{\alpha, \beta_2, \gamma_2 + \gamma_3, \delta_3\}$.

Then S and T have a unique common fixed point.

2. Main Results

In this section, we formulate some fixed point results for onto expansive type mapping in a complete dq b-metric space.

Theorem 2.1. *Let (X,d) be a complete dq b-metric space and T be an onto self-mapping on X such that*

$$d(Ta,Tb) \geq k \min\left\{\alpha\, d(a,b); \beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Ta,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1 \frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right\} \quad (2.1)$$

for all $a,b \in X$ *with* $d(a,b) \neq 0, k > 1$, *nonnegative real numbers* $\alpha, \beta_i, \gamma_j, \delta_j, \lambda_j$ *for* $i = 1,2;\ j = 1,2,3$ *and* $\frac{1}{k} = min\{\alpha, \beta_2, \gamma_2 + \gamma_3, k^2\delta_2(\delta_1 + \delta_2) + k\delta_3, \lambda_3\}$. *Then T has a unique fixed point.*

Proof. Let us take $\theta(a,b) = \min\left\{\alpha\, d(a,b); \beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Ta,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1 \frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right\}$.

For $a_0 \in X$, since T are onto, there exist $a_0 \in X$ such that $a_0 = Ta_1$. Continuing this process, we define a sequence $\{a_n\}$ in X with $a_{n-1} = Ta_n$, for all $n \in \mathbb{N}$.

The cases listed below will occur.

Case (i). If $\theta(a,b) = \alpha\, d(a,b)$, then

$$d(Ta,Tb) \geq k\, \alpha\, d(a,b), \text{for all } a,b \in X. \quad (2.2)$$

Now, using (2.2), we get

$$d(a_{n-1},a_n) = d(Ta_n,Ta_{n+1}) \geq k\, \alpha\, d(a_n,a_{n+1})$$
$$\text{i.e.}, d(a_n,a_{n+1}) \leq \frac{1}{k\, \alpha} d(a_{n-1},a_n).$$

Let $\tau = \frac{1}{k\,\alpha} < 1$. Then from the above inequality, we have

$$d(a_n,a_{n+1}) \leq \tau\, d(a_{n-1},a_n).$$

Also,

$$d(a_{n+1},a_{n+2}) \leq \tau\, d(a_n,a_{n+1}) + \tau^2\, d(a_{n-1},a_n).$$

From this we get,

$$d(a_n,a_{n+1}) \leq \tau^n\, d(a_0,a_1).$$

For $j > i$,

$$\begin{aligned} d(a_i,a_j) &\leq s\, d(a_i,a_{i+1}) + s^2\, d(a_{i+1},a_{i+2}) + \cdots + s^{j-i}\, d(a_{j-1},a_j) \\ &\leq s\, \tau^i d(a_0,a_1) + s^2\tau^{i+1} d(a_0,a_1) + \cdots + s^{j-i}\tau^{j-1} d(a_0,a_1) \\ &\leq \left[s\, \tau^i + s^2\tau^{i+1} + \cdots\right] d(a_0,a_1) \\ &= s\, \tau^i[1 + s\tau + (s\tau)^2 + \cdots] d(a_0,a_1) \end{aligned}$$

$$= \frac{s\,\tau^{i}}{1-s\tau}\, d(a_0,a_1) \to 0 \text{ as } i,j \to \infty$$

Therefore $\{a_n\}$ is a b-Cauchy sequence in X. Since X is complete, there exists $u \in X$ such that $\lim_{n\to\infty} a_n = u$. Since, T is onto, we can find $p \in X$ such that $Tp = u$.

Now, for all $n \in \mathbb{N}$.

$$d(u,a_n) = d(Tp,Ta_{n+1}) \geq k\,\alpha\, d(p,a_{n+1}).$$

Taking limit superior as $n \to \infty$, and using Lemma 1.3, we get

$$\frac{1}{s} d(p,u) \leq \frac{k\,\alpha}{s} \lim_{n\to\infty} \sup d(p,a_{n+1}) \leq \lim_{n\to\infty} \sup d(u,a_n) \leq s\, d(u,u).$$

From Lemma. 1.4, we get $d(p,u) = 0$ and similarly $d(u,p) = 0$.

Thus, $d(p,u) = d(u,p) = 0$. So, $p = u$.

Uniqueness. Let $v(\neq u)$ be another fixed point of T. Then

$$d(u,v) = d(Tu,Tv) \geq k\,\alpha\, d(u,v), \text{i.e.}, (1-k\,\alpha)d(u,v) \leq 0$$

which gives us $d(u,v) = 0$. Similarly, we can prove that $d(v,u) = 0$, and that $u = v$.

Case (ii). $\theta(a,b) = \beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b)$, then

$$d(Ta,Tb) \geq k\left[\beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b)\right] \tag{2.3}$$

Now, using (2.3), we get

$$\begin{aligned} d(a_{n-1},a_n) = d(Ta_n,Ta_{n+1}) &\geq k\beta_1 \frac{d(Ta_n,a_n)d(Ta_{n+1},a_{n+1})}{d(a_n,a_{n+1})} + k\beta_2 d(a_n,a_{n+1}) \\ &= k\beta_1 \frac{d(a_{n-1},a_n)d(a_n,a_{n+1})}{d(a_n,a_{n+1})} + k\beta_2 d(a_n,a_{n+1}) \\ &\geq k\beta_2 d(a_n,a_{n+1}). \end{aligned}$$

i.e., $d(a_n,a_{n+1}) \leq \frac{1}{k\beta_2} d(a_{n-1},a_n)$ which implies that $d(a_n,a_{n+1}) \leq \vartheta\, d(a_{n-1},a_n)$,

where $\vartheta = \frac{1}{k\beta_2} < 1$. Proceeding similar to ***Case (i)***, we get $\{a_n\}$ is a b-Cauchy sequence in X, which converges to some $u \in X$, which can be shown to be unique fixed point of T.

Case (iii). $\theta(a,b) = \gamma_1 d(Ta,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b)$, then

$$d(Ta,Tb) \geq k[\gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b)] \tag{2.4}$$

Now, using (2.4), we get

$$\begin{aligned} d(a_{n-1},a_n) &= d(Ta_n,Ta_{n+1}) \\ &\geq k\gamma_1 d(Ta_n,a_n) + k\gamma_2\, d(Ta_{n+1},a_{n+1}) + k\gamma_3\, d(a_n,a_{n+1}) \\ &= k\gamma_1 d(a_{n-1},a_n) + k\gamma_2\, d(a_n,a_{n+1}) + k\gamma_3\, d(a_n,a_{n+1}) \\ &\geq k[\gamma_2+\gamma_3] d(a_n,a_{n+1}) \end{aligned}$$

i.e., $d(a_n,a_{n+1}) \leq \frac{1}{k[\gamma_2+\gamma_3]} d(a_{n-1},a_n)$ implies that $d(a_n,a_{n+1}) \leq \rho\, d(a_{n-1},a_n)$, where $\rho = \frac{1}{k[\gamma_2+\gamma_3]} < 1$. Proceeding similar to ***Case (i)***, we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique fixed point of T.

Case (iv). $\theta(a,b) = \delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)$, then

$$d(Ta,Sb) \geq k[\delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)] \tag{2.5}$$

From (2.5), we get

$$\begin{aligned} d(a_{n-1},a_n) &= d(Ta_n,Ta_{n+1}) \\ &\geq k\delta_1 d(Ta_n,a_{n+1}) + k\delta_2\, d(Ta_{n+1},a_n) + k\delta_3\, d(a_n,a_{n+1}) \\ &= k\delta_1 d(a_{n-1},a_{n+1}) + k\delta_2\, d(a_n,a_n) + k\delta_3\, d(a_n,a_{n+1}) \\ &\geq k\delta_2\, d(a_n,a_n) + k\delta_3 d(a_n,a_{n+1}) \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} d(a_n,a_n) &= d(Ta_{n+1},Ta_{n+1}) \\ &\geq k\delta_1 d(Ta_{n+1},a_{n+1}) + k\delta_2\, d(Ta_{n+1},a_{n+1}) + k\delta_3\, d(a_{n+1},a_{n+1}) \\ &\geq k\delta_1 d(a_n,a_{n+1}) + k\delta_2\, d(a_n,a_{n+1}) \\ &= k[\delta_1+\delta_2] d(a_n,a_{n+1}). \end{aligned}$$

From the inequality (2.6), we get

$d(a_{n-1},a_n) \geq [k^2\delta_2(\delta_1+\delta_2) + k\,\delta_3] d(a_n,a_{n+1})$ which implies that

i.e., $d(a_n,a_{n+1}) \leq \frac{1}{k^2\delta_2(\delta_1+\delta_2)+k\,\delta_3} d(a_{n-1},a_n)$ implies that $d(a_n,a_{n+1}) \leq \omega\, d(a_{n-1},a_n)$, where $\omega = \frac{1}{k^2\delta_2(\delta_1+\delta_2)+k\,\delta_3} < 1$. Proceeding similar to ***Case (i)***, we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique fixed point of T.

Case (v). $\theta(a,b) = \lambda_1 \frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)$, then

$$d(Ta,Tb) \geq k\left[\lambda_1 \frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right] \tag{2.7}$$

Now, using (2.6), we get

$$\begin{aligned} d(a_{n-1},a_n) &= d(Ta_n,Ta_{n+1}) \\ &\geq k\,\lambda_1 \frac{d(Ta_n,a_{n+1})d(Ta_{n+1},a_{n+1})}{d(a_n,a_{n+1})} + k\,\lambda_2 \frac{d(Ta_{n+1},a_n)d(Ta_{n+1},a_{n+1})}{d(a_n,a_{n+1})} \\ &\quad + k\,\lambda_3\, d(a_n,a_{n+1}) \\ &= k\,\lambda_1 \frac{d(a_{n-1},a_{n+1})d(a_n,a_{n+1})}{d(a_n,a_{n+1})} + k\,\lambda_2 \frac{d(a_n,a_n)d(a_n,a_{n+1})}{d(a_n,a_{n+1})} + k\,\lambda_3\, d(a_n,a_{n+1}) \\ &\geq k\lambda_3 d(a_n,a_{n+1}). \end{aligned}$$

i.e., $d(a_n, a_{n+1}) \leq \frac{1}{k\lambda_3} d(a_{n-1}, a_n)$ which implies that $d(a_n, a_{n+1}) \leq \varphi\, d(a_{n-1}, a_n)$, where $\varphi = \frac{1}{k\lambda_3} < 1$. Proceeding similar to ***Case (i),*** we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique fixed point of T.

The following is an example in support of Theorem 2.1.

Example 2.2**.** Let $X = \mathbb{R}^+$. We define $d: X \times X \to \mathbb{R}^+$ by $d(a,b) = |a-b|^2 + |a|^2$. Then clearly, (X, d) is a complete dp b-metric space with $s = 2$. We define self-mappings $T: X \to X$ by

$$T(a) = a(a+2), \qquad \text{for all } a \in X.$$

We take $k = \frac{3}{2}, \alpha = \beta_2 = \gamma_3 = \delta_3 = \lambda_3 = 1, \beta_1 = \gamma_1 = \gamma_2 = \delta_1 = \delta_2 = \lambda_1 = \lambda_2 = 0.$

Without loss of generality we assume that $a \geq b$. We consider

$$\begin{aligned} d(Sa, Tb) &= |Ta - Tb|^2 + |Sa|^2 \\ &= (a^2 + 2a - b^2 - 2b)^2 + (a^2 + 2a)^2 \\ &= (a-b)^2(a+b+2)^2 + a^2(\mathrm{a}+2)^2 \\ &\geq \frac{3}{2}[(a-b)^2 + a^2] \\ &= k \min\Big\{\alpha\, d(a,b); \beta_1 \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Ta,a) + \gamma_2\, d(Tb,b) \\ &\quad + \gamma_3\, d(a,b); \delta_1 d(Ta,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1 \frac{d(Ta,b)d(Tb,b)}{d(a,b)} \\ &\quad + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\Big\} \end{aligned}$$

$a = b$ values	$kd(a,b) = \frac{3}{2}a^2$	$d(Ta,Tb) = (a^2+2a)^2$
0	0	0
0.1	0.015	0.0441
0.2	0.06	0.1936
0.3	0.135	0.4761
0.4	0.24	0.9216
0.5	0.375	1.5625
0.6	0.54	2.4336
0.7	0.735	3.5721
0.8	0.96	5.0176
0.9	1.215	6.8121
1	1.5	9

Table 1: Demonstration of the inequality (2.1), using some numerical values.

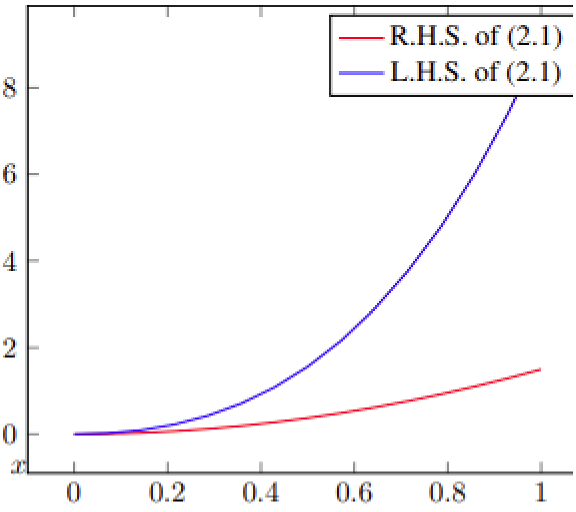

Figure 2

Table 1 and Figure 2, illustrates the condition (2.1) of Theorem 2.1, with blue line representing the left part of the condition and red line representing the right part of the condition. Thus, all the conditions of Theorem 2.8 are satisfied. So T has a unique common fixed point, which is clearly 0 here.

Corollary 2.3. *Let (X, d) be a complete dq b-metric space and T be an onto self-mapping on X such that $d(Ta, Tb) \geq kd(a, b)$ for all $a, b \in X$ with $d(a, b) \neq 0,\ k > 1$. Then T has a unique fixed point.*

Proof. By taking $\alpha = \beta_2 = \gamma_3 = \delta_3 = \lambda_3 = 1$ and $\beta_1 = \gamma_1 = \gamma_2 = \delta_1 = \delta_2 = \lambda_1 = \lambda_2 = 0$ in Theorem 2.1, the result follows easily.

Corollary 2.4. *Let (X, d) be a complete dq b-metric space and T be an onto self-mapping on X such that*

$$\begin{aligned} d(Ta, Tb) \geq k \min\Bigg\{ & d(a, b); \frac{d(Ta, a)d(Tb, b)}{d(a, b)} + d(a, b); d(Ta, a) + d(Tb, b) \\ & + d(a, b); d(Ta, b) + d(Tb, a) + d(a, b); \frac{d(Ta, b)d(Tb, b)}{d(a, b)} + \frac{d(Tb, a)d(Tb, b)}{d(a, b)} \\ & + d(a, b)\Bigg\} \end{aligned}$$

for all $a, b \in X$ with $d(a, b) \neq 0,\ k > 1$. Then T has a unique fixed point.

Proof. Putting $\alpha = \beta_1 = \beta_2 = \gamma_1 = \gamma_2 = \gamma_3 = \delta_1 = \delta_2 = \delta_3 = \lambda_1 = \lambda_2 = \lambda_3 = 1$ in Theorem 2.1, the result follows easily.

Theorem 2.5. *Let (X, d) be a complete dq b-metric space and T be an onto self-mapping on X such that*

$$d(Ta,Tb) \geq \alpha_1 d(a,b) + \alpha_2\left[\frac{d(Ta,a)d(Tb,b)}{d(a,b)} + d(a,b)\right] + \alpha_3[d(Ta,a) + d(Tb,b) + d(a,b)]$$
$$+ \alpha_4[d(Ta,b) + d(Tb,a) + d(a,b)]$$
$$+ \alpha_5\left[\frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + d(a,b)\right] \tag{2.8}$$

for all $a,b \in X$ with $d(a,b) \neq 0$, $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 > 1$. Then T has a unique fixed point.

Proof. Let $\theta(a,b) = \min\left\{d(a,b); \frac{d(Ta,a)d(Tb,b)}{d(a,b)} + d(a,b); d(Ta,a) + d(Tb,b) + d(a,b); d(Ta,b) + d(Tb,a) + d(a,b); \frac{d(Ta,b)d(Tb,b)}{d(a,b)} + \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + d(a,b)\right\}$. (2.9)

Using (2.8) and (2.9), we get

$$d(Ta,Tb) \geq \alpha_1\theta(a,b) + \alpha_2\theta(a,b) + \alpha_3\theta(a,b) + \alpha_4\theta(a,b) + \alpha_5\theta(a,b)$$
$$= (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)\theta(a,b).$$

Then the inequality becomes $d(Ta,Tb) \geq k\,\theta(a,b)$, where $k > 1$. Therefore by Corollary 2.4, we conclude the proof.

Remark 2.6. Theorem 2.1 and Example 2.2 extend and generalize Theorem 1.5 to $dq\ b$\$-metric spaces by taking $\lambda_i = 0, i = 1,2,3$ in Theorem 2.1.

Remark 2.7. Corollary 2.3 extend and generalize the result of [24] in the framework of $dq\ b$-metric space.

In the following, we deduce a common fixed point theorem to a pair of onto expansive type self-mappings.

Theorem 2.8. *Let (X,d) be a complete $dq\ b$-metric space and S,T be two onto self-mapping on X such that*

$$d(Sa,Tb) \geq k\min\left\{\alpha\, d(a,b); \beta_1\frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b)\right.$$
$$+ \gamma_3\, d(a,b); \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1\frac{d(Sa,b)d(Tb,b)}{d(a,b)}$$
$$\left. + \lambda_2\frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right\} \tag{2.10}$$

and

$$d(Ta,Sb) \geq k\min\left\{\alpha\, d(a,b); \beta_1\frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b)\right.$$
$$+ \gamma_3\, d(a,b); \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1\frac{d(Sa,b)d(Tb,b)}{d(a,b)}$$
$$\left. + \lambda_2\frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right\} \tag{2.11}$$

for all $a, b \in X$ *with* $d(a,b) \neq 0, k > 1$, *nonnegative real numbers* $\alpha, \beta_i, \gamma_j, \delta_j, \lambda_j$ *for* $i = 1,2;\ j = 1,2,3$ *and* $\frac{1}{k} = min\{\alpha, \beta_2, \gamma_2 + \gamma_3, \delta_3, \lambda_3\}$. *Then S and T have a unique common fixed point.*

Proof. Let us take $\theta(a,b) = \min\Big\{\alpha\, d(a,b); \beta_1 \frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1 \frac{d(Sa,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\Big\}$.

For $a_0 \in X$, since S, T are onto, there exist $a_0, a_1 \in X$ such that $a_0 = Sa_{1,}, a_1 = Ta_2$. Continuing this process, we define a sequence $\{a_n\}$ by

$$\begin{aligned} Sa_{2n-1} &= a_{2n-2} \\ Ta_{2n} &= a_{2n-1}, \text{for all } n \in \mathbb{N}. \end{aligned} \tag{2.12}$$

The following cases will arise.

Case (i). If $\theta(a,b) = \alpha\, d(a,b)$, then

$$d(Ta,Sb) \geq k\,\alpha\, d(a,b), \text{for all } a,b \in X. \tag{2.13}$$

Now, using (2.12) and (2.13), we get

$$d(a_{2n}, a_{2n+1}) = d(Sa_{2n+1}, Ta_{2n+2}) \geq k\,\alpha\, d(a_{2n+1}, a_{2n+2})$$
$$\text{i.e.}, d(a_{2n+1}, a_{2n+2}) \leq \frac{1}{k\,\alpha} d(a_{2n}, a_{2n+1}).$$

Let $\tau = \frac{1}{k\,\alpha} < 1$. Then from the above inequality, we have

$$d(a_{2n+1}, a_{2n+2}) \leq \tau\, d(a_{2n}, a_{2n+1}).$$

Also, from (2.11),

$$d(a_{2n}, a_{2n+1}) \leq \tau\, d(a_{2n-1}, a_{2n}).$$

So,

$$d(a_{2n+1}, a_{2n+2}) \leq \tau^2\, d(a_{2n-1}, a_{2n}).$$

From this, we get

$$d(a_n, a_{n+1}) \leq \tau^n\, d(a_0, a_1).$$

For $j > i$,

$$\begin{aligned} d(a_i, a_j) &\leq s\, d(a_i, a_{i+1}) + s^2\, d(a_{i+1}, a_{i+2}) + \cdots + s^{j-i}\, d(a_{j-1}, a_j) \\ &\leq s\,\tau^i d(a_0, a_1) + s^2 \tau^{i+1} d(a_0, a_1) + \cdots + s^{j-i}\tau^{j-1} d(a_0, a_1) \\ &\leq \left[s\,\tau^i + s^2\tau^{i+1} + \cdots\right] d(a_0, a_1) \\ &= s\,\tau^i [1 + s\tau + (s\tau)^2 + \cdots] d(a_0, a_1) \\ &= \frac{s\,\tau^i}{1 - s\tau}\, d(a_0, a_1) \to 0 \text{ as } i, j \to \infty \end{aligned}$$

Therefore $\{a_n\}$ is a b-Cauchy sequence in X. Since X is complete, there exists $u \in X$ such that $\lim_{n\to\infty} a_n = u$. Since, S and T are onto, we can find $p, q \in X$ such that $Sp = Tq = u$.

Now, for all $n \in \mathbb{N}$.

$$d(u, a_{2n+1}) = d(Sp, Ta_{2n+2}) \geq k\,\alpha\, d(p, a_{2n+2}).$$

Taking limit superior as $n \to \infty$, and using Lemma 1.3, we get

$$\frac{1}{s} d(p,u) \leq \frac{k\,\alpha}{s} \lim_{n\to\infty} \sup d(p, a_{2n+2}) \leq \lim_{n\to\infty} \sup d(u, a_{2n+1}) \leq s\, d(u,u).$$

From Lemma. 1.4, we get $d(p,u) = 0$ and similarly $d(u,p) = 0$.

Thus, $d(p,u) = d(u,p) = 0$. So, $p = u$.

Uniqueness. Let $v (\neq u)$ be another common fixed point of S and T. Then

$$d(u,v) = d(Su, Tv) \geq k\,\alpha\, d(u,v), \text{i.e.}, (1 - k\,\alpha) d(u,v) \leq 0$$

which gives us $d(u,v) = 0$. Similarly, we can prove that $d(v,u) = 0$, and that $u = v$.

Case (ii). $\theta(a,b) = \beta_1 \frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b)$, then

$$d(Ta, Sb) \geq k\left[\beta_1 \frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b)\right] \tag{2.14}$$

Now, using (2.12) and (2.14), we get

$$\begin{aligned} d(a_{2n}, a_{2n+1}) = d(Sa_{2n+1}, Ta_{2n+2}) &\geq k\beta_1 \frac{d(a_{2n}, a_{2n+1}) d(a_{2n+1}, a_{2n+2})}{d(a_{2n+1}, a_{2n+2})} + k\beta_2 d(a_{2n+1}, a_{2n+2}) \\ &\geq k\beta_2 d(a_{2n+1}, a_{2n+2}) \end{aligned}$$

$\text{i.e.}, d(a_{2n+1}, a_{2n+2}) \leq \frac{1}{k\beta_2} d(a_{2n}, a_{2n+1})$ which implies that $d(a_{2n+1}, a_{2n+2}) \leq \vartheta\, d(a_{2n}, a_{2n+1})$, where $\vartheta < 1$. Proceeding similar to *Case (i)*, we get $\{a_n\}$ is a b-Cauchy sequence in X, which converges to some $u \in X$, which can be shown to be unique common fixed point of S and T.

Case (iii). $\theta(a,b) = \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b)$, then

$$d(Ta, Sb) \geq k[\gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b)] \tag{2.15}$$

Now, using (2.12) and (2.15), we get

$$\begin{aligned} d(a_{2n}, a_{2n+1}) &= d(Sa_{2n+1}, Ta_{2n+2}) \\ &\geq k\gamma_1 d(a_{2n}, a_{2n+1}) + k\gamma_2\, d(a_{2n+1}, a_{2n+2}) + k\gamma_3\, d(a_{2n+1}, a_{2n+2}) \\ &\geq k[\gamma_2 + \gamma_3] d(a_{2n+1}, a_{2n+2}) \end{aligned}$$

i.e., $d(a_{2n+1}, a_{2n+2}) \leq \frac{1}{k[\gamma_2+\gamma_3]} d(a_{2n}, a_{2n+1})$ implies that $d(a_{2n+1}, a_{2n+2}) \leq \rho\, d(a_{2n}, a_{2n+1})$, where $\rho < 1$. Proceeding similar to *Case (i)*, we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique common fixed point of S and T.

Case (iv). $\theta(a,b) = \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)$, then

$$d(Ta,Sb) \geq k[\delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b)] \tag{2.16}$$

Now, using (2.12) and (2.16), we get

$$\begin{aligned} d(a_{2n},a_{2n+1}) &= d(Sa_{2n+1},Ta_{2n+2}) \\ &\geq k\delta_1 d(a_{2n},a_{2n+2}) + k\delta_2\, d(a_{2n+1},a_{2n+1}) + k\delta_3\, d(a_{2n+1},a_{2n+2}) \\ &\geq k\delta_3 d(a_{2n+1},a_{2n+2}) \end{aligned}$$

i.e., $d(a_{2n+1},a_{2n+2}) \leq \frac{1}{k\delta_3} d(a_{2n},a_{2n+1})$ implies that $d(a_{2n+1},a_{2n+2}) \leq \omega\, d(a_{2n},a_{2n+1})$, where $\omega < 1$. Proceeding similar to *Case (i)*, we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique common fixed point of S and T.

Case (v). $\theta(a,b) = \lambda_1 \frac{d(Sa,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)$, then

$$d(Ta,Sb) \geq k\left[\lambda_1 \frac{d(Sa,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right] \tag{2.17}$$

Now, using (2.12) and (2.17), we get

$$\begin{aligned} d(a_{2n},a_{2n+1}) &= d(Sa_{2n+1},Ta_{2n+2}) \\ &\geq k\lambda_1 \frac{d(a_{2n},a_{2n+2})d(a_{2n+1},a_{2n+2})}{d(a_{2n+1},a_{2n+2})} + k\lambda_3\, d(a_{2n+1},a_{2n+2}) \\ &\geq k\lambda_3 d(a_{2n+1},a_{2n+2}) \end{aligned}$$

i.e., $d(a_{2n+1},a_{2n+2}) \leq \frac{1}{k\lambda_3} d(a_{2n},a_{2n+1})$ which implies that $d(a_{2n+1},a_{2n+2}) \leq \varphi\, d(a_{2n},a_{2n+1})$, where $\varphi < 1$. Proceeding similar to *Case (i)*, we get $\{a_n\}$ is a b-Cauchy sequence in X, and that converges to some $u \in X$, which is a unique common fixed point of S and T.

Example 2.9. Let $X = \mathbb{R}^+$. We define $d: X \times X \to \mathbb{R}^+$ by $d(a,b) = |a-b|^2 + |a|^2$. Then clearly, (X,d) is a complete dp b-metric space with $s = 2$. We define self-mappings $S,T: X \to X$ by

$$S(a) = a(a+2) \text{and } T(a) = 2a, \text{for all } a \in X.$$

We take $k = \frac{3}{2}, \alpha = \beta_2 = \gamma_3 = \delta_3 = \lambda_3 = 1, \beta_1 = \gamma_1 = \gamma_2 = \delta_1 = \delta_2 = \lambda_1 = \lambda_2 = 0.$

Without loss of generality we assume that $a \geq b$. We consider

$$\begin{aligned} d(Sa,Tb) &= |Sa - Tb|^2 + |Sa|^2 \\ &= (a^2 + 2a - 2b^2)^2 + (a^2 + 2a)^2 \\ &\geq (2a - 2b)^2 + 2a^2 \\ &\geq \frac{3}{2}[(a-b)^2 + a^2] \end{aligned}$$

$$= k \min\left\{\alpha\, d(a,b); \beta_1 \frac{d(Sa,a)d(Tb,b)}{d(a,b)} + \beta_2 d(a,b); \gamma_1 d(Sa,a) + \gamma_2\, d(Tb,b) + \gamma_3\, d(a,b); \delta_1 d(Sa,b) + \delta_2 d(Tb,a) + \delta_3 d(a,b); \lambda_1 \frac{d(Sa,b)d(Tb,b)}{d(a,b)} + \lambda_2 \frac{d(Tb,a)d(Tb,b)}{d(a,b)} + \lambda_3 d(a,b)\right\}$$

$a = b$ values	$kd(a,b) = \frac{3}{2}a^2$	$d(Sa,Tb) = a^4 + (a^2+2a)^2$
0	0	0
0.1	0.015	0.0442
0.2	0.06	0.1952
0.3	0.135	0.4842
0.4	0.24	0.9472
0.5	0.375	1.625
0.6	0.54	2.5632
0.7	0.735	3.8122
0.8	0.96	5.4272
0.9	1.215	7.4682
1	1.5	10

Table 2: Demonstration of the inequalities (2.10) and (2.11), using some numerical values.

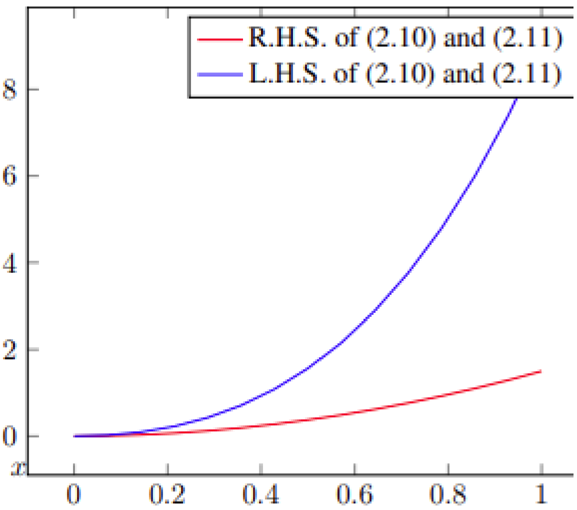

Figure 3

Table 2 and Figure 3, illustrates the condition (2.10) and (2.11) of Theorem 2.8, with blue line representing the left part of the condition and red line representing the right part of the condition. Thus, all the conditions of Theorem 2.8 are satisfied. So, S and T have a unique common fixed point in X which is clearly 0 here.

***Remark* 2.10.** Theorem 2.8 and Example 2.9 extend and generalize Theorem 1.6 to dq b-metric spaces.

Corollary 2.11. *Let (X,d) be a complete dq b-metric space and S,T be two onto self-mappings on X such that $d(Sa,T\,b) \geq k\,d(a,b)$, for all $a,b \in X$ with $k > 1$. Then S and T have a unique common fixed point in X.*

3. Application to nonlinear integral equations

Let $\Omega = C[a,b]$ be a set of real valued continuous functions on $[a,b]$,where $[a,b]$ is closed and bounded integral in $\mathbb{R}$. We define $d\colon \Omega\times\Omega \to \mathbb{R}^+$ by $d(\xi,\eta) = \max_{a\leq t\leq b}\{|\xi(t)-\eta(t)|^p + |\xi(t)|^p\}$, where $p > 1$ a real number, for all $\xi,\eta \in \Omega$. Therefore (Ω,d) is a complete b-metric space with

$s = 2^p$. Many author's studied unique solution of a nonlinear integral equations [4-7]. In this section, we obtain the existence of unique solution of a nonlinear integral equation of Fredholm type defined by

$$\xi(t) = f(t) + \mu\int_a^b \mathcal{M}(t,r,\xi(r))dr \qquad (3.1)$$

Where $\xi \in \Omega$ is the unknown function, μ in $\mathbb{R}$, $t,r \in [a,b]$, $\mathcal{M}\colon [a,b]\times[a,b]\times\mathbb{R}\to\mathbb{R}$ and $f\colon [a,b]\to\mathbb{R}$ are continuous functions. Let $\mathcal{F}\colon \Omega\to\Omega$ be a mapping defined by

$$\mathcal{F}(\xi(t)) = f(t) + \mu\int_a^b \mathcal{M}(t,r,\xi(r))dr \qquad (3.2)$$

Theorem 3.1. *Let $\mathcal{F}\colon\Omega\to\Omega$ be a mapping defined by* (3.2) *and there exists a constant $K > 1$ such that for all $t,r \in [a,b]$ and $\xi_1,\xi_2 \in \Omega$ with $|\mu| \geq 1$, the following condition is satisfied:*

$\left|\int_a^b[\mathcal{M}(t,r,\xi_1(r)) - \mathcal{M}(t,r,\xi_2(r))]dr\right|^p \geq K|\xi_1(t)-\xi_2(t)|^p + K|\xi_1(t)|^p$. *Then the system of nonlinear integral equations* (3.1) *has a unique solution in* Ω .

Proof. Let $\xi_1,\xi_2 \in \Omega$ and for all $t \in [a,b]$, we have

$$\begin{aligned}
d(\mathcal{F}\xi_1,\mathcal{F}\xi_2) &= \max_{a\leq t\leq b}\{|\mathcal{F}\xi_1(t)-\mathcal{F}\xi_2(t)|^p + |\mathcal{F}\xi_1(t)|^p\}\\
&\geq |\mu|^p \max_{a\leq t\leq b}\left\{\left|\int_a^b \mathcal{M}(t,r,\xi_1(r))dr - \int_a^b \mathcal{M}(t,r,\xi_2(r))dr\right|^p \right.\\
&\quad \left. + \left|\int_a^b \mathcal{M}(t,r,\xi_1(r))dr\right|^p\right\}\\
&\geq |\mu|^p \max_{a\leq t\leq b}\left\{\left|\int_a^b [\mathcal{M}(t,r,\xi_1(r)) - \mathcal{M}(t,r,\xi_2(r))]dr\right|^p\right\}\\
&\geq K\max_{a\leq t\leq b}\{|\xi_1(t)-\xi_2(t)|^p + K|\xi_1(t)|^p\}\\
&= Kd(\xi_1,\xi_2)
\end{aligned}$$

which implies that

$$d(\mathcal{F}\xi_1,\mathcal{F}\xi_2) \geq Kd(\xi_1,\xi_2).$$

Therefore, all the conditions of Corollary 2.3 are satisfied, and hence $\mathcal{F}$ has a unique solution for nonlinear integral equations defined in (3.1).

4. Conclusion and future work

In this paper, we studied fixed point results for expansive mappings in dislocated quasi-b-metric spaces. Using similar approaches, it can be studied new fixed point results on metric and some generalized metric spaces. The investigation of certain circumstances to exclude the identity map of X from Theorem 2.1 and Theorem 2.8 and related results is a worthwhile problem for future effort.

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