Communications on Applied Nonlinear Analysis
ISSN: 1074-133X
Vol 32 No. 10s (2025)

On Irregular Coloring of Middle Graph of Snake Graphs

Dr. G. Jothilakshmi.[1], Dr. K. Rajam [2], Dr. S. Manimekalai [3], Dr. U. S. Monolisa[4], K. Nandhini[5]

[1,5] Assistant Professor, Department of Mathematics, RVS College of Arts and Science (Affiliated to Bharathiar University) Coimbatore, India. Email: jothikrishna11@gmail.com

[2,] Assistant Professor (SS), Department of Mathematics, Dr. NGP Institute of Technology (Affiliated to Anna University) Coimbatore, India, Email: rajee.mat@gmail.com

[3,] Associate Professor, Department of Mathematics, Dr.N.G.P Arts and Science College (Affiliated to Bharathiar University) Coimbatore, India Email:Manimekalai@drngpasc.ac.in

[4] Assistant Professor, Department of Mathematics, Dr. Mahalingam College of Engineering and Technology (Affiliated to Anna University) Coimbatore, India Email: monolisamat@gmail.com

Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract: An irregular coloring is a proper coloring in which distinct vertices have different color codes. In this paper we obtain the irregular chromatic number of middle graph of certain snake graphs of snake graph families.

Keywords: Double triangular snake graph, Diamond snake graph, Graph coloring, Irregular chromatic number, Irregular coloring , Middle graph ,Triangular snake graph

Classification number: 05C15, 05C76

1. Introduction

Let G be a finite, undirected graph [1] with no loops and multiple edges. The graph G has the vertex set V (G) and the edge set E (G). Graph coloring [5] is coloring of G such that no two adjacent vertices share the same color.

A proper coloring c is an irregular coloring [1] if no two like-colored vertices have the same color code. i.e., for every pair of vertices u and w; code(u) ≠code(w) whenever c(u) = c(w). Thus, an irregular coloring distinguishes each vertex from each of other vertex by its color or by its color code.

Mary Radcliffe and Ping Zhang[8,9] established sharp upper and lower bounds for the irregular chromatic number of a disconnected graph in terms of the irregular chromatic numbers of its components. A.Rohini and M.Venkatachalam [11] discussed the irregular coloring of wheel related graphs. Further this paper exhibits the irregular coloring of snake graph families.

The **central graph** C(G) is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

The vertex set of **middle graph** M(G) is V(G)∪E(G) in which two elements are adjacent in M(G) if the following conditions hold.

(i) x,y ∈ E(G) and x, y are adjacent in G. (ii) x ∈V(G), y ∈ E(G) and y is incident on x in G.

A **triangular snake graph** T_n [3] is obtained from a path $v_1, v_2, v_3, \ldots v_{n+1}, u_1, u_2 \ldots u_n$ .It has 2n+1 vertices and 3n edges, where n is the number of blocks in the triangular snake and a **double triangular snake graph** $D(T_n)$ [3] is obtained from a path $v_1, v_2, v_3, \ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex w_i to another new vertex u_i . A **diamond snake graph** D_n [3] is obtained from a path $v_1, v_2, v_3, \ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex w_i to another new vertex u_i

2. Structural properties of middle graph of M (T_n), M [D(T_n)] and M[D_n]

- Number of vertices in M (T_n), p= 5n+1
- Number of vertices in M[D(T_n)] ,p=8n+1
- Number of vertices in M[D_n] ,p=7n+1
- Maximum degree in M (T_n) and M[D_n], Δ=6
- Maximum degree in M[D(T_n)], Δ=7
- Minimum degree in M(T_n),M[D_n], δ =2
- Minimum degree in M[D(T_n)], $\delta = 3$

3. IRREGULAR COLORING OF M (T_n), M [D(T_n)] and M[D_n]

Theorem 3.1 : For M(T_n), χ_{ir}[[M (T_n)] = $2n + 2$, n≥2

Proof:

Now by definition of middle graph, each edge of the triangular snake graph T_n is sub-divided by a new vertex.

Assume that each edge (v_i,v_{i+1}) and the line joining v_i and v_{i+1} to a vertex u_i ,i=1,2,3,….n are sub-divided by the vertices w_i, e_{jj} and e_{jj+1}, j=1,2,3,….n respectively.

Assign the color c_1 to v_i, u_i and $c_2, c_3 \ldots c_{2n+1}$ to $e_1, e_2 \ldots e_{2n}$

Assign the color $c_{n+3}, c_{n+4} \ldots c_{2n+2}$ to $e_{2n+1}, e_{2n+2} \ldots e_{3n}$

To prove (2n+2)- coloring is an irregular coloring of M (T_n),

since deg(u_i) ≠ deg(e_i),it shows that code(u_i) ≠ code (e_i)

Since each u_i's are adjacent to e_i's but v_i's are not adjacent to u_i's.

Hence code (e_i) ≠code(v_i) .Thus χ_{ir}[M (T_n)] ≤ $2n + 2$

By the definition of middle graph{v,e_i}induces a clique of order 2n+2 in M(T_n) [9]

It follows that,χ_{ir}[M (T_n)] ≥ χ[M (T_n)] = $2n + 2$. Hence χ_{ir}[[M (T_n)] = $2n + 2$, n≥2

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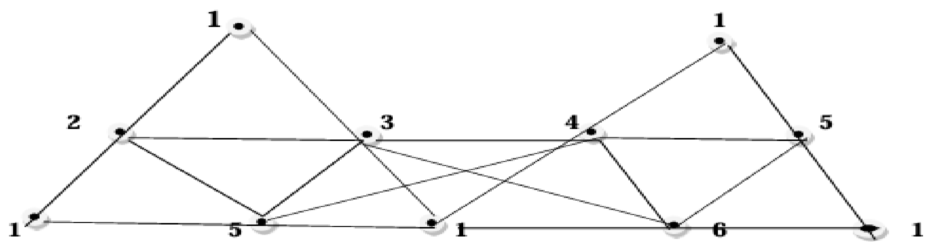

Figure 1: Middle graph of Triangular snake graph M(T_2)

Theorem 3.2 :

For [M(D (T_n))] ,the irregular - chromatic number is 6. $(i.e)$ χ_{ir} [M(D (T_n))] = $3n+3$ n ≥ 2

Proof:

Let {v_1,v_2………v_{n+1} , u_1,u_2………u_n ,w_1,w_2……w_n} be the vertices of the double triangular snake graph D(T_n).

Assume that each edge (v_i,v_{i+1}) (u_i , v_j)and (v_j , w_i) , i=1,2,3,….n and j=1,2,3…n+1 is sub-divided by the vertices x_i, e_{ij} and f_{ij} for , i=1,2,3,….n and j=1,2,3…n+1 respectively.[9]

Color the vertices v_1,v_2………v_{n+1} ,u_1,u_2………u_n and w_1,w_2……w_n with c_1.

Color the elements e_1,e_2……e_{2n} with c_2, c_3…. c_{2n+1} and e_{2n+1},e_{2n+2}……e_{4n} with c_5, c_6…..c_{2n+4}

Atlast, Color the sub-divided vertices $v_{i,i+1}$,$v_{i+1,i+2}$….$v_{n,i+n}$ with c_{2n+4}, c_{2n+5}…. c_{3n+3}

Hence χ_{ir} [M (D(T_n)] = $3n+3$, n≥2

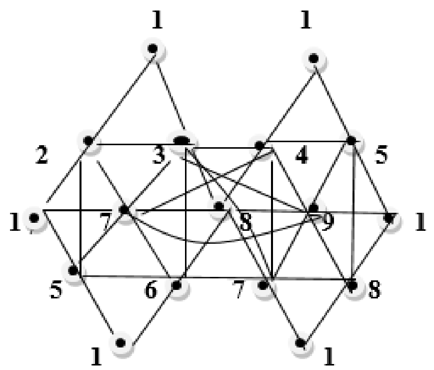

Figure 2: Middle graph of Double Triangular snake graph M(D(T_2))

Theorem 3.3:

For [M (D_n)], the irregular -chromatic number is 2n+3 $(i.e)$ χ_{ir} [M (D_n)] =2n+3, n≥2

Proof:

Color the vertices {v_1,v_2………v_{n+1} , u_1,u_2………u_n ,w_1,w_2,….w_n }with c_1.

Color the elements e_{11},e_{12}……e_{2n} with c_2, c_3. …. c_{2n+1}

Color the elements $e_{2n+1}, e_{2n+2} \ldots\ldots e_{4n}$ with $c_4, c_5 \ldots\ldots c_{2n+3}$.

To prove (2n+3)- coloring is an irregular coloring of M (D_n),

since $\deg(u_i) \neq \deg(e_i)$,it shows that $code(u_i) \neq code(e_i)$

Since each u_i's are adjacent to e_i's but v_i's are not adjacent to u_i's.

Hence $code(e_i) \neq code(v_i)$.

Thus $\chi_{ir}[M(D_n)] \leq 2n+3$

By the definition of middle graph,{v,e_i} induces a clique of order 2n+3 in $M(D_n)$[9]

It follows that,$\chi_{ir}[M(D_n)] \geq \chi[M(D_n)] = 2n+3$

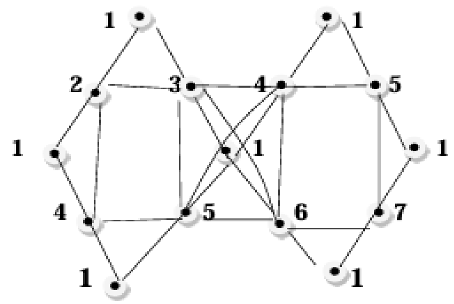

Figure 3: Middle graph of Double Triangular snake graph $M(D_2)$.

5. Conclusion

An irregular coloring play an important role in clustering , automatic reading system and distributed system.The investigation of similar results for different graphs as well in the context of various graph coloring problems is an open area of research.

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