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Neutrosophic Fuzzy α Composition

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Abstract:
We discussed the methods for finding the optimal (maximum and minimum) solution of inverse problem of Neutrosophic fuzzy relation equation of the form R ◦ Q = T where for both cases R and Q are kept unknown interchangeably using alpha operator.

Keywords: Fuzzy set, Neutrosophic fuzzy set, alpha operator, composition.

1. Introduction

In 1965, Zadeh[1] introduced the concept of fuzzy set which takes the values between 0 &1. Later many authors developed different algebras , concepts and results in fuzzy sets. In vast theories we are studied few literature on algebraic structures**.** Uzair Ahmed developed Optimal Solution of Fuzzy Relation Equations. In fuzzy set only true values are included. But in some situations indeterminacy and falsity included. To counter this situation. Smarandache introduced and developed concepts of Neutrosophic sets. Later many authors start working on NS sets. Hemabala Srinivasa Kumar[14-16] developed algebrac structures in Neutrosophic multi fuzzy sets. In this paper particularly emphasize finding the maximum solution of Neutrosophic fuzzy relation equations through the use of alpha operator and inverse relations.

2. Preliminaries

Fuzzy Set 2.1 Fuzzy set was first defined by Lofti A.Zadeh[1] in 1965. He defined a fuzzy set as a collection of objects with membership values in the interval [0,1]. These membership values represent the grades of membership with the properties and distinct features of the collection.

Fuzzy sets are mathematically defined as follows: “A subset A of universe X with the membership function μ(x) which may take any value in the interval [0,1] is called fuzzy set”.

μ_A : X → [0,1] is called a fuzzy subset of X.

Neutrosophic fuzzy set 2.2

Let X be a non empty set. A neutrosophic fuzzy set $\mathcal{N}$ on X can be defined as follows

$\mathcal{N}=\{< x,(t_{\mathcal{N}}(x), i_{\mathcal{N}}(x), f_{\mathcal{N}}(x) >/x \in X\}$

Where $t_{\mathcal{N}}(x)$: X →[0,1]

$i_{\mathcal{N}}(x)$: X →[0,1]

$f_{\mathcal{N}}(x)$: X →[0,1]

$0 \leq \sup t_{\mathcal{N}}(x) + sup\, i_{\mathcal{N}}(x) + \sup f_{\mathcal{N}}(x) \leq 3$

$t_{\mathcal{N}}(x)$ is the truth membership value, $i_{\mathcal{N}}(x)$ is the indeterminacy membership value and $f_{\mathcal{N}}(x)$falsity membership value.

Properties 2.3:

Let X be a non empty set and $\mathcal{N}$ and $\mathcal{M}$ neutrosophic fuzzy sets of X. Then

1. $\mathcal{N} \cup \mathcal{M} = \{< x, t_{\mathcal{N} \cup \mathcal{M}}(x), i_{\mathcal{N} \cup \mathcal{M}}(x), f_{\mathcal{N} \cup \mathcal{M}}(x) >/x \in X \}$,

   where $t_{\mathcal{N} \cup \mathcal{M}}(x) = \max(t_{\mathcal{N}}(x), t_{\mathcal{M}}(x))$
   $i_{\mathcal{N} \cup \mathcal{M}}(x) = \min(i_{\mathcal{N}}(x), i_{\mathcal{M}}(x))$
   $f_{\mathcal{N} \cup \mathcal{M}}(x) = \min(f_{\mathcal{N}}(x), f_{\mathcal{M}}(x))$,

2. $\mathcal{N} \cap \mathcal{M} = \{< x, t_{\mathcal{N} \cap \mathcal{M}}(x), i_{\mathcal{N} \cap \mathcal{M}}(x), f_{\mathcal{N} \cap \mathcal{M}}(x) >/x \in X \}$,

   where $t_{\mathcal{N} \cap \mathcal{M}}(x) = \min(t_{\mathcal{N}}(x), t_{\mathcal{M}}(x))$
   $i_{\mathcal{N} \cap \mathcal{M}}(x) = \max(i_{\mathcal{N}}(x), i_{\mathcal{M}}(x))$
   $f_{\mathcal{N} \cap \mathcal{M}}(x) = \max(f_{\mathcal{N}}(x), f_{\mathcal{M}}(x))$,

Definition 2.4: A neutrosophic set $\mathcal{N}$ is contained in another neutrosophic set $\mathcal{M}$ if
$t_{\mathcal{N}}(x) \leq t_{\mathcal{M}}(x), i_{\mathcal{N}}(x) \geq i_{\mathcal{M}}(x)\ , f_{\mathcal{N}}(x) \geq f_{\mathcal{M}}(x)$

Definition 2.5: Two neutrosophic sets $\mathcal{N}$ and $\mathcal{M}$ are said to be equal if
$t_{\mathcal{N}}(x) = t_{\mathcal{M}}(x), i_{\mathcal{N}}(x) = i_{\mathcal{M}}(x)\ , f_{\mathcal{N}}(x) = f_{\mathcal{M}}(x)$

Definition 2.6:

A lattice is a partially ordered set (poset) L in which any two elements “x” and “y” have a greatest lower bound (inf) denoted by $x \wedge y = \min(x, y)$ and a least upper bound (sup) denoted by $x \vee y = \max(x, y)$.

Definition 2.7:

A brouwerian lattice is a lattice L [2] in which for any given elements “a” and “b”, the set of all $x \in L$ such that $a \vee x \leq b$ contains a greatest element, denoted “a α b”, the relative pseudo complement of a in b.

Definition 2.8:

For any given a and b in lattice L ∈ [0,1], α - operator is defined as

$$a \alpha b = \begin{cases} 1 \; if \; a \leq b \\ b \; if \; a > b \end{cases}$$

It is also called Sanchez operator

Example 1:

For different values of a and b , α - operator will defned as:

$0.8\alpha0.5 = 0.5$, $0.5\alpha0.6 = 1$, $0.3\alpha0.3 = 1$, $0.6\alpha0.3 = 0.3$

Properties of α - Operator 2.9:

1. If b = 0 then “a α b” will be given as

   $$a \alpha 0 = \begin{cases} 1 \; if \; a = b \\ b \; if \; a > b \end{cases}$$

2. If a = 0 then “a α b” will be given as:

   0 α b = 1hh

3. If b = 1 then “a α b” will be given as:

   a α 1 = 1

4. If a = 1 then “a α b” will be given as:

   1 α b =b

5. α - operator is not commutative.

   $a \alpha b \neq b \alpha a$ (Not Commutative)

6. α - operator is not associative.

a α (b α c) ≠ (a α b) α c (Not Associative)

3. Neutrosophic composition of the α – operator type

In this section we will discuss the methods of finding the maximal solution of neutrosophic fuzzy relation equation with respect to unknowns R and Q. We will discuss the methods for finding the maximal "Q" and maximal "R" respectively for NFRE of the form R ∘ Q = T. Here "∇" denotes the maximal solution.

Definition 3.1:

Consider two neutrosophic fuzzy relations R ⊆ X × Y and Q ⊆ Y × Z. Relationship between these two neutrosophic fuzzy relations when using @ composition is defined as

R @ Q ⊆ X × Z with the membership function defined as:

$$\mu_{R@Q}(x,z) = \left(\wedge\{\mu_{RT}(x,y)\alpha\mu_{QT}(y,z)\}, \vee\{\mu_{RI}(x,y)\alpha\mu_{QI}(y,z)\}, \vee\{\mu_{RF}(x,y)\alpha\mu_{QF}(y,z)\}\right)$$

∀ x ∈ X, y ∈ Y and z ∈ Z

Example 2:

let X = {x_1, x_2, x_3}, Y = {y_1, y_2, y_3} and Z = {z_1, z_2, z_3}

Consider two fuzzy relations R ⊆ X × Y and Q ⊆ Y × Z which are given below respectively .We are to compute T ⊆ X × Z using "@" composition

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} (0.1,0.2,0.7) & (0,0,0) & (0.5,0.4,0.1) \\ (1,0,0) & (0.6,0.3,0.1) & (0.8,0.1,0.1) \\ (0.3,0.3,0.4) & (0.4,0.2,0.4) & (0.1,0.5,0.4) \end{bmatrix} \end{array}$$

$$Q = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} (1,0,0) & (0.9,0,0.1) & (0.6,0.4,0) \\ (0.5,0.2,0.3) & (0,0,0) & (0.2,0.1,0.1) \\ (0,0,0.1) & (0.6,0.3,0.1) & (0.1,0.2,0.3) \end{bmatrix} \end{array}$$

T(x, z) = $\mu_{R@Q}(x,z)$ ∀ x ∈ X and z ∈ Z

$R@Q= (\wedge\{(0.1\alpha1),(0\ \alpha0.5),(0.5\ \alpha0)\},\vee\ \{(0.2\alpha0),(0\ \alpha0.2),(0.4\ \alpha0)\},\ \vee$
$\{(0.7\alpha0),(0\ \alpha0.3),(0.1\ \alpha0.1)\}]=(0,1,1)$

$R@Q= (\wedge\{(0.1\alpha0.9),(0\ \alpha0),(0\ \alpha0.6)\},\vee\ \{(0.2\alpha0),(0\ \alpha0),(0.4\ \alpha0.3)\},\ \vee$
$\{(0.7\alpha0.1),(0\ \alpha0),(0\ \alpha0.1)\}]=(1,1,1)$

$R@Q= (\wedge\{(0.1\alpha0.6),(0\ \alpha0.2),(0\ \alpha0.1)\},\vee\ \{(0.2\alpha0.4),(0\ \alpha0.1),(0.4\ \alpha0.2)\},\ \vee$
$\{(0.7\alpha0),(0\ \alpha0.1),(0\ \alpha0.3)\}]=(0.1,1,1)$

$R@Q= (\wedge\{(1\alpha1),(0.6\ \alpha0.5),(0.8\ \alpha0)\},\vee\ \{(0\alpha0.1),(0.3\ \alpha0.2),(0.1\ \alpha0)\},\ \vee$
$\{(0\alpha0),(0.1\ \alpha0.3),(0.1\ \alpha0.1)\}]=(0.5,1,1)$

$R@Q= (\wedge\{(1\alpha0.9),(0.6\ \alpha0),(0.8\ \alpha0.6)\},\vee\ \{(0\alpha0),(0.3\ \alpha0),(0.1\ \alpha0.3)\},\ \vee$
$\{(0\alpha0.1),(0\ \alpha0),(0.1\ \alpha0.1)\}]=(0,1,1)$

$R@Q= (\wedge\{(1\alpha0.6),(0.6\ \alpha0.2),(0.8\ \alpha0.1)\},\vee\ \{(0\alpha0.4),(0.3\ \alpha0.1),(0.1\ \alpha0.2)\},\ \vee$
$\{(0\alpha0),(0.1\ \alpha0.1),(0.1\ \alpha0.3)\}]=(0.1,1,1)$

$R@Q= (\wedge\{(0.3\alpha1),(0.4\ \alpha0.5),(0.1\ \alpha0)\},\vee\ \{(0.3\alpha0.1),(0.2\ \alpha0.2),(0.5\ \alpha0)\},\ \vee$
$\{(0.4\alpha0),(0.4\ \alpha0.3),(0.4\ \alpha0.1)\}]=(0,1,0.3)$

$R@Q= (\wedge\{(0.3\alpha0.9),(0.4\ \alpha0),(0.1\ \alpha0.6)\},\vee\ \{(0.3\alpha0),(0.2\ \alpha0),(0.5\ \alpha0.3)\},\ \vee$
$\{(0.4\alpha0.1),(0.4\ \alpha0),(0.4\ \alpha0.1)\}]=(0,0,0.1)$

$R@Q= (\wedge\{(0.3\alpha0.6),(0.4\ \alpha0.2),(0.1\ \alpha0.1)\},\vee\ \{(0.3\alpha0.4),(0.2\ \alpha0.1),(0.5\ \alpha0.2)\},\ \vee$
$\{(0.4\alpha0),(0.4\ \alpha0.1),(0.4\ \alpha0.3\}]=(0.2,1,0.3)$

$$T(x, z) = \mu_{R@Q}(x,z)= \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} (0,1,1) & (1,1,1) & (0.1,1,1) \\ (0.5,1,1) & (0,1,1) & (0.1,1,1) \\ (0,1,0.3) & (0,0,0.1) & (0.2,1,0.3) \end{bmatrix} \end{array}$$

Lemma 3.2:

If we have two fuzzy relations R ⊆ X × Y and Q ⊆ Y × Z then the following inclusion will hold: $\mu_Q^T \le \mu^T_{\mathrm{R-1@(R\,\circ\,Q)}}$, $\mu^I_{\mathrm{R-1@(R\,\circ\,Q)}} = 1$, $\mu^F_{\mathrm{R-1@(R\,\circ\,Q)}} = 1$ if R ⊆ Q or Q ⊆ R

where "◦" denotes the max min composition and "@" is the composition made by α - operator.

Proof:

Let A be the neutrosophic fuzzy set

Consider, A = R^{-1}@(R ◦ Q) ⊆ Y × Z.

$$\mu_A(y,z) = \{\mu_{R^{-1}}(y,x)\alpha\mu_{RoQ}(x,z)\} \qquad \forall\ x \in X$$

$$= \{\mu_R(x,y)\alpha\mu_{RoQ}(x,z)\} \qquad \forall\ x \in X$$

$$= \{\mu_R^T(x,y)\alpha\left(\vee\left(\mu_R^T(x,t)\wedge\mu_Q^T(t,z)\right)\right),1,1\} \qquad \text{since} \quad R \subseteq Q \text{ and } \quad \alpha \text{ operator}$$

$$(\mu_A^T(y,z),\mu_A^I(y,z),\mu_A^F(y,z)) \qquad = \{\mu_R^T(x,y)\alpha(\left(\mu_R^T(x,y)\wedge\mu_Q^T(y,z)\right)\wedge\vee_{t\neq y}\left(\vee\,\mu_R^T\left((x,t)\wedge\mu_Q^T(t,z)\right)\right),1,1\}$$

$$\mu_A^T(y,z) \quad = \{\mu_R^T(x,y)\alpha(\left(\mu_R^T(x,y)\wedge\mu_Q^T(y,z)\right)\wedge\vee_{t\neq y}\left(\vee\,\mu_R^T\left((x,t)\wedge\mu_Q^T(t,z)\right)\right)$$

$\mu_A^I(y,z)$=1

$\mu_A^F(y,z)$=1

$$\mu_A^T(y,z) \quad \geq\{\mu_R(x,y)\alpha((\mu_R(x,y)\wedge\mu_Q(y,z))\}$$

since a α (a ∧ b) ≥ b

$$\mu_A(y,z) \geq \mu_Q(y,z)$$

Example 3:

Consider X = {x_1, x_2, x_3}, Y = {y_1, y_2, y_3} and Z = {z_1, z_2, z_3}.

$$\begin{array}{cc} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ R = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} (0.5,0.9,0.8) & (0.6,0.3,0.2) & (0.4,0.5,0.1) \\ (0.3,0.2,0.3) & (0.2,0.7,0.6) & (0.8,0.2,0.2) \\ (1,0.3,0.2) & (0.0,0.7,0.6) & (0.6,0.3,0.2) \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ Q = \begin{array}{c} y_1 \\ y_2 \\ y_3 \end{array} & \begin{bmatrix} (0.7,0.8,0.6) & (0.8,0.2,0.1) & (0.5,0.4,0) \\ (0.5,0.1,0.2) & (0.4,0.5,0.5) & (0.9,0.1,0.1) \\ (1,0.2,0.1) & (0.3,0.6,0.5) & (0.8,0.2,0.1) \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ R^{-1} = \begin{array}{c} y_1 \\ y_2 \\ y_3 \end{array} & \begin{bmatrix} (0.5,0.9,0.8) & (0.3,0.2,0.3) & (1,0.3,0.2) \\ (0.6,0.3,0.2) & (0.2,0.7,0.6) & (0.0,0.7,0.6) \\ (0.4,0.5,0.1) & (0.8,0.2,0.2) & (0.6,0.3,0.2) \end{bmatrix} \end{array}$$

$$
RoQ = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} (0.5,0.3,0.2) & (0.5,0.5,0.5) & (0.6,0.3,0.2) \\ (0.8,0.2,0.2) & (0.3,0.2,0.3) & (0.8,0.2,0.2) \\ (0.7,0.3,0.2) & (0.8,0.3,0.2) & (0.6,0.3,0.2) \end{bmatrix} \end{array}
$$

$$
R^{-1} @ RoQ = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} (0.7,1,1) & (0.8,1,1) & (0.5,1,1) \\ (0.5,1,1) & (0.5,1,1) & (0.6,1,1) \\ (1,1,1) & (0.3,1,1) & (1,1,1) \end{bmatrix} \end{array}
$$

So this example shows that $\mu_Q^T \leq \mu^T_{\mathrm{R-1@(R\,\circ\,Q)}}$, $\mu^I_{\mathrm{R-1@(R\,\circ\,Q)}} = 1$, $\mu^F_{\mathrm{R-1@(R\,\circ\,Q)}} = 1$ Hence it clearly satisfies the lemma

Lemma 3.3:

Assume that we have two fuzzy relations R ⊆ X × Y and T ⊆ X × Z then the following inclusion holds:

R ∘ (R $^{-1}$@T) ⊂ T where "∘" denotes the max-min composition and "@" is the composition of α -operator. The proof of this lemma is analogous to the proof of lemma

Lemma 3.4:

Consider we have two fuzzy relations R ⊆ X × Y and Q ⊆ Y × Z then the following inclusion holds:

R ⊆ (Q@ (R ∘ Q) $^{-1}$) $^{-1}$

Lemma 3.5:

Consider we have two fuzzy relations Q ⊆ Y × Z and T ⊆ X × Z then the following inclusion holds:

(Q@T $^{-1}$) $^{-1}$ ∘ Q ⊂ T

Theorem 3.6:

Let R ⊆ X × Y and T ⊆ X × Z be the two fuzzy relations, S(Q) be the set of fuzzy relations Q ∈ Y × Z such that R ∘ Q = T. S(Q) = {Q ∈ Y × Z | R ∘ Q = T} $\neq \varphi$, if and only if R^{-1}@T ∈ S(Q) then "R^{-1}@T" is the the greatest element in S(Q).

Theorem 3.7:

Let R ⊆ X × Y and T ⊆ X × Z be the two fuzzy relations, the set of fuzzy relations Q ∈ Y × Z such that R ∘ Q ⊆ T contains a greatest element R^{-1}@T.

Proof:

Let $S(Q) = \{Q \in (Y \times Z) \mid R \circ Q \subseteq T\} \neq \varphi$.

let $Q \subseteq S(Q)$: $R \circ Q = T$

then we have $R^{-1}@ (R \circ Q) \subseteq R^{-1}@T$,

but $Q \subset R^{-1}@ (R \circ Q)$

then it shows

$Q \subset R^{-1}@T$

we have $R^{-1}@T \in S(Q)$.

Then it shows that $R-1@T \in S(Q)$.

then $R^{-1}@T$ will be the greatest element in S(Q).

Hence $R^{-1}@T$ be the greatest element in S(Q).

Then $Q^{\triangledown} = R^{-1}@ T$ which is the maximum relation "Q" satisfying the equation $R \circ Q = T$

4. Conclusion:

Our findings suggest that the identification of an optimal solution among various alternatives in a given problem can be achieved through the utilization of nonlinear fuzzy regression equation. After conducting extensive calculations using NFREs in the context of our proposed scenarios in civil engineering, we have been able to discern the most favourable outcome among the presented options. The application of fuzzy relation operations played a crucial role in assessing and determining the best project outcome. A potential avenue for future research could involve optimizing the influence factors rather than solely focusing on identifying the best project.

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