

Exploring Bifurcation Theory in Epidemiological Models: A Case Study on COVID-19 Spread

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Article History:

Received: 12-01-2025

Revised: 15-02-2025

Accepted: 01-03-2025

Abstract:

Bifurcation theory plays a crucial role in understanding the nonlinear dynamics of infectious disease transmission particularly in the context of global pandemics such as COVID-19. This study explores the application of bifurcation analysis to epidemiological models specifically the Susceptible-Infected-Recovered-Susceptible (SIRS) framework to identify critical threshold conditions that govern disease spread. Existing epidemiological models often rely on deterministic or stochastic approaches but lack a

systematic bifurcation analysis that explicitly captures transitions between controlled outbreaks and widespread pandemics. This research bridges this gap by conducting a comprehensive stability analysis of equilibrium states, examining the impact of key parameters such as the recruitment rate and the basic reproduction number (R_0). By integrating bifurcation theory with real-world data, we demonstrate how small changes in public health policies such as vaccination strategies and non-pharmaceutical interventions, can induce significant shifts in epidemic trajectories. The findings reveal that timely interventions, guided by bifurcation thresholds could have substantially altered the course of the COVID-19 pandemic, reinforcing the necessity of mathematical modeling in public health decision-making. This study also highlights the challenges in existing methodologies including the limitations of homogeneous modeling approaches that fail to account for population heterogeneity and policy compliance variations. Our methodology employs numerical bifurcation analysis, phase plane analysis and eigenvalue computations to systematically explore stability transitions within the SIRS framework. The practical implications of this research extend to optimizing control measures, refining epidemic forecasting and informing global health strategies for future outbreaks. By advancing the theoretical understanding of bifurcations in epidemiological systems, this study contributes to the broader field of applied nonlinear analysis and underscores the need for interdisciplinary approaches in mathematical epidemiology.

Keywords: Communication Analysis, Political Communication, Community Aspiration.

Background

Bifurcation theory plays a crucial role in understanding the dynamics of infectious disease spread particularly during significant public health events such as the COVID-19 pandemic. A bifurcation is defined as a qualitative or quantitative change in the dynamics of a system that occurs when a parameter crosses a critical threshold. This phenomenon can result in substantial shifts in the course of an epidemic, transitioning from a controlled outbreak to a major pandemic [1] [2] In the context of epidemiological modeling, bifurcations are essential for identifying the conditions under which an outbreak may escalate or diminish. For example, by studying the bifurcation structures of models like the SIRS (Susceptible-Infected-Recovered-Susceptible) system, researchers can analyze how variations in parameters impact the dynamics of disease transmission [3][4] The two-dimensional parameter space, specifically, allows for a detailed examination of key variables that govern epidemic behavior. One such critical parameter is the recruitment rate of individuals into the population which can be influenced by factors such as immigration policies [5][6]. Understanding how these parameters interact and lead to bifurcations can inform public health strategies aimed at controlling the spread of diseases. The implications of these changes are significant as they can guide the development of more effective control measures during outbreaks [7]. During the COVID-19 pandemic, for instance, analyses revealed that effective control measures could have led to better management of the disease spread. Graphical representations of deterministic models with and without control measures illustrated the potential outcomes based on various scenarios. The basic reproduction number, an important indicator in epidemiology, being less than 1 suggests that the disease could decline under certain controlled conditions [8][9][10]

Bifurcation Theory in Epidemiology

Bifurcation theory is crucial in understanding the dynamics of infectious diseases as it describes qualitative or quantitative changes that occur in a system when a parameter crosses a critical threshold. This phenomenon can significantly alter the course of an epidemic, potentially transforming a controlled outbreak into a widespread pandemic[11][12] By analyzing these bifurcations, researchers can develop more effective strategies for managing infectious diseases.

SIRS Model and Parameter Analysis

In the context of epidemiological modeling, the SIRS (Susceptible-Infectious-Recovered-Susceptible) model serves as a foundational framework. It allows for the examination of bifurcation structures in a two-dimensional parameter space specifically focusing on the parameters that influence epidemic dynamics. Notably, parameter A is instrumental as it governs the recruitment rate of individuals into the population, a factor that can be influenced by immigration policies. The selection of the appropriate parameter space for bifurcation analysis is essential as it enables researchers to identify conditions leading to significant shifts in disease spread.

Integration of Qualitative and Bifurcation Analysis

A comprehensive approach that integrates qualitative analysis with bifurcation analysis enhances the understanding of both social dynamics and mathematical modeling of disease transmission. This interdisciplinary methodology draws insights from social sciences alongside epidemiological modeling, thereby improving the reliability of predictions. By justifying the use of both qualitative and bifurcation analyses, researchers can effectively bridge theoretical modeling with practical public health strategies particularly in regions like Bangladesh where such dynamics are increasingly relevant[12][13]

Stability Analysis and Control Strategies

The stability of equilibrium points within the SIRS model is determined by analyzing the eigenvalues associated with the Jacobian matrix at fixed points. If the absolute values of the eigenvalues are less than one, the equilibrium point is considered asymptotically stable. Conversely, if at least one eigenvalue exceeds one in absolute value, the equilibrium point becomes unstable[14][15]. This analysis is typically numerical due to the complexities of nonlinear systems, though some analytical results can still be established. Future research directions involve a deeper exploration of bifurcation curves and their implications for control strategies. Investigating the stability domain of periodic points such as period-doubling bifurcations, poses analytical challenges but may yield critical insights for optimizing control measures during epidemics[16]. Understanding these bifurcations and their consequences is pivotal for translating theoretical findings into actionable public health interventions aimed at controlling infectious disease spread.

COVID-19 Case Study

The spread of COVID-19 has been extensively analyzed using various mathematical models to understand its dynamics and inform public health interventions. Researchers have utilized modified SIR (Susceptible, Infected, Recovered) and SIRS (Susceptible, Infected, Recovered and Susceptible) models to estimate infection rates and evaluate strategies for controlling the rapid spread of the

virus[17]. These models enable a detailed exploration of how changes in parameters can affect the epidemic's progression.

Mathematical Modeling Approaches

Many studies have adopted different modeling approaches, including continuous and discrete-time models as well as stochastic modeling to simulate the dynamics of COVID-19 transmission[18]. The SIRS epidemic model, in particular, has been highlighted for its ability to capture the cyclic nature of infection dynamics, where individuals may transition back to a susceptible state after recovery

Bifurcation Analysis

Bifurcation analysis has been a crucial tool in examining the stability and behavior of the SIRS model under varying parameters. Researchers have identified critical points at which small changes can lead to significant shifts in the system's dynamics thereby providing insights into the conditions that could lead to either outbreaks or declines in infection rates[19]. This analysis not only aids in understanding the existing data but also helps in predicting future trends in COVID-19 spread.

Application to Real-World Data

Numerical simulations have been conducted to enhance the understanding of stability regions within the parameter space. These simulations have utilized real-world data from countries such as Algeria and the USA to compare observed infection rates against model predictions[20][21]. By employing next-generation matrix approaches, researchers have calculated the basic reproduction number (R_0) which serves as a key indicator of the transmission potential of the virus[22]. This number has been essential in guiding policymakers to implement effective public health measures.

Vaccination Strategies and Their Impact

Vaccination plays a crucial role in controlling the spread of infectious diseases including COVID-19. Vaccination campaigns exert both direct and indirect effects, significantly influencing disease dynamics within populations[23]. The development and deployment of COVID-19 vaccines such as Pfizer, Moderna astraZeneca (AZD1222) and Johnson & Johnson (J&J Ad26.COVS) marked a significant milestone in the fight against the virus taking over one and a half years from the inception of research to the first vaccines being approved for public use in March 2021[18]. The efficacy of these vaccines has been reported to be high, with figures around 94% for Moderna, 95% for Pfizer, 81.3% for AstraZeneca and 85% for J&J. These vaccines not only help prevent severe disease and mortality but also enhance the immunity of the vaccinated population against the virus, contributing to overall public health safety. In conjunction with vaccination, public health measures such as lockdowns, social distancing and mask-wearing have also been shown to reduce the growth rate of confirmed COVID-19 cases. Studies indicate that these non-pharmaceutical interventions can be more effective when combined with vaccination strategies[27]. For instance, effective lockdowns were found to be associated with varying levels of compliance and the implementation of other measures highlighting the need for a multifaceted approach to epidemic control. Moreover, understanding the dynamics of vaccination is essential as it relates to the concepts of bifurcation within epidemiological models.

Bifurcation theory helps in analyzing how changes in vaccination coverage can lead to significant shifts in epidemic outcomes such as transitioning from a controlled outbreak to a pandemic state. By

investigating these dynamics, researchers can better develop strategies to optimize vaccination campaigns and improve public health responses

Conclusion and Future Direction

Bifurcation theory provides significant insights into the dynamics of infectious disease spread particularly in the context of the COVID-19 pandemic. It highlights how changes in model parameters can lead to qualitative shifts in epidemic behavior such as transitions from controlled outbreaks to widespread pandemics. Understanding these bifurcations is crucial for developing effective public health interventions and control strategies. Numerical simulations have been conducted to explore the stability regions of various parameters particularly focusing on the influence of the recruitment rate of individuals into the population which can be modulated through immigration policies. Such parameters are pivotal in determining the basic reproduction number (R_0) which is a critical metric for assessing the potential for an epidemic to grow or decline. By analyzing these stability regions, researchers can predict potential outbreaks and formulate timely responses. This systematic review identified several limitations in the existing studies on public health interventions. The heterogeneous methodologies employed, along with a lack of control groups in many studies, hinder the accurate evaluation of the individual impact of interventions. Most studies were categorized as having moderate quality, with some showing low levels of evidence. Furthermore, the simultaneous implementation of multiple interventions complicates the assessment of their individual effectiveness, potentially leading to misestimations of their overall impact on COVID-19 transmission dynamics.

Future research should focus on the need for rigorous methodologies particularly experimental and quasi-experimental designs to accurately assess the outcomes of non-pharmaceutical interventions and public health measures. Such studies will be vital for refining epidemic models and improving the efficacy of strategies aimed at controlling the spread of infectious diseases including COVID-19. Understanding the underlying bifurcation structures in these models will not only enhance predictive capabilities but also assist policymakers in making informed decisions regarding public health interventions.

References:

- [1]. Akhtar, M. W., Bashir, Z., & Malik, M. G. (2025). Exploring Bifurcations and Chaos in an Eco-epidemiological Prey-predator Model with Infected Prey: Optimization with RBFNN. *Journal of Dynamical and Control Systems*, 31(1), 1-35.
- [2]. Nadim, S. S., & Chattopadhyay, J. (2020). Occurrence of backward bifurcation and prediction of disease transmission with imperfect lockdown: A case study on COVID-19. *Chaos, Solitons & Fractals*, 140, 110163.
- [3]. Kiseleva, O., Yakovlev, S., Chumachenko, D., & Kuzenkov, O. (2024). Exploring bifurcation in the compartmental mathematical model of COVID-19 transmission. *Computation*, 12(9), 186.
- [4]. Ahmed, M., Masud, M. A. B., & Sarker, M. M. A. (2023). Bifurcation analysis and optimal control of discrete SIR model for COVID-19. *Chaos, Solitons & Fractals*, 174, 113899.
- [5]. Idisi, O. I., Yusuf, T. T., Owolabi, K. M., & Ojokoh, B. A. (2023). A bifurcation analysis and model of Covid-19 transmission dynamics with post-vaccination infection impact. *Healthcare Analytics*, 3, 100157.

- [6]. Kuddus, M. A., Mohiuddin, M., Paul, A. K., & Rahman, A. (2024). Insights from qualitative and bifurcation analysis of COVID-19 vaccination model in Bangladesh. *PloS one*, 19(11), e0312780.
- [7]. Wanjala, H., Okongo, M., & Ochwach, J. (2024). Bifurcation Analysis Of An Infectious Respiratory Disease With Lockdown And Social Distancing. *ARRUS Journal of Mathematics and Applied Science*, 4(2), 118-134.
- [8]. Sharbayta, S. S., Desta, H. D., & Abdi, T. (2023). Mathematical Modelling of COVID-19 Transmission Dynamics with Vaccination: A Case Study in Ethiopia. *Discrete Dynamics in Nature and Society*, 2023(1), 2972164.
- [9]. Radha, M., & Balamuralitharan, S. (2020). A study on COVID-19 transmission dynamics: stability analysis of SEIR model with Hopf bifurcation for effect of time delay. *Advances in Difference Equations*, 2020, 1-20.
- [10]. Wang, X., Wang, S., Wang, J., & Rong, L. (2022). A multiscale model of COVID-19 dynamics. *Bulletin of Mathematical Biology*, 84(9), 99.
- [11]. Kotola, B. S., Teklu, S. W., & Abebaw, Y. F. (2023). Bifurcation and optimal control analysis of HIV/AIDS and COVID-19 co-infection model with numerical simulation. *Plos one*, 18(5), e0284759.
- [12]. Small, M., & Cavanagh, D. (2020). Modelling strong control measures for epidemic propagation with networks—A COVID-19 case study. *IEEE access*, 8, 109719-109731.
- [13]. Samma, F. M. A., Avinash, N., Chellamani, P., Albasheir, N. A., Gargouri, A., Britto Antony Xavier, G., & Almazah, M. M. (2024). Exploring Symmetry in an Epidemiological Model: Numerical Analysis of Backward Bifurcation and Sensitivity Indices. *Symmetry*, 16(12), 1579.
- [14]. Ullah, I., Ali, N., Haq, I. U., Ahmad, I., Albalwi, M. D., & Ali Biswas, M. H. (2024). Analysis of COVID-19 Disease Model: Backward Bifurcation and Impact of Pharmaceutical and Nonpharmaceutical Interventions. *International Journal of Mathematics and Mathematical Sciences*, 2024(1), 6069996.
- [15]. Alqahtani, H., Badshah, Q., Rahman, G. U., Baleanu, D., & Sakhi, S. (2023). Threshold dynamics and bifurcation analysis of the epidemic model of MERS-CoV. *Fractals*, 31(10), 2340167.
- [16]. Zhang, J., & Qiao, Y. (2023). Bifurcation analysis of an SIR model considering hospital resources and vaccination. *Mathematics and Computers in Simulation*, 208, 157-185.
- [17]. Khan, A. Q., Tasneem, M., & Almatrafi, M. B. (2021). Discrete-time covid-19 epidemic model with bifurcation and control. *Math Biosci Eng*, 19, 1944-1969.
- [18]. Yaseen, R. M., Mohsen, A. A., Al-Husseiny, H. F., & Hattaf, K. (2023). Stability and Hopf bifurcation of an epidemiological model with effect of delay the awareness programs and vaccination: analysis and simulation. *Commun. Math. Biol. Neurosci.*, 2023, Article-ID.
- [19]. Zerefe, S. B., Ayele, T. K., & Tilahun, S. L. (2024). Mathematical Modeling of COVID-19 Disease Dynamics With Contact Tracing: A Case Study in Ethiopia. *Journal of Applied Mathematics*, 2024(1), 5556734.
- [20]. Wangari, I. M. (2024). Emergence of a reversed backward bifurcation, reversed hysteresis effect, and backward bifurcation phenomenon in a COVID-19 mathematical model. *Mathematical Methods in the Applied Sciences*, 47(4), 2250-2272.

- [21]. Ahmad, W., Rafiq, M., Butt, A. I. K., Ahmad, N., Ismaeel, T., Malik, S., ... & Asif, Z. (2024). Analytical and numerical explorations of optimal control techniques for the bi-modal dynamics of Covid-19. *Nonlinear Dynamics*, 112(5), 3977-4006.
- [22]. Zhang, J., Qiao, Y., & Zhang, Y. (2023). Bifurcation analysis and optimal control of COVID-19 with exogenous reinfection and media coverages. *International Journal of Biomathematics*, 16(03), 2250086.
- [23]. Nazarimehr, F., Pham, V. T., & Kapitaniak, T. (2020). Prediction of bifurcations by varying critical parameters of COVID-19. *Nonlinear dynamics*, 101, 1681-1692.
- [24]. Kumar, A., Gupta, A., Dubey, U. S., & Dubey, B. (2023). Stability and bifurcation analysis of an infectious disease model with different optimal control strategies. *Mathematics and Computers in Simulation*, 213, 78-114.
- [25]. Borah, M., Gayan, A., Sharma, J. S., Chen, Y., Wei, Z., & Pham, V. T. (2022). Is fractional-order chaos theory the new tool to model chaotic pandemics as Covid-19?. *Nonlinear dynamics*, 109(2), 1187-1215.
- [26]. Belili, M. C., Sahari, M. L., Kebiri, O., & Zeghdoudi, H. (2024). Analyzing bifurcations and optimal control strategies in SIRS epidemic models: insights from theory and COVID-19 data. *Mathematical and Computational Applications*, 29(4), 69.
- [27]. Sun, T. C., DarAssi, M. H., Alfwzan, W. F., Khan, M. A., Alqahtani, A. S., Alshahrani, S. S., & Muhammad, T. (2023). Mathematical modeling of COVID-19 with vaccination using fractional derivative: A case study. *Fractal and Fractional*, 7(3), 234.
- [28]. Ajbar, A., Alqahtani, R. T., & Boumaza, M. (2021). Dynamics of an SIR-based COVID-19 model with linear incidence rate, nonlinear removal rate, and public awareness. *Frontiers in Physics*, 9, 634251.
- [29]. Ramzan, S., Rashid, S., Shah, M. A., & Elagan, S. K. (2024). Exploring the dynamical bifurcation and stability analysis of Nipah virus; novel perspectives utilizing fractional calculus. *Modeling Earth Systems and Environment*, 10(4), 5427-5448.
- [30]. Verma, T., & Gupta, A. K. (2021). Network synchronization, stability and rhythmic processes in a diffusive mean-field coupled SEIR model. *Communications in Nonlinear Science and Numerical Simulation*, 102, 105927.