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Generalised Four Step Adams-Moulton Second Derivative Method for the Solution Stiff Ordinary Differential Equations

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Abstract:

A uniform order ten second derivative block method is presented for solving stiff ordinary differential equations. The proposed method was derived by using the approach of collocation and interpolation of four step Adams-Moulton method. Four individual schemes that made up the block method are obtained at step number, *k = 4*. The stability properties of the new method have been ascertained and it has shown to be consistent, zero–stable and A–stable. The solutions of two problems have been computed and compared with the corresponding exact and other existing solutions. Solutions are presented on graphs and the associated absolute errors are compared with some existing solution-errors in tables.

Keywords: Second Derivative, Adams-Moulton Method, Collocation and Interpolation, Block Methods, Stiff Ordinary Differential Equations, Stability Properties, Absolute Error.

1.0 Introduction

Differential equations are mathematical models of problems emanating from almost every field of study, where measurement can be taken. They are of different types and exhibit different phenomenon. Stiff ordinary differential equations are special class of problems of differential equation which are mostly found in engineering sciences and other areas of studies, [1]. So many researchers are involved in finding solution to the initial valued problems of stiff ordinary differential equations of the form:

$$y' = f(x,y), \;\; y(x_0) = y_0, \; x \in [a,b], y \in \mathbb{R} \qquad \textbf{(1.1)}$$

as can be seen in [1] and [2]. As it has been stated in [2], about the availability of the analytical solution of (1), there is a need for a suitable numerical method to solve (1.1).

Dauda *et. al.*, [3], presented a second derivative block hybrid method suitable for the continuous integration of stiff ordinary differential equations and was stated to compete favourably with some strong stability stiff integrators. In [4], Rufai *et. al.*, derive a new continuous hybrid block method using Chebyshev polynomials of the first kind as basis function and the new proposed method for the solution of initial value problems of systems of stiff ordinary differential equations. It can also be seen in Skwame *et. al.*, [5], Kumleng et. al., [6], Ajie *et. al.*, [7] and Kumleng *et. al.*, [8], how new methods are presented for the solution of problems of the form (1.1).

Interpolation and collocation approach has been adopted in many literatures for the construction of block method of solving differential equations. Kumleng *et. al.*, [8], adopted the approach and methods of uniform orders 10 and 11 were constructed. In a similar approach, [1] generated a block method of order 7. It can also be seen in [9], where a new block method of order 4 has been formed. Other methods in which this approach is adopted are [10], [11], [12]. Kayode *et at*., [14], presened a continuous two step trigonometrically-fitted second order method for the solution of linear and nonlinear initial value oscillatory problems. The coefficients of the developed approaches are determined by the approximate solution's frequency and step size, a discrete trigonometrically -fitted second order ordinary differential equation was recovered as a by-product. , The stability and other properties qualities are described to demonstrate the method's usefulness and efficiency. And real-world oscillatory problems of ordinary differential equations implemented to the suitability of the new method.

We present a second derivative block method for the solution (1.1). The new block method is derived from a four step Adams-Moulton method.

2.0 Derivation of the Method

Harnessing the concept of interpolation and collocation technique, we consider the following method for our new method:

$$\bar{y}(x) = \alpha_3(x)y_{n+3} + h\sum_{j=0}^{4}\beta_j(x)f_{n+j}(x)) + h^2\sum_{j=0}^{4}\gamma_j(x)g_{n+j} \qquad \mathbf{(2.1)}$$

where

(2.1) is the approximation of a continuously differentiable solution of (1.1)

$\alpha_3(x)$, $\beta_j(x)$ and $\gamma_j(x)$ are the continuous coefficients

h is the step size of the method with a step number, $k = 4$

y_{n+j} is the approximation to the theoretical solution of (1.1) at x_{n+j} and,

the first and second derivative of y_{n+j} are $f_{n+j} \equiv f(x_{n+j}y_{n+j})$ and $g_{n+j} \equiv g(x_{n+j}, y_{n+j}, f_{n+j})$, respectively.

Adopting the technique in [8] and other work, we obtain a matrix defined by D, as follows:

$$D = \begin{pmatrix} D_1 & D_3 \\ D_2 & D_4 \end{pmatrix}$$

where,

$$D_1 = \begin{pmatrix} 1 & x_n + 3h & (x_n + 3h)^2 & (x_n + 3h)^3 & (x_n + 3h)^4 & (x_n + 3h)^5 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\ 0 & 1 & 2(x_n + h) & 3(x_n + h)^2 & 4(x_n + h)^3 & 5(x_n + h)^4 \\ 0 & 1 & 2(x_n + 2h) & 3(x_n + 2h)^2 & 4(x_n + 2h)^3 & 5(x_n + 2h)^4 \\ 0 & 1 & 2(x_n + 3h) & 3(x_n + 3h)^2 & 4(x_n + 3h)^3 & 5(x_n + 3h)^4 \\ 0 & 1 & 2(x_n + 4h) & 3(x_n + 4h)^2 & 4(x_n + 4h)^3 & 5(x_n + 4h)^4 \end{pmatrix}$$

$$D_2 = \begin{pmatrix} 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 \\ 0 & 0 & 2 & 6(x_n + h) & 12(x_n + h)^2 & 20(x_n + h)^3 \\ 0 & 0 & 2 & 6(x_n + 2h) & 12(x_n + 2h)^2 & 20(x_n + 2h)^3 \\ 0 & 0 & 2 & 6(x_n + 3h) & 12(x_n + 3h)^2 & 20(x_n + 3h)^3 \\ 0 & 0 & 2 & 6(x_n + 4h) & 12(x_n + 4h)^2 & 20(x_n + 4h)^3 \end{pmatrix}$$

$$D_3 = \begin{pmatrix} (x_n+3h)^6 & (x_n+3h)^7 & (x_n+3h)^8 & (x_n+3h)^9 & (x_n+3h)^{10} \\ 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 & 10x_n^9 \\ 6(x_n+h)^5 & 7(x_n+h)^6 & 8(x_n+h)^7 & 9(x_n+h)^8 & 10(x_n+h)^9 \\ 6(x_n+2h)^5 & 7(x_n+2h)^6 & 8(x_n+2h)^7 & 9(x_n+2h)^8 & 10(x_n+2h)^9 \\ 6(x_n+3h)^5 & 7(x_n+3h)^6 & 8(x_n+3h)^7 & 9(x_n+3h)^8 & 10(x_n+3h)^9 \\ 6(x_n+4h)^5 & 7(x_n+4h)^6 & 8(x_n+4h)^7 & 9(x_n+4h)^8 & 10(x_n+4h)^9 \end{pmatrix}$$

$$D_4 = \begin{pmatrix} 30x_n^4 & 42x_n^5 & 56x_n^6 & 72x_n^7 & 90x_n^8 \\ 30(x_n+h)^4 & 42(x_n+h)^5 & 56(x_n+h)^6 & 72(x_n+h)^7 & 90(x_n+h)^8 \\ 30(x_n+2h)^4 & 42(x_n+2h)^5 & 56(x_n+2h)^6 & 72(x_n+2h)^7 & 90(x_n+2h)^8 \\ 30(x_n+3h)^4 & 42(x_n+3h)^5 & 56(x_n+3h)^6 & 72(x_n+3h)^7 & 90(x_n+3h)^8 \\ 30(x_n+4h)^4 & 42(x_n+4h)^5 & 56(x_n+4h)^6 & 72(x_n+4h)^7 & 90(x_n+4h)^8 \end{pmatrix}$$

From the inverse of the D matrix, and by putting $x - x_n = \mu$, the values of the continuous coefficients in (2.1) are obtained and presented as follows:

$$\left.\begin{array}{c} \alpha_3 = 1 \\ h\beta_0 = -\frac{1}{4354560h^9}(58509h^7 - 102771h^6\mu - 122274h^5\mu^2 + 457350h^4\mu^3 - 446265h^3\mu^4 + 198135h^2\mu^5 - 40810h\mu^6 + 3150\mu^7)(3h-\mu)^3 \\ h\beta_1 = \frac{1}{272160h^9}(1260\mu^7 - 14420\mu^6 h + 57015\mu^5h^2 - 88065\mu^4h^3 + 38910\mu^3h^4 + 5526\mu^2h^5 + 8289\mu h^6 + 8289h^7)(3h-\mu)^3 \\ h\beta_2 = -\frac{3}{560h^8}(216h^6 + 216h^5\mu + 144h^4\mu^2 - 480h^3\mu^3 + 810h^2\mu^4 - 315h\mu^5 + 35\mu^6)(h-\mu)^3 \\ h\beta_3 = -\frac{1}{272160h^9}(57591h^9 + 19197h^8\mu + 6399h^7\mu^2 - 320427h^6\mu^3 + 807111h^5\mu^4 - 850515h^4\mu^5 + 466335h^3\mu^6 - 138075h^2\mu^7 + 20860h\mu^8 - 1260\mu^9)(3h-\mu) \\ h\beta_4 = -\frac{1}{4354560h^9}(3699h^7 + 3699h^6\mu + 2466h^5\mu^2 - 57990h^4\mu^3 + 115065h^3\mu^4 - 87255h^2\mu^5 + 28490h\mu^6 - 3150\mu^7)(3h-\mu)^3 \\ h^2\gamma_0 = -\frac{1}{725760h^8}(1017h^7 + 1017h^6\mu - 12762h^5\mu^2 + 24270h^4\mu^3 - 20205h^3\mu^4 + 8379h^2\mu^5 - 1666hz^6 + 126z^7)(3h-\mu)^3 \\ h^2\gamma_1 = \frac{1}{90720h^8}(837h^7 + 837h^6\mu + 558h^5\mu^2 - 17610h^4\mu^3 + 24795h^3\mu^4 - 13293h^2\mu^5 + 3052h\mu^6 - 252\mu^7)(3h-\mu)^3 \\ h^2\gamma_2 = \frac{1}{320h^8}(3h-\mu)^3(3h^4 + 12h^3\mu + 29h^2\mu^2 - 16hz^3 + 2\mu^4)(h-\mu)^3 \\ h^2\gamma_3 = \frac{1}{90720h^8}(1647h^8 + 1098h^7\mu + 549h^6\mu^2 - 17676h^5\mu^3 + 39675h^4\mu^4 - 35874h^3\mu^5 + 15729h^2\mu^6 - 3248h\mu^7 + 252\mu^8)(3h-\mu)^2 \\ h^2\gamma_4 = \frac{1}{725760h^8}(135h^7 + 135h^6\mu + 90h^5\mu^2 - 2190h^4\mu^3 + 4365h^3\mu^4 - 3339h^2\mu^5 + 1106h\mu^6 - 126\mu^7)(3h-\mu)^3 \end{array}\right\}$$

(2.2)

To obtain the continuous form of the new method, we substitute the values of the continuous coefficients into (2.1). This is presented in the following form:

$$\begin{aligned} \bar{y}(x) = y_{n+3} &+ \left(-\frac{1}{4354560h^9}(58509h^7 - 102771h^6\mu - 122274h^5\mu^2 + 457350h^4\mu^3 - 446265h^3\mu^4 + 198135h^2\mu^5 - 40810h\mu^6 + 3150\mu^7)(3h-\mu)^3\right) f_n \\ &+ \left(\frac{1}{272160h^9}(1260\mu^7 - 14420\mu^6h + 57015\mu^5h^2 - 88065\mu^4h^3 + 38910\mu^3h^4 + 5526\mu^2h^5 + 8289\mu h^6 + 8289h^7)(3h-\mu)^3\right) f_{n+1} \\ &+ \left(-\frac{3}{560h^8}(216h^6 + 216h^5\mu + 144h^4\mu^2 - 480h^3\mu^3 + 810h^2\mu^4 - 315h\mu^5 + 35\mu^6)(h-\mu)^3\right) f_{n+2} \\ &+ \left(-\frac{1}{272160h^9}(57591h^9 + 19197h^8\mu + 6399h^7\mu^2 - 320427h^6\mu^3 + 807111h^5\mu^4 - 850515h^4\mu^5 + 466335h^3\mu^6 - 138075h^2\mu^7 + 20860h\mu^8 - 1260\mu^9)(3h-\mu)\right) f_{n+3} \\ &+ \left(-\frac{1}{4354560h^9}(3699h^7 + 3699h^6\mu + 2466h^5\mu^2 - 57990h^4\mu^3 + 115065h^3\mu^4 - 87255h^2\mu^5 + 28490h\mu^6 - 3150\mu^7)(3h-\mu)^3\right) f_{n+4} \\ &+ \left(-\frac{1}{725760h^8}(1017h^7 + 1017h^6\mu - 12762h^5\mu^2 + 24270h^4\mu^3 - 20205h^3\mu^4 + 8379h^2\mu^5 - 1666hz^6 + 126z^7)(3h-\mu)^3\right) g_n \\ &+ \left(\frac{1}{90720h^8}(837h^7 + 837h^6\mu + 558h^5\mu^2 - 17610h^4\mu^3 + 24795h^3\mu^4 - 13293h^2\mu^5 + 3052h\mu^6 - 252\mu^7)(3h-\mu)^3\right) g_{n+1} \\ &+ \left(\frac{1}{320h^8}(3h-\mu)^3(3h^4 + 12h^3\mu + 29h^2\mu^2 - 16hz^3 + 2\mu^4)(h-\mu)^3\right) g_{n+2} \\ &+ \left(\frac{1}{90720h^8}(1647h^8 + 1098h^7\mu + 549h^6\mu^2 - 17676h^5\mu^3 + 39675h^4\mu^4 - 35874h^3\mu^5 + 15729h^2\mu^6 - 3248h\mu^7 + 252\mu^8)(3h-\mu)^2\right) g_{n+3} \\ &+ \left(\frac{1}{725760h^8}(135h^7 + 135h^6\mu + 90h^5\mu^2 - 2190h^4\mu^3 + 4365h^3\mu^4 - 3339h^2\mu^5 + 1106h\mu^6 - 126\mu^7)(3h-\mu)^3\right) g_{n+4} \end{aligned}$$

(2.3)

The continuous scheme (2.3) is interpolated at some points for $\mu = 0, h, 2h$ and $4h$, thereby, yielding 4 discrete methods. These can be presented as:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+1}\\y_{n+2}\\y_{n+3}\\y_{n+4}\end{bmatrix}=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-3}\\y_{n-2}\\y_{n-1}\\y_n\end{bmatrix}$$

$$+h\left(\begin{bmatrix}\frac{89371}{272160}&\frac{103}{630}&\frac{38341}{272160}&\frac{59681}{4354560}\\\frac{6616}{8505}&\frac{208}{315}&\frac{1576}{8505}&\frac{1153}{68040}\\\frac{921}{1120}&\frac{81}{70}&\frac{711}{1120}&\frac{411}{17920}\\\frac{8192}{8505}&\frac{416}{315}&\frac{8192}{8505}&\frac{3202}{8505}\end{bmatrix}\begin{bmatrix}f_{n+1}\\f_{n+2}\\f_{n+3}\\f_{n+4}\end{bmatrix}+\begin{bmatrix}0&0&0&\frac{1539551}{4354560}\\0&0&0&\frac{24463}{68040}\\0&0&0&\frac{6501}{17920}\\0&0&0&\frac{3202}{8505}\end{bmatrix}\begin{bmatrix}f_{n+3}\\f_{n+2}\\f_{n+1}\\f_n\end{bmatrix}\right)$$

$$+h^2\left(\begin{bmatrix}-\frac{31207}{90720}&-\frac{81}{320}&-\frac{1243}{18144}&-\frac{2237}{725760}\\-\frac{152}{567}&-\frac{2}{5}&-\frac{248}{2835}&-\frac{43}{11340}\\-\frac{279}{1120}&-\frac{81}{320}&-\frac{183}{1120}&-\frac{9}{1792}\\-\frac{512}{2835}&0&\frac{512}{2835}&-\frac{116}{2835}\end{bmatrix}\begin{bmatrix}g_{n+1}\\g_{n+2}\\g_{n+3}\\g_{n+4}\end{bmatrix}+\begin{bmatrix}0&0&0&\frac{26051}{725760}\\0&0&0&\frac{421}{11340}\\0&0&0&\frac{339}{8960}\\0&0&0&\frac{116}{2835}\end{bmatrix}\begin{bmatrix}g_{n-3}\\g_{n-2}\\g_{n-1}\\g_n\end{bmatrix}\right) \quad (2.4)$$

3.0 Analysis of the Block Methods

A summary on the order, error constant and the convergence of the proposed block method are given using the technique presented in [1]. The block method (2.4) is represented by a matrix finite difference equation in block form as:

$$A^{(0)}Y_m = A^{(1)}y_m + h\{B^{(0)}F_m + B^{(1)}f_m\} + h^2\{C^{(1)}g_m + C^{(0)}G_m\} \qquad \textbf{(3.1)}$$

where,

$Y_m = [y_{n+1} \quad y_{n+2} \quad y_{n+3} \quad y_{n+4}]^T$, $y_m = [y_{n-3} \quad y_{n-2} \quad y_{n-1} \quad y_n]^T$, $F_m = [f_{n+1} \quad f_{n+2} \quad f_{n+3} \quad f_{n+4}]^T$, $f_m = [f_{n-3} \quad f_{n-2} \quad f_{n-1} \quad f_n]^T$, $g_m = [g_{n-3} \quad g_{n-2} \quad g_{n-1} \quad g_n]^T$ and $G_m = [g_{n+1} \quad g_{n+2} \quad g_{n+3} \quad g_{n+4}]^T$

and the matrices $A^{(1)}, A^{(0)}, B^{(1)}, B^{(0)}, C^{(0)}$and $C^{(1)}$ are 4 by 4 matrices.

3.1 Zero-stability

As in (12), the roots of the first characteristics polynomial, $\rho(\lambda)$, defined by:

$$\rho(\lambda) = |\lambda A^{(0)} - A^{(1)}|$$

satisfies $|r| \leq 1$, are, $|r| = 0, 0, 0,$ or 1. Therefore, the new block method is zero-stable.

3.2 Order and Error Constants

The new block method is of uniform order $p = [10 \quad 10 \quad 10 \quad 10]^T$, with an error constant of $C_{p+1} = \left[\frac{551}{314344800} \quad \frac{4}{1964655} \quad \frac{1}{431200} \quad \frac{8}{1964655}\right]^T$

3.3 Consistency

By definition, the order is $p \geq 1$ implies that the new block method is consistence, see [1].

3.4 Absolute stability region

The absolute stability region of the new block method is obtained using [13]. The method, (2.4) is reformulated as a general linear method and is plotted using the Matlab program as revealed in the figure 1 to be A-stable.

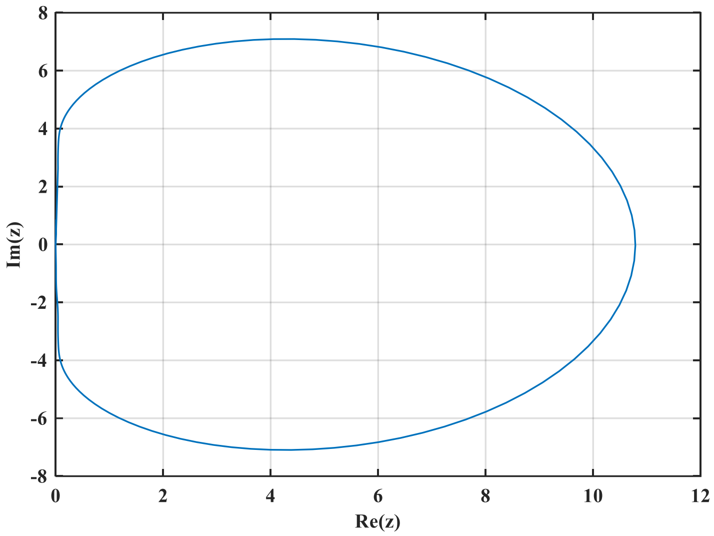

Figure 1: Region of Absolute stability of the new block method

3.5 Convergence

According to [1], the new block method is convergent, since it’s consistent and zero-stable.

4.0 Numerical experiments

To demonstrate the performance of the new block method, we consider two linear stiff ordinary differential equations of the form (1.1). We present the calculated absolute errors in tables and the solution curves are shown in figures.

Example 1: This problem is taken from [1]:

on the range, $0 \leq x \leq 1$ and a step size, $h = 0.1$:

$$y' = -1000y + 999e^{-x}, \qquad y(0) = 1$$

With exact solution: $y(x) = e^{-x}$.

Example 2: This problem is taken from [2]:

$$y'(x) = \begin{bmatrix} 21 & 19 & -21 \\ 19 & -21 & 20 \\ 40 & -40 & 40 \end{bmatrix} \begin{bmatrix} y_1(x) \\ y_2(x) \\ y_3(x) \end{bmatrix}, \qquad y(0) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

the exact solutions for $0 \leq x \leq 10$, are:

$$y(x) = \begin{bmatrix} e^{-x} \\ -e^{-x} \end{bmatrix}$$

Result:

Table 1: Errors in New block method and in [1] with Exact solution: Example 1

x	Error in [1]	Error in New block method
0	0	0
0.100000000000000	1.88737914186277e-15	0
0.200000000000000	2.10942374678780e-15	0
0.300000000000000	8.88178419700125e-16	1.11022302462516e-16

0.400000000000000	1.55431223447522e-15	1.11022302462516e-16
0.500000000000000	9.99200722162641e-16	1.11022302462516e-16
0.600000000000000	1.22124532708767e-15	0
0.700000000000000	1.49880108324396e-15	0
0.800000000000000	9.99200722162641e-16	0
0.900000000000000	4.99600361081320e-16	0
1	8.32667268468867e-16	1.11022302462516e-16

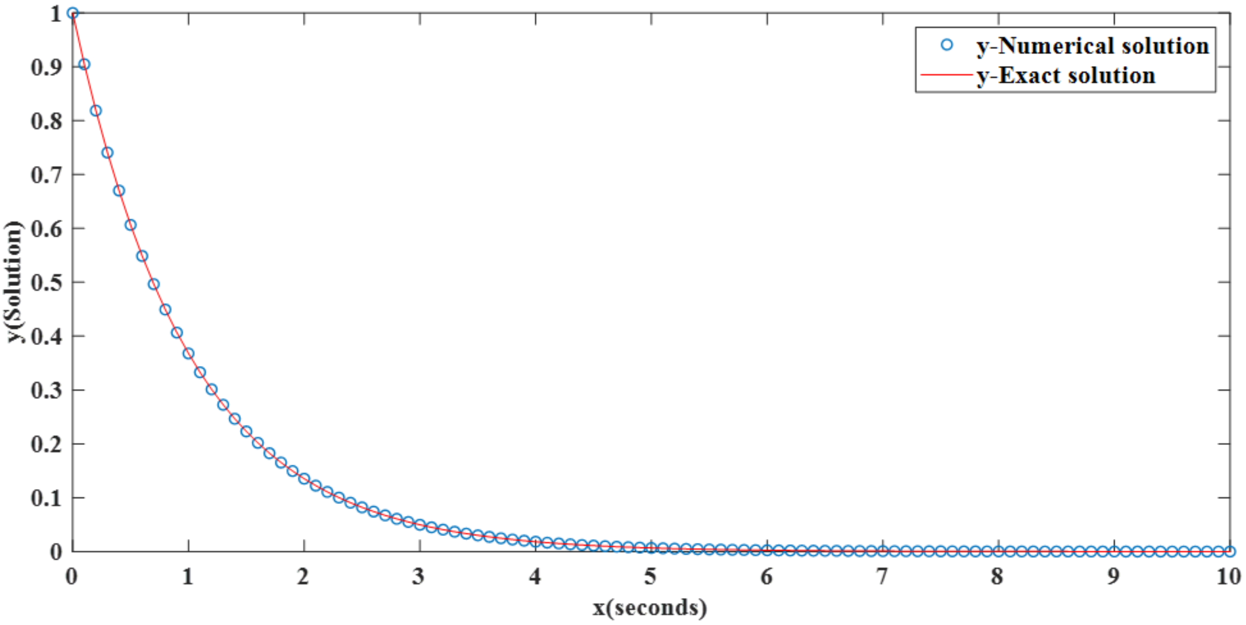

Figure 2: Showing the New block method with Exact solutions: Example 1

Table 2: Comparison of error in the new block method with the exact solution in Example 2

x	**Error of y1**	**Error of y2**	**Error of y3**
0	0	0	0
0.1000	0	0.0260987027814653	0.00206310803809157
0.2000	0.0122373815672981	0.000719747107311319	0.00107577664370589
0.3000	0.000387854389990783	3.24631645365336e-06	5.40015326179073e-05
0.4000	6.54313444264920e-06	9.83441426322118e-07	9.66591002920572e-07
0.5000	9.51042133906510e-07	3.83220316313437e-08	3.02159582896780e-08

0.6000	3.644035531658O4e-08	4.77281492106130e-10	2.72539843546488e-09
0.7000	4.43050762743980e-10	2.67659783226293e-11	8.80135840652426e-11
0.8000	2.69109318162819e-11	1.81354931072519e-12	5.81808897278642e-13
0.9000	1.84649517898094e-12	6.90142387682613e-14	8.58072112004154e-14
1	3.18495230189342e-14	1.68198788230711e-14	4.72864743371380e-15

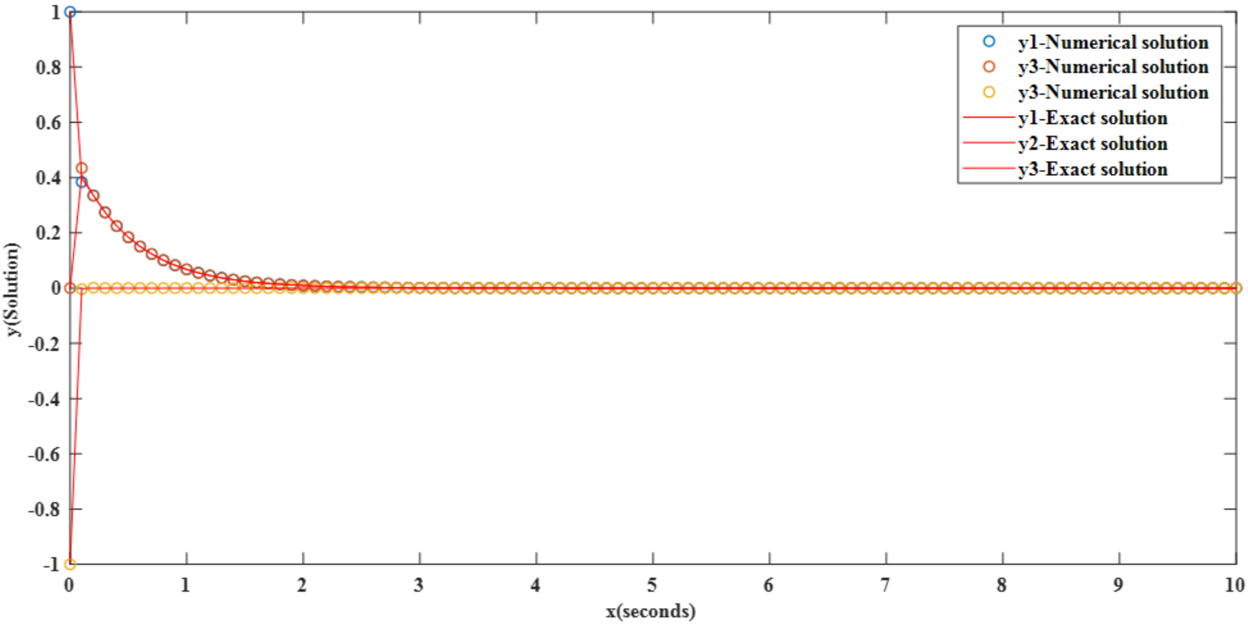

Figure 3: Showing the New block method with Exact solutions: Example 2

5. DISCUSSION OF RESULTS

The confirmation of the region of absolute stability is presented in figure 1. The region containing the entire left-half plane, reveals that the new block method is A-stable, thereby ensuring unconditional stability for all step sizes. The region of absolute stability is larger than that of the fourth-order Runge-Kutta (RK4) method, making it more stable for a wider range of problems.

Table 1 shows the comparisons between the error in the proposed methods with the presented in [1]. With the same step number and step size, the new method has slightly more error stability compared to the error in [1]. The numerical and exact solutions for example 1 are presented in figure 2, with the numerical solution competing favorably with the exact solution.

The representation of the absolute errors in the new method with the exact solution, in table 2, shows a healthy competition with the exact solution. Similar favoritism is observed in figure 3 of this work. Furthermore, convergence of the new method ensures a reduction in inherent error, in comparison with other existing methods.

6. CONCLUSION

The generalized four step Adam Moulton Method has been presented in this research. Following the technique adopted in [1], four discrete methods were constructed. Stability analysis demonstrates that the method satisfies essential properties such as consistency, zero-stability; and the stability region is plotted using MATLAB, visually confirming A-stability. Furthermore, numerical results obtained in examples 1and 2 confirm the method's ability to handle stiff ODEs efficiently. Tables and graphs are presented to provide a clear comparison between the new method and existing approaches. The paper does not analyze the computational cost of the proposed method. Including execution time, function evaluations, and memory usage.

7. DECLARATIONS

Availability of data and material: All data generated or analyzed during this study are included in this published article and its supplementary references.

Conflicts of interest/Competing interests: All authors declare that they have no conflicts of interest.

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Authors' contributions: We understand that the Corresponding Author is the sole contact for the Editorial process (including Editorial Manager and direct communications with the office). He is responsible for communicating with the other authors about progress, submissions of revisions and final approval of proofs. We confirm that we have provided a current, correct email address which is accessible by the Corresponding Author. All authors read and approved the final manuscript.

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Code availability: 'Not applicable.

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