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Dynamical Aspects of Bianchi Type VI0 Cosmological Model with Cosmic String and Dark Energy in $f(T)$ Gravity

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Abstract:

This paper deals with the investigations of a spatially homogeneous and anisotropic Bianchi-type VI_0 dark energy coupled string cosmological model within the context of $f(T)$ gravity, where T denotes the torsion scalar. We have obtained the exact solutions of highly non-linear field equations considering some physically plausible conditions. Some physical and kinematical properties of the constructed model have been discussed and presented graphically. Additionally, we have calculated a jerk parameter that fits the ΛCDM model and discussed the physical stability and acceptability of constructed models.

Keywords: Bianchi type VI_0 space-time, Cosmic String, Dark Energy, *f(T)* Gravity.

1 Introduction

To explain the present accelerated expansion of the universe, modified theories of gravity are playing a vital role which is an attempt to modify Einstein's theory of gravity whose field equations are derived from the Einstein-Hilbert action principle instead of Einstein's field equations. Amongst them, $f(T)$ theory of gravitation is a viable candidate which is based on the modification of the TEGR i.e teleparallel equivalent of general relativity. In recent years $f(T)$ gravity theory has received immense interest from researchers. Initially, $f(T)$ gravity theory was proposed to study inflation [1,2], then cosmic acceleration [3-6], its perturbations [7-9], and reconstructions [10-14]. Toward the scientific curiosity of the universe, many cosmologists have taken significant efforts in $f(T)$ gravity by constructing different cosmological models [15-23].

We are well known for the fact that measurements from high redshift supernovae observed by researchers provide direct evidence for an accelerating universe [24-31]. The cause of this cosmic acceleration is unknown and is commonly referred to as the dark energy (DE) problem which is due to the large negative pressure of the universe. Observational data indicate that our universe is filled with 70% of DE and 30% of non-relativistic matter both the baryons and dark matter. In this regards many researchers have constructed various DE models such as quintessence [32-34], quintom [35,36], phantom [37], k-essence [38,39], and tachyon [40] as well as holographic DE [41-44], Ricci DE [45], new age graphic DE [46,47], Chaplygin gas [48], extended Chaplygin gas [49,50] and the generalized Chaplygin gas [51,52], etc to fulfill their curiosity in studying cosmology.

The cosmic strings build appreciable interest as these are believed to play an important role during the early evolution and late time accelerating expansion of the universe [53-57], one cannot predict the count of visible cosmic strings in the universe but their existence may play an important role [58,59]. The cosmic strings hold stress energy and coupled gravitational field, which increases one's interest to study the gravitational effect which originates from strings. Letelier [60,61], initiated the general relativistic operation of strings and investigated the solution of Einstein's field equations for a cloud of strings with spherical, plane, and cylindrical symmetry along with the cloud string models, their couplings, and Kantowski-Sachs-type cosmological model. Stachel [62], developed the model for the thickening of string perfect dust, its generalization to null strings, and to perfect fluid of strings. The distinct features of the string scenario, galaxy formations, baryon-dominated universe, neutrino-dominated universe, the axion-dominated universe, and some observational effects of strings were studied by Vilenkin [63]. Pawar et al. [64,65] studied string coupled cosmological models using the Bianchi type V and VI_0 space-times in modified gravity. As well, in recent years there have been a lot of investigations of DE and cosmic string on the anisotropic features of the universe [66-72].

Motivated by the cited literature and studies we have considered a spatially homogeneous and anisotropic Bianchi Type VI_0 metric within the context of $f(T)$ gravity. In our work, we have calculated some physical and kinematical parameters and discussed their physical behavior with existing cosmological models. Lastly, the jerk parameter and stability factor for the DE model is discussed.

This paper is divided into several sections: In Sec. 2 we provide a brief discussion of $f(T)$ gravity, metric, and field equations. In Sec. 3 we find the solutions of the field equation with some physical and kinematical parameters of the considered space-time along with the jerk parameter and discussed the physical stability of the derived model. And lastly, in Sec. 4 we concluded the investigations.

2 Basics of $f(T)$ Gravity, Metric, and Field Equations

We consider a spatially homogeneous and anisotropic Bianchi type-VI_0 line element, given by

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2e^{2x}dz^2, \tag{1}$$

where the scale factors $A,\ B,$ and C are functions of cosmic time t only.

In teleparallel gravity the tetrad is given by $g_{\mu\nu} = \eta_{ij}e^i_\mu e^j_\nu$, where μ and ν are coordinate indices. For a given metric $\sqrt{-g} = \det[e^i_\mu] = e$ and the dynamic fields of the theory are represented by the infinite tetrad fields e^α_μ which satisfy $e^\mu_i e^i_\nu = \delta^\mu_\nu$ or $e^\mu_i e^j_\mu = \delta^j_i$. Now the set of diagonal tetrads related to the metric (1) is given as

$$[e^\nu_\mu] = diag\ [1, A, Be^{-x}, Ce^{x}]. \tag{2}$$

The determinant of the matrix (1) is

$$e = ABC. \tag{3}$$

Now we consider the action of generalizing the teleparallel gravity, i.e. $f(T)$ theory as

$$S=\int[f(T)+L_{Matter}]ed^4x, \qquad (4)$$

where $f(T)$ denotes an algebraic function of the torsion scalar T and the torsion scalar is defined using the contraction which is similar to the scalar curvature in general relativity as

$$T=T^{\alpha}{}_{\mu\nu}S_{\alpha}{}^{\mu\nu}. \qquad (5)$$

where the tensors $T^{\alpha}{}_{\mu\nu}$, and $S_{\alpha}{}^{\mu\nu}$ are respectively as follows,

$$T^{\alpha}{}_{\mu\nu}=\Gamma^{\alpha}{}_{\mu\nu}-\Gamma^{\alpha}{}_{\nu\mu}=e_i^{\alpha}(\partial_{\mu}e_{\nu}^{i}-\partial_{\nu}e_{\mu}^{i}), \qquad (6)$$

$$S_{\alpha}{}^{\mu\nu}=\frac{1}{2}\left(K^{\mu\nu}{}_{\alpha}+\delta_{\alpha}^{\mu}T^{\beta\nu}{}_{\beta}-\delta_{\alpha}^{\nu}T^{\beta\mu}{}_{\beta}\right), \qquad (7)$$

in which the Weitzenbocks connection components for a manifold, where the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) having only the existence of non-zero torsion terms, and con-torsion tensor components are respectively defined as

$$\Gamma^{\alpha}{}_{\mu\nu}=e_i^{\alpha}\partial_{\nu}e_{\mu}^{i}=-e_{\mu}^{i}\partial_{\nu}e_i^{\alpha}, \qquad (8)$$

$$K^{\mu\nu}_{\alpha}=-\frac{1}{2}\left(T^{\mu\nu}{}_{\alpha}-T^{\nu\mu}{}_{\alpha}-T_{\alpha}{}^{\mu\nu}\right). \qquad (9)$$

So from (5), we have obtained the torsion scalar as

$$T=-2\left\{\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-1\right\}. \qquad (10)$$

Now the equations of motion are obtained by functional variation of the action (4) with respect to the tetrads as

$$S_{\mu}{}^{\nu\rho}\left(\partial_{\rho}T\right)f_{TT}+\left\{e^{-1}e_{\mu}^{i}\partial_{\rho}\left(ee_i^{\alpha}S_{\alpha}{}^{\nu\rho}\right)+T^{\alpha}{}_{\lambda\mu}S_{\alpha}{}^{\nu\lambda}\right\}f_T+\frac{1}{4}\delta_{\mu}^{\nu}f=4\pi\left\{T_{\mu}^{\nu(CS)}+\overline{T}_{\mu}^{\nu(DE)}\right\}$$

(11)

where $T_{\mu}^{\nu(CS)}$ and $\overline{T}_{\mu}^{\nu(DE)}$ are the energy-momentum tensors for cosmic string (CS) and dark energy (DE) respectively, f_T and f_{TT} denotes the respective first and second-order derivatives of $f(T)$ with respect to T and by setting $f(T)=a_0=$ constant the equations of motion (11) are the same as that of the teleparallel gravity with a cosmological constant, and this is dynamically equivalent to the general relativity. These equations depend on the choice made for the set of tetrads.

We have considered the string of clouds fluid as matter content. So the energy-momentum tensor for a string of clouds is defined as

$$T_{\mu}^{\nu(CS)} = (p+\rho)u^{\nu}u_{\mu} - pg_{\mu}^{\nu} + \lambda x^{\nu}x_{\mu}, \tag{12}$$

where u^{ν} are the four velocities of the cloud string, x^{ν} are the normal space-like four-vector, p is the pressure, ρ and λ are the proper energy density and tension density of the cloud string respectively. An orthonormalization of u^{ν} and x^{ν} is given by

$$u^{\nu}u_{\nu} = 1, \quad u^{\nu}x_{\nu} = 0, \quad x^{\nu}x_{\nu} = -1, \tag{13}$$

where, $u^{\nu} = (0,0,0,1)$ and $x^{\nu} = (A^{-1},0,0,0)$.

The energy-momentum tensor for dark energy as described by Vinutha [73] is given by

$$\begin{aligned} \bar{T}_{\mu}^{\nu(DE)} &= diag\ [\rho_{\Lambda}, -p_{\Lambda}, -p_{\Lambda}, -p_{\Lambda}] \\ &= diag\ [1, -\omega_{\Lambda}, -\omega_{\Lambda}, -\omega_{\Lambda}]\ \rho_{\Lambda}, \end{aligned} \tag{14}$$

where ω_{Λ} is the equation of state (EoS) parameter and ρ_{Λ} is the energy density for dark energy.

We obtained the field equations for Bianchi type- VI_0 space-time (1), from (11),(12), and (14) in the framework of $f(T)$ gravity as

$$f + 2f_T\left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - 2\right) + 2\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{T}\ f_{TT} = -16\pi\left(p + \lambda + \omega_{\Lambda}\rho_{\Lambda}\right), \tag{15}$$

$$f + 2f_T\left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{A}\dot{C}}{AC}\right) + 2\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)\dot{T}\ f_{TT} = -16\pi\left(p + \omega_{\Lambda}\rho_{\Lambda}\right), \tag{16}$$

$$f + 2f_T\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{T}\ f_{TT} = -16\pi\left(p + \omega_{\Lambda}\rho_{\Lambda}\right), \tag{17}$$

$$f + 4f_T\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) = 16\pi\left(\rho + \rho_{\Lambda}\right), \tag{18}$$

$$\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) f_T = 0, \tag{19}$$

where the overhead dot $(\dot{\square})$ denotes derivatives with respect to cosmic time t. Here we have five highly non-linear differential field equations with nine unknowns, namely; f, A, B, C, p, ρ, λ, ρ_{Λ}, and ω_{Λ}. Since $f(T)$ is non-vanishing in TEGR i.e. teleparallel equivalent of general relativity, from (19) one can get $B = k\,C$ which yields $B = C$ for $k = 1$. Then we obtain

$$f+4f_T\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-1\right)+4\left(\frac{\dot{B}}{B}\right)\dot{T}\,f_{TT}=-16\pi\left(p+\lambda+\omega_\Lambda\rho_\Lambda\right), \tag{20}$$

$$f+2f_T\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+3\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}\,f_{TT}=-16\pi\left(p+\omega_\Lambda\rho_\Lambda\right), \tag{21}$$

$$f+4f_T\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)=16\pi\left(\rho+\rho_\Lambda\right). \tag{22}$$

Finally, we obtained only three non-linear differential equations with eight unknowns, namely; f, A, B, p, ρ, λ, ρ_Λ, and ω_Λ. The solution to these equations is discussed in the next section.

3 Solutions of Field Equations

To obtain the exact solution of highly non-linear differential equations (20) – (22) we have considered the following physically plausible conditions.

(i) We consider the linear form of $f(T)$ model as

$$f(T)=T. \tag{23}$$

(ii) We assume the analytic relation between the metric coefficients as

$$A=B^n. \tag{24}$$

(iii) We consider the value of the average scale factor as

$$a=te^t. \tag{25}$$

(iv) Lastly, we consider that the energy-momentum tensors, i.e., the CS and DE, interact minimally and are conserved separately. Therefore, the energy conservation equation of the CS and DE respectively leads to

$$\dot{\rho}+3H\rho=0, \tag{26}$$

$$\dot{\rho}_\Lambda+3H\left(\omega_\Lambda+1\right)\rho_\Lambda=0. \tag{27}$$

We find some kinematical space-time quantities, as follows:

The average scale factor (a) and the volume (V) respectively as

$$V=ABC=a^3, \qquad a=\sqrt[3]{B^{n+2}}. \tag{28}$$

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter (H) given by

$$H=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{1}{3}\left(H_1+H_2+H_3\right), \tag{29}$$

where $H_1, H_2,$ and H_3 denotes the directional Hubble's parameters.

From Eqns. (28) and (29), we get

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{\dot{a}}{a}. \tag{30}$$

To analyze, whether the model approaches isotropy or not, we discuss the mean anisotropy parameter (A_m) as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H}-1\right)^2. \tag{31}$$

The expansion scalar (θ) and the shear scalar $\left(\sigma^2\right)$ are respectively defined as

$$\theta=u^{\mu}_{;\mu}=3H, \tag{32}$$

$$\sigma^2=\frac{3}{2}A_mH^2. \tag{33}$$

The deceleration parameter is defined as

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{34}$$

From (25) and (28) we have obtained the metric coefficients $A, B,$ and C as

$$A=\left(te^t\right)^{\frac{3n}{(n+2)}},\quad B=C=\left(te^t\right)^{\frac{3}{(n+2)}}. \tag{35}$$

From (35) into (1), we obtain

$$ds^2=dt^2-\left(te^t\right)^{\frac{6n}{(n+2)}}dx^2-\left(te^t\right)^{\frac{6}{(n+2)}}e^{-2x}dy^2-\left(te^t\right)^{\frac{6}{(n+2)}}e^{2x}dz^2. \tag{36}$$

From (10) we have obtained the torsion scalar as

$$T=\frac{2\left(n+2\right)^2t^2-18\left(2n+1\right)\left(t+1\right)^2}{\left(n+2\right)^2t^2}. \tag{37}$$

3.1 Some Kinematical and Physical Parameters

We have determined the volume (V), the mean Hubble's parameter (H), the expansion scalar (θ), the mean anisotropy parameter (A_m), the shear scalar $\left(\sigma^2\right)$, and the deceleration parameter (q) respectively as

$$V=\left(te^{t}\right)^{3}, \tag{38}$$

$$H=\frac{t+1}{t}, \tag{39}$$

$$\theta=\frac{3(t+1)}{t}, \tag{40}$$

$$A_m=\frac{2(n-1)^2}{(n+2)^2}, \tag{41}$$

$$\sigma^2=\frac{3(n-1)^2(t+1)^2}{(n+2)^2 t^2}, \tag{42}$$

$$q=-1+\frac{1}{(t+1)^2}. \tag{43}$$

As depicted in Fig.1 at an initial epoch when $t=0$ the volume (V) of the universe is zero and starts increasing exponentially with an increase in cosmic time and diverges when $t\to\infty$, while the mean Hubble's parameter (H), the expansion scalar (θ), and the shear scalar (σ^2) having large value initially but decreases as cosmic time increases and approach to a constant value. This means that the expansion of the universe is significantly faster at the beginning of the universe and subsequently becomes slower through time. The mean anisotropy parameter (A_m) is constant throughout the evolutions and the deceleration parameter (q) which is a decreasing function of cosmic time is observed to be in range $-1\le q\le 0$ showing the accelerating phase of the universe. Also, the ratio $\sigma^2/\theta^2\neq 0$ shows that the constructed model doesn't approach isotropy.

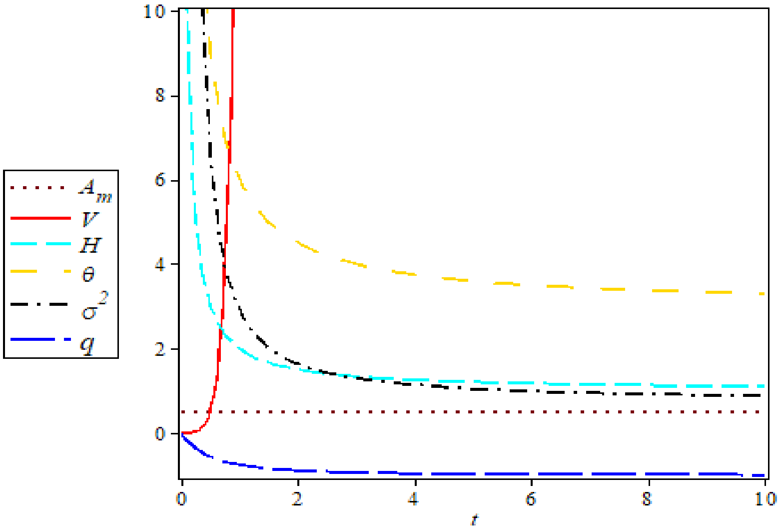

Figure 1: Variation of V, H, θ, A_m, σ^2, and q Vs. t for $n=0.01$

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Now from (39) into (26) and solving, we can obtain the energy density as

$$\rho = \frac{1}{(te^t)^3}. \quad (44)$$

Fig.2 represents the energy density of CS versus cosmic time, in which the energy density is the inverse function of time; initially, it is having infinitely large value when time $t = 0$ and vanishes at later times.

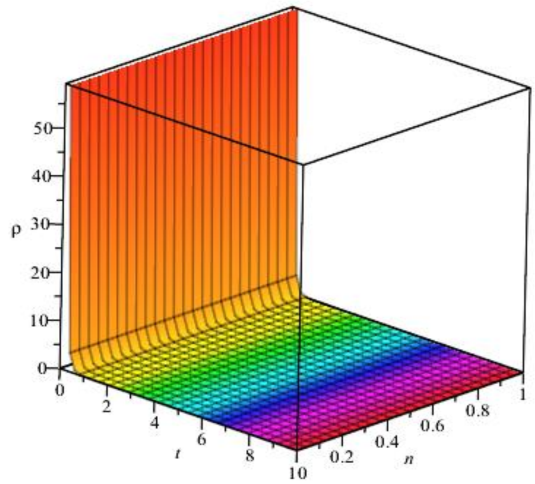

Figure 2: Variation of ρ Vs. t

From (23), (35), (20), and (21) we have obtained the value of tension density as

$$\lambda = -\frac{(11n-5)t^2 + 6(n-1)(3t+1)}{8\pi(n+2)t^2}. \quad (45)$$

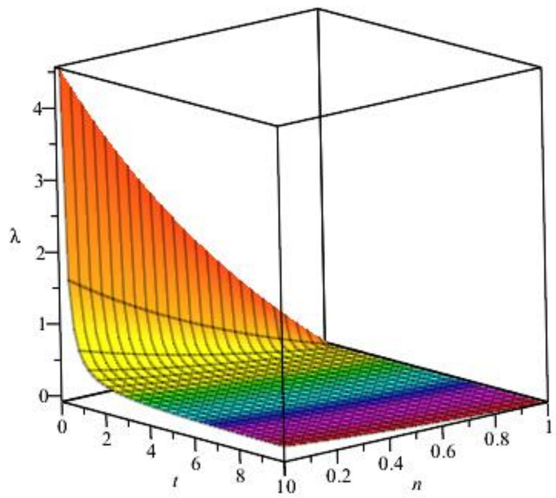

Figure 3: Variation of λ Vs. t

As shown in Fig.3 the tension density of CS initially seems to be large for small n and decreases from positive to approach a constant value with increasing cosmic time but as n increases, the initial tension density decreases and approaches a small constant value.

From (23), (35) and (44) into (22), we have obtained the value of DE density as

$$\rho_\Lambda = \frac{\left\{\left(n^2+22n+13\right)t^2+18\left(2n+1\right)t+9\left(2n+1\right)\right\}te^{3t}-8\pi\left(n+2\right)^2}{8\pi\left(n+2\right)^2t^3e^{3t}}. \tag{46}$$

The behavior of the DE density versus cosmic time is depicted in Fig.4, in which it is observed that the DE density increases intensely from negative and approaches a constant value when $t\to\infty$.

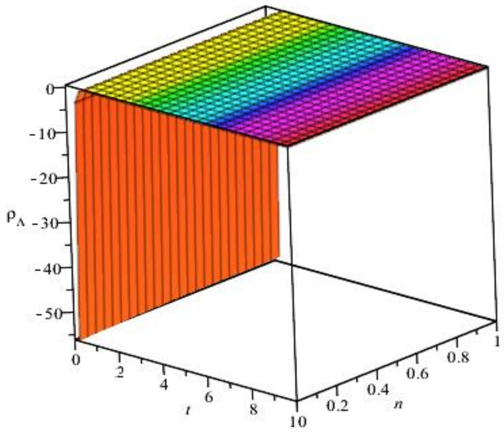

Figure 4: Variation of ρ_Λ Vs. t

From (39) and (46) into (27), we have obtained the EoS parameter as

$$\omega_\Lambda = -\frac{\left\{\left(n^2+22n+13\right)t^2+18\left(2n+1\right)t+3\left(2n+1\right)\right\}te^{3t}}{\left\{\left(n^2+22n+13\right)t^2+18\left(2n+1\right)t+9\left(2n+1\right)\right\}te^{3t}-8\pi\left(n+2\right)^2}. \tag{47}$$

The graphical representation of the EoS parameter versus cosmic time is depicted in Fig.5, for the cosmic accelerated phase. For this model, the EoS parameter is observed to increase in positive slowly for a short moment and then immediately decrease showing a transition for the period $0<t<1$ and then increasing in negative to approach -1 when $t\to\infty$, which is the same as the cosmological constant. As reported in the figure the variation n reflects the depth of the transition phase. As the EoS is time-dependent, the DE influences Cosmic Microwave Background (CMB) initially. The type Ia supernovae data submits EoS parameter in range $-1.67<\omega_\Lambda<-0.62$ [24] while the expected limit for EoS parameter accompanying the type Ia supernovae data with CMB anisotropy and galactic clustering census is $-1.33<\omega_\Lambda<-0.79$ [74]. It can be observed from the figure that at early times the EoS is positive, i.e. the matter-dominated universe, and later times EoS gives a negative epoch which is in good agreement with type Ia supernovae and CMB observational data.

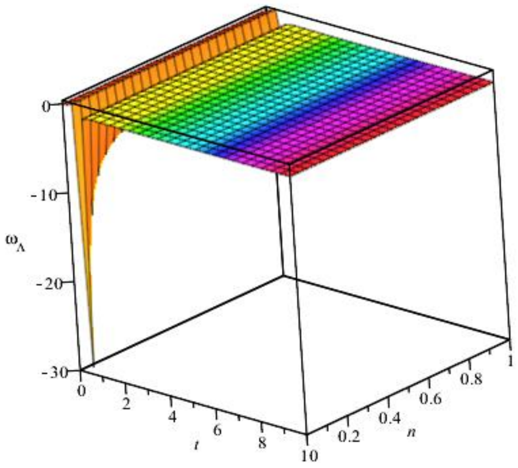

Figure 5: Variation of ω_Λ **Vs.** t

From (23), (35), (46), (47), into (21) the pressure can be obtained as

$$p = -\frac{3n(n-1)\left\{3t^2 + 6t + 2\right\}}{8\pi(n+2)^2 t^2}. \tag{48}$$

The variation of pressure with respect to cosmic time is depicted in Fig.6, which demonstrates that for this model the pressure is always positive throughout the cosmic evolution and approach to constant when $t \to \infty$.

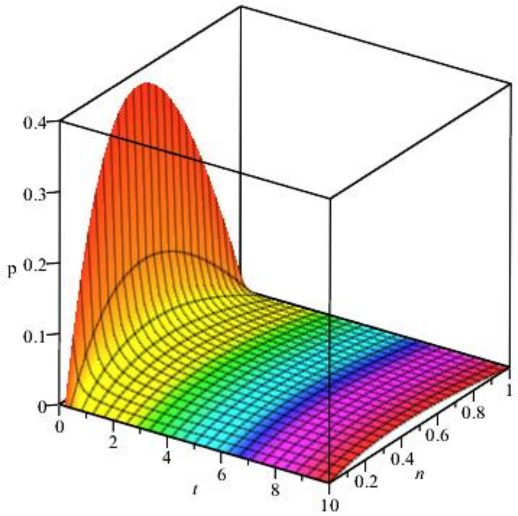

Figure 6: Variation of p **Vs.** t

3.2 Jerk Parameter

In early times, cosmic expansion was slow due to the low density of DE, which overcomes the gravity of the matter in the universe. With the cosmic expansion, the era of DE dominance began and the density of matter decreased. Many researchers have explained the realignment from decelerating to an accelerated phase of the universe due to the cosmic jerk that eventually occurs for distinct models of the universe with a positive value of the jerk parameter and a negative value of the deceleration parameter.

The jerk parameter is defined and obtained as

$$j(t)=\frac{\dddot{a}}{aH^3}=\frac{(t+3)t^2}{(t+1)^3}. \tag{49}$$

The variation of the jerk parameter versus cosmic time is depicted in Fig.7. In our model, it is observed that the jerk parameter is an increasing function of cosmic time and approaches a constant value $j=1$ when $t\to\infty$, it is in good agreement with the ΛCDM model and the recent observational data [75-78].

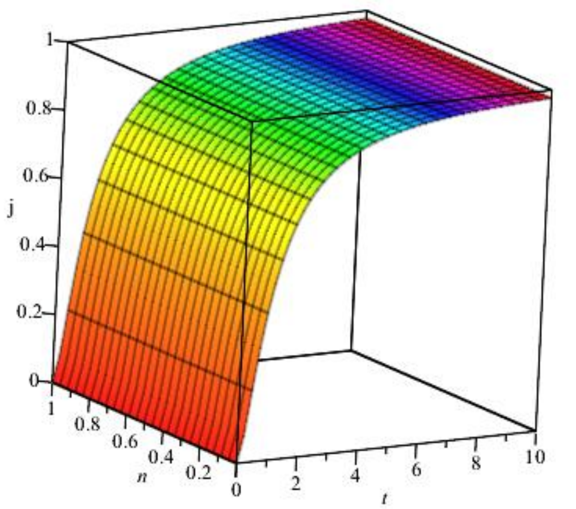

Figure 7: Variation of j Vs. t

3.3 Stability Factor

We now contemplate the important quantity advised in cosmology to investigate the physical acceptability and stability analysis of any DE model known as the squared speed of sound marked by, ϑ_s^2. The derived model with $\vartheta_s^2>0$ is stable while with $\vartheta_s^2<0$ are unstable. As we recognize that the sound speed (ϑ_s^2) is less than light speed (c), for a gravitational unit we hold $c=1$. Therefore, the model which lies in between 0 to 1 is physically viable.

We can define and obtain the squared sound speed as

$$\vartheta_s^2=\frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda}=-\frac{(2n+1)\{3t+1\}te^{3t}}{(t+1)\left\{3(2n+1)te^{3t}-4\pi(n+2)^2\right\}}. \tag{50}$$

It is seen from Fig.8 that for $0<t<1$ sound speed $\vartheta_s^2 > 0$ and with the increasing cosmic time $\vartheta_s^2 < 0$. Hence the stability and physical acceptability of the universe is viable for a short amount of cosmic time but later on, the model becomes unstable which is in good agreement with [79].

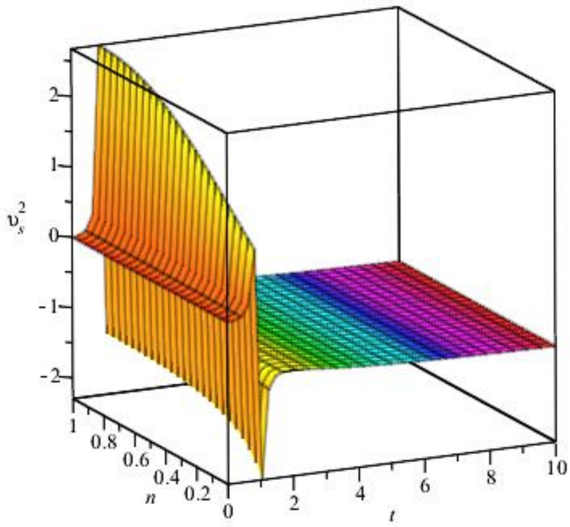

Figure 8: Variation of ϑ_s^2 Vs. t

4 Conclusions

We have investigated a spatially homogeneous and anisotropic Bianchi Type – VI_0 dark energy cosmological model coupled with cosmic string in $f(T)$ gravity, where T denotes the torsion scalar. We have discussed some physical and kinematical properties of the universe. This model is an accelerating model of the universe, the expansion of this model is significantly faster in the beginning and subsequently becomes slower through time. The investigations also show that the model is purely anisotropic throughout the expansion of the universe. The EoS parameter is initially positive, i.e. gives the matter-dominated universe for a short amount of time but later on becomes negative which is in good agreement with type Ia supernovae and the CMB observational data [24,72]. We have also calculated the jerk parameter and observed that the jerk parameter approaches a constant value, i.e. $j=1$ when $t \to \infty$, which is in good agreement with the ΛCDM model and the recent observational data [73-76]. The investigated stability parameter confirms the physical acceptability of this model at an initial phase however at a later time, this model becomes unstable which is in good agreement with [79].

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