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LRS Bianchi Type-I Magnetized Dark Energy Cosmological Model in Bimetric Theory of Relativity with Cosmological Term – Λ

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Abstract:

In this paper, to study Locally Rotationally Symmetric (LRS) Bianchi type-I magnetized dark energy cosmological models with cosmological constant -Λ, we have calculated Rosen's field equations for LRS Bianchi type -I spacetimes in the presence of dark energy and discuss their geometrical as well as physical significances using two cases (i) $\rho + \lambda = 0$ (ii) $\rho - \lambda = 0$. To obtain the determinate solution of the model, we considered i) scalar expansion θ is directly proportional to share σ ii) $\Lambda = \frac{\alpha}{AB^2}$. In the presence of a magnetic field for the conditions $\rho + \lambda = 0$, scalar expansion θ attains its minimum and increases with increase in T indicating that the universe is expanding with increment in T, for the case $\rho - \lambda = 0$ scalar expansion θ attains its maximum and decreases with increase in T indicate that slows down the rate of expansion.

Keywords: LRS Bianchi Type I; Bimetric; Gravitation; Dark Energy; Dark Matter.

1. Introduction

It presently appears that the cosmos is composed of 4.9% baryonic (ordinary) matter, 26.8% dark matter, and 68.3% dark energy in the form of energy densities. Recently, various cosmological theories including dark matter as well as dark energy have been investigated. Due to the development of the dark energy problem, the constraints of general relativity have once more become apparent. It has been assumed that dark energy will explain how the universe is clearly expanding at an acceleration rate. Dark energy has been suggested to be responsible for of the universe's observed acceleration in expansion. Dark energy (DE) is a speculative type of energy acting appallingly which allows all of space and will in general speed up the extension of the cosmos. This sped-up extension of the cosmos was affirmed by the perceptions like Supernovae type I, LSS, SDS Survey, WMA Probe, BAO and CMBR Some authors have discussed this aspect viz; Borkar et al.(1), Mamon et al.(2), Riess et al.(3), Perlmutter et al.(4), Spergel et al.(5), Tegmark et al.(6), Hinshaw et al.(7), Nolta et al.(8), Hinshaw et al.(9), Anderson et al.[10]. Cosmologists such as Kamenshchik et al.[11], Katore et al.[12], Borkar et al.[13] presented several contenders for dark energy based on these senses and their theories about it. The cosmological steady up-and-comer experiences two notable issues, specifically, the tweaking and the grandiose incidental problems studied by Copeland et al.[14]. Positive energy thickness and negative tension are combined in the pith dynamical dark energy model. The basic scenario is further inspired by the tracker field this study was done by Saha et al.[15] and Sing et al.[16]. The Chaplygin gas model, which is seen as the combined effect of dark matter along with dark energy, is another option to the dynamical dark energy model studied by Dev et al.[17], Bento et al.[18], Abdusattar et al.[19]. Rosen[20],[21] Investigated the importance of attractiveness in the cosmos is important, given that the attractive field exists in the space between universes and plays a very important role in cosmology. Melvin proposes that matter was severely ionized during the creation of the cosmos also, because of the expansion of the cosmos, this matter is once more flawlessly united with the attractive field and builds unbiased matter. Therefore, it makes sense that there would be an attracting field throughout the cosmos. The incredible result of general relativity did not stop any other explanations for the universe's expanding pace and the existence of dark matter from being put forth. In actuality, a short while after Einstein's theory was published, numerous elective speculations have arisen telling the best way to expand, how to connect with other theories, and how to consolidate. The bimetric theory of gravity proposed by Rosen is one of the other alternatives, which is free of singularities as well as black holes which where appear in the Big Bang theory of the cosmological model. The theory is consistent with the most recent observational evidence on GR, and it substantially agrees with the covariance and equivalence principles of GR. Therefore, Rosen's BTG is significant and is dependent on two matrices namely fundamental metric and flat metric. The metric tensor g_{ij} is the Riemannian metric tensor depicts the gravitational potential whereas the metric tensor γ_{ij} is background metric tensor depicts the inertial forces related to the speed increase of the reference frame and it has no direct actual importance except for shows up in the field equation. The gravitational potential represented by the metric tensor $g_{ij}$is not fixed by the field equation or by interactions between matter and gravity. In this way, the flat space-time metric collaborates with the Riemannian metric yet not straightforwardly with matter. Without a trace of matter, one would have $g_{ij}= \gamma_{ij}$ fulfilling the standards covariance as well as the equivalence of the development of GR. In this manner at each point of space-time in Bimetric relativity, there are two matrices as follows.

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

$$dn^2 = \gamma_{ij}dx^i dx^j \tag{2}$$

The bimetric theory of gravitation via Rosen's field equations is

$$N_i^j - \frac{1}{2}N\delta_i^j - \Lambda g_i^j = -8\pi k T_i^j \tag{3}$$

$N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj}g_{si|p}\right)_{|r}$, Here $(\,|\,)$ denotes γ–covariant derivative and T_i^j represents energy–momentum tensor.

$N = N_i^i$, $k = \sqrt{g/\gamma}$ together with$g = determinant(g_{ij})$ and $\gamma = determinant(\gamma_{ij})$.

Numerous specialists have fostered the hypothesis and examined numerous models of the cosmos, in BTG and concentrated on their way of behaving geometrically as well as physically and this was studied by Karade et al.[22], Reddy et al.[23], Gaikwad et al.[24]. In this examination note, we will focus on the Locally Rotationally Symmetric Bianchi type-I magnetized dark energy cosmological model in the bimetric theory of gravity (BTG) since it has a great deal more fidelity to the physics of spatial homogeneous cosmology.

According to theories of gravitation, the main issue with the advancement of the cosmos is focusing on the singularity's situation and knowing its nature. Rosen[20],[21] joined the γ_{ij} along with g_{ij} to keep away from the singularities in his BTG. Any actual hypothesis should be released from singularity since its presence suggests the breakdown of the rules propounded by the theory. This has led to the development of the bimetric theory of gravity, which is now free of singularities. Numerous experts, including Borkar et al.[22], Karade et al.[23)], Kruskal et al.[24], and so forth in GR are focused on it. There are two types of singularity one is removable and the other is non-removable. The nonremovable singularity problem has attracted the attention of experts in several gravitational theories studied by Penrose et al.[25], Karade et al.[26], Borkar et al.[27], Pradhan et al.[28]. To depict the gravitational field around large things with singularities, Schwarzschild deduced an alternative arrangement for Einstein's vacuum spacetime field equation Borkar et al.[27]. There is exactly one singularity at r = 0 in BTG for circularly symmetric static spacetime in the case of emptiness, and it would disappear in the unlikely event that the molecule represented by a matter tensor differs from zero in a specific area given by Rosen[20],[21].

In this work, we study the LRS Bianchi type-I magnetized dark energy cosmological model in BTG by solving Rosen's equations and also study physical as well as geometrical significance.

2. Line Element & Field Equations

LRS Bianchi type - I space-time can be considered as

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \tag{4}$$

where A and B rely upon enormous time t as it were.

The flat metric relating to metric (4) is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{5}$$

Where, T_j^i is energy-momentum tensor for a cloud string with a magnetic field in the presence of perfect fluid is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j + E_i^j \tag{6}$$

$v^i v_i = -x_i x^i = -1$ and $v^i x_i = 0$

Here, ρ, λ, x_i, v_i is proper energy density, string tension density, unit space-like vector unit time-like vector respectively.

We choose a comoving coordinate system as $v^i = (0,0,0,1);\ x^i = (\frac{1}{A}, 0,0,0)$

We consider particle density ρ_p of configuration as

$$\rho = \rho_p + \lambda$$

E_i^j is the electromagnetic field defined as

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \tfrac{1}{2} g_i^j\right) - h_i h^j\right] \quad (7)$$

Where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector characterized as

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j$$

Here F^{kl} and $\in_{ijkl}$ are electromagnetic field tensor and Levi-Cevita tensor density respectively.

We can write a set of Maxwell's equations as

$F_{[ij,k]} = 0$ yield F_{23} = H (*say*) = constant

Because it's believed that electrical conductivity never ends, so we have

$$F_{14} = F_{24} = F_{34} = 0$$

The one element that does not vanish of F_{ij} is F_{23}.

So that, $h_1 = \frac{AH}{\bar{\mu}B^2}$ and $|h|^2 = \frac{H^2}{\bar{\mu}^2 B^4}$

The electromagnetic field's constituent parts E_i^j are given by

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = -\frac{H^2}{\bar{\mu}^2 B^4} \quad (8)$$

Equation (6) of energy-momentum tensor yield

$$T_1^1 = \left(p - \lambda - \frac{H^2}{2\bar{\mu}B^4}\right), T_2^2 = T_3^3 = \left(p + \frac{H^2}{2\bar{\mu}B^4}\right), T_4^4 = \left(-\rho - \frac{H^2}{2\bar{\mu}B^4}\right) \quad (9)$$

For the metric (4) and (5) using equation (9) Rosen's field equations (3) gives,

$$\frac{-A_{44}}{A} + \frac{2B_{44}}{B} + \frac{{A_4}^2}{A^2} - \frac{2{B_4}^2}{B^2} = 16\pi AB^2\left(-p + \lambda + \frac{H^2}{2\bar{\mu}B^4}\right) - 2\Lambda \quad (10)$$

$$\frac{A_{44}}{A} - \frac{{A_4}^2}{A^2} = 16\pi AB^2\left(-p - \frac{H^2}{2\bar{\mu}B^4}\right) - 2\Lambda \quad (11)$$

$$\frac{A_{44}}{A} + \frac{2B_{44}}{B} - \frac{{A_4}^2}{A^2} - \frac{2{B_4}^2}{B^2} = 16\pi AB^2\left(\rho + \frac{H^2}{2\bar{\mu}B^4}\right) - 2\Lambda \quad (12)$$

3. Solution of Field Equations With Magnetic Field

The field equation (10) (11) and (12) are the three equations along with six unknowns A, B, Λ, λ, ρ and p. Therefore, the system is initially unknown, and to obtain the system's full solution, we need three more equations. Therefore, we supposed that scalar expansion θ is directly proportional to share σ.

$$A = B^n \qquad \text{and} \quad \Lambda = \frac{\alpha}{AB^2}$$

Under the two scenarios listed above, we assume the following cases:

$$\text{(i) } \rho + \lambda = 0 \text{ and (ii) } \rho - \lambda = 0 \tag{13}$$

Case I: $\rho + \lambda = 0$

After adding equations (10) and (12) we get

$$\frac{4B_{44}}{B} - \frac{4{B_4}^2}{B^2} = 16\pi AB^2\left(-p + \frac{H^2}{\bar{\mu}B^4}\right) - 4\Lambda \tag{14}$$

Using equations (13) and (11) in equation (14) we get

$$\frac{B_{44}}{B} - \frac{{B_4}^2}{B^2} = \frac{48\pi KB^{n-2}}{4-n} - \frac{2\alpha}{(4-n)B^{n+2}} \tag{15}$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (15) we obtain

$$\frac{df^2}{dB} - \frac{2f^2}{B} = \frac{96\pi KB^{n-1}}{4-n} - \frac{4\alpha}{(4-n)B^{n+1}} \tag{16}$$

On integration, equation (16) leads to

$$f^2 = \frac{96\pi KB^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)B^n} + MB^2 \tag{17}$$

Here M is the integration constant.

From (17), we get

$$\int \frac{dB}{\sqrt{\frac{96\pi KB^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)B^n} + MB^2}} = t + V \tag{18}$$

Here V is the constant of integration. The value of B can be calculated from (18).

Through the proper coordinate transformation

$B = T\,, x = X\,, y = Y \; and \; z = Z$ metric (4) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{96\pi KT^n}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)T^n} + MT^2\right]} + T^{2n}dX^2 + T^2(dY^2 + dZ^2) \tag{19}$$

Case II: $\rho - \lambda = 0$

On subtracting equations (10) from (12) we get,

$$2\left[\frac{A_{44}}{A} - \frac{A_4^2}{A^2}\right] = 16\pi AB^2P \tag{20}$$

Using equations (13) and (11) in equation (20) we get

$$B_{44} - \frac{B_4^2}{B} = \frac{-16\pi KB^{n-1}}{3n} - \frac{2\alpha}{3nB^{n+1}} \tag{21}$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (21) we obtain

$$\frac{df^2}{dB} - \frac{2f^2}{B} = \frac{-32\pi KB^{n-1}}{3n} - \frac{4\alpha}{3nB^{n+1}} \tag{22}$$

On integrating equation (22) we get,

$$f^2 = \frac{-32\pi K B^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)B^n} + NB^2 \tag{23}$$

Here N is the constant of integration.

$$\int \frac{dB}{\sqrt{\frac{-32\pi K B^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)B^n} + NB^2}} = t + U \tag{24}$$

Where U is the integration constant. The value of B can be calculated from (24)

Through the proper coordinate transformation $B = T, x = X, y = Y \; and \; z = Z$ metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{-32\pi K T^n}{3n(n-2)} + \frac{4\alpha}{3n(n+2)T^n} + NT^2\right]} + T^{2n}dX^2 + T^2(dY^2 + dZ^2) \tag{25}$$

3.1 Geometrical and Physical Characteristics With Magnetic Field

For the model (19) all the physical quantities are given by

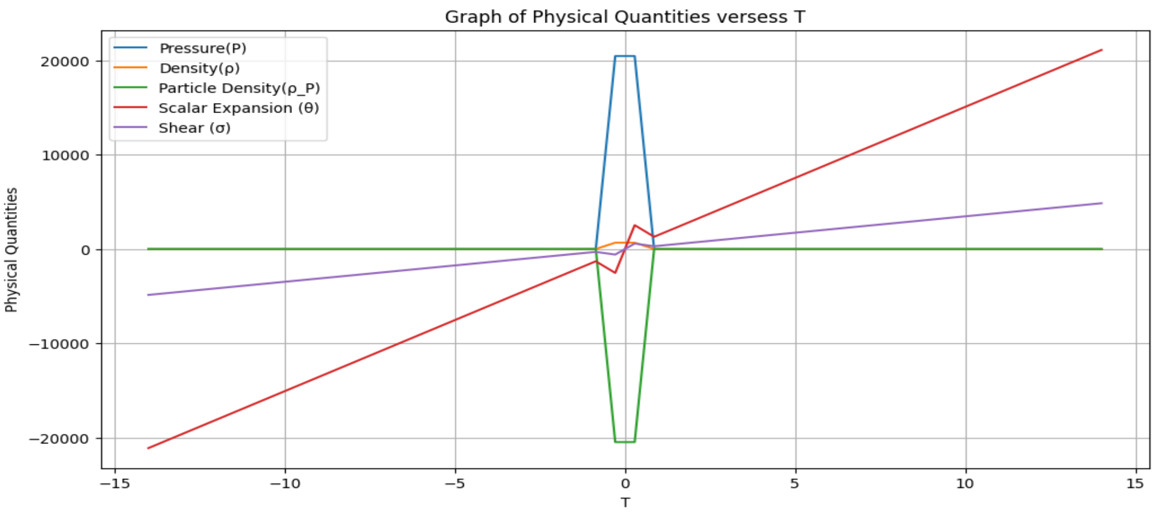
$$\rho = -\lambda = \frac{K(4n+2)}{(4-n)T^4} - \frac{\alpha(n-1)}{4\pi(4-n)T^{2(n+2)}} \tag{26}$$

$$\rho_p = \frac{2K(4n+2)}{(4-n)T^4} - \frac{\alpha(n-1)}{2\pi(4-n)T^{2(n+2)}} \tag{27}$$

$$P = \frac{-(2n+4)K}{(4-n)T^4} + \frac{\alpha(n-2)}{4\pi(4-n)T^{2(n+2)}} \tag{28}$$

$$\theta = (n+2)\left[\frac{96\pi K T^{n-2}}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)T^{n+2}} + M\right]^{\frac{1}{2}} \tag{29}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{96\pi K T^{n-2}}{(4-n)(n-2)} + \frac{4\alpha}{(4-n)(n+2)T^{n+2}} + M\right]^{\frac{1}{2}} \tag{30}$$



From above it is clear that with magnetic field for the conditions $\rho+\lambda=0$ the physical quantities $P, \rho,$ and ρ_p Follows straight line path with slight deviation near $T=0$, also scalar expansion θ attains its minimum and increases with an increase in T and tends to ∞ as $T\to\infty$.

For the model (25) all physical quantities are given by

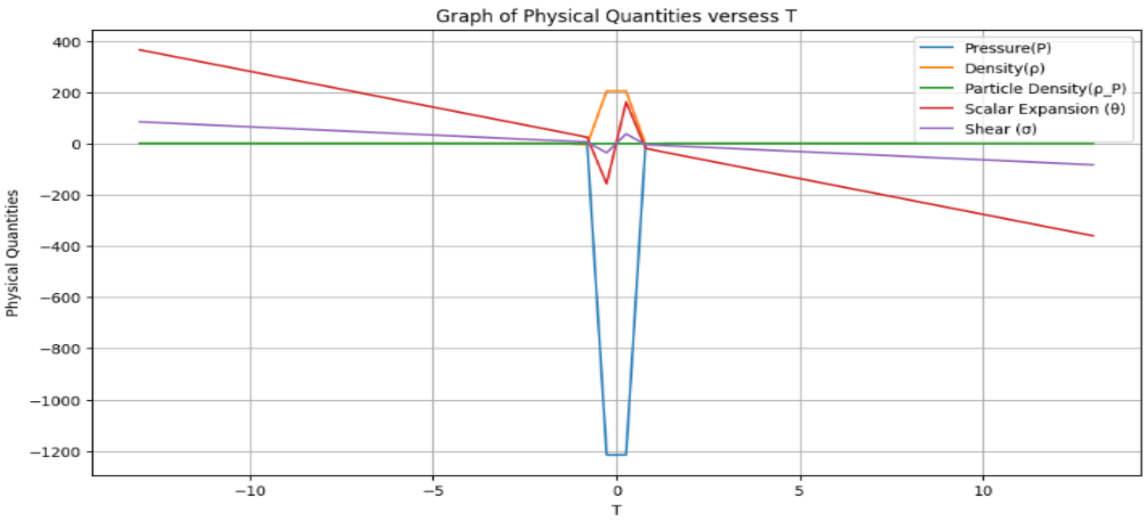
$$\rho=\lambda=\frac{\alpha(n-1)}{12\pi n T^{2(n+2)}}-\frac{2K(2n+1)}{3nT^4} \tag{31}$$

$$\rho_p=0 \tag{32}$$

$$P=-\frac{\alpha}{12\pi T^{2(n+2)}}-\frac{2K}{3T^4} \tag{33}$$

$$\theta=(n+2)\left[\frac{4\alpha}{3n(n+2)T^{n+2}}-\frac{32\pi K T^{n-2}}{3n(n-2)}+N\right]^{\frac{1}{2}} \tag{34}$$

$$\sigma=\frac{(n-1)}{\sqrt{3}}\left[\frac{4\alpha}{3n(n+2)T^{n+2}}-\frac{32\pi K T^{n-2}}{3n(n-2)}+N\right]^{\frac{1}{2}} \tag{35}$$



From above it is clear that with magnetic field for the conditions $\rho-\lambda=0$ the physical quantities $P, \rho,$ and ρ_p Follows a straight line path with slight deviation near $T=0$, also the scalar expansion θ goes on decreasing with an increase in the value of T.

4. Solution of Field Equations Without Magnetic Field

Case I: $\rho+\lambda=0$

When the magnetic field is not present equation (19) becomes

$$ds^2=-\frac{dT^2}{\left[\frac{4\alpha}{(4-n)(n+2)T^n}+MT^2\right]}+T^{2n}dX^2+T^2(dY^2+dZ^2) \tag{36}$$

Case II: $\rho-\lambda=0$

When the magnetic field is not present equation (25) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{4\alpha}{3n(n+2)T^n} + NT^2\right]} + T^{2n}dX^2 + T^2(dY^2 + dZ^2) \quad (37)$$

4.1 Geometrical and Physical Characteristics Without Magnetic Field

For the model (36) all physical quantities are given by

$$\rho = -\lambda = -\frac{\alpha(n-1)}{4\pi(4-n)T^{2(n+2)}} \quad (38)$$

$$\rho_p = -\frac{\alpha(n-1)}{2\pi(4-n)T^{2(n+2)}} \quad (39)$$

$$P = \frac{\alpha(n-2)}{4\pi(4-n)T^{2(n+2)}} \quad (40)$$

$$\theta = (n+2)\left[\frac{4\alpha}{(4-n)(n+2)T^{n+2}} + M\right]^{\frac{1}{2}} \quad (41)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{4\alpha}{(4-n)(n+2)T^{n+2}} + M\right]^{\frac{1}{2}} \quad (42)$$

For the model (37) all physical quantities are given by

$$\rho = \lambda = \frac{\alpha(n-1)}{12\pi n T^{2(n+2)}} \quad (43)$$

$$P = -\frac{\alpha}{12\pi T^{2(n+2)}} \quad (44)$$

$$\rho_p = 0 \quad (45)$$

$$\theta = (n+2)\left[\frac{4\alpha}{3n(n+2)T^{n+2}} + N\right]^{\frac{1}{2}} \quad (46)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left[\frac{4\alpha}{3n(n+2)T^{n+2}} + N\right]^{\frac{1}{2}} \quad (47)$$

When the magnetic field is not present for both the cases $\rho + \lambda = 0$ and $\rho - \lambda = 0$ all the quantities show slight deviation from the straight line at $T = 0$. For with and without magnetic field for both the cases, also $\sigma/\theta = constant$ shows that the model does not approach isotropy in general studied by Pradhan et al.[(28)].

5. Quintessence Model Of Dark Energy Without Magnetic Field

One of the models of dark energy is the Quintessence model which obeys the equation of state.

$$P_q = \omega_q \rho_q \,, -1 \leq \omega_q \leq 0 \quad (48)$$

in which ω_q is the equation of state parameter correlated with the quintessence model.

For case (i) ω_q is given by $\omega_q = -\frac{(n-2)}{(n-1)}$ (49)

For case (ii) ω_q is given by $\omega_q = -\frac{n}{(n-1)}$ (50)

We are interested in three values of ω_q which are $-1, 0, \frac{1}{3}$ and 1. $\omega_q = -1$ corresponds to the vacuum fluid, $\omega_q = 0$ corresponds to the dust fluid, $\omega_q = \frac{1}{3}$ corresponds to the radiating fluid, $\omega_q = 1$ stiff dominated universe.

6. Conclusion

In this work, we examine the Locally Rotationally Symmetric Bianchi type-I magnetized dark energy cosmological model in BTG by solving Rosen's field equations. The study investigates the model's geometrical and physical characteristics with and without a magnetic field in two cases (i) $\rho + \lambda = 0$ (ii) $\rho - \lambda = 0$.

1. In the Presence of a magnetic field for the case $\rho + \lambda = 0$ the increasing scalar expansion θ and Shear σ over time indicate (fig 1) that the expansion of the universe. This concept is similar to the Big Bang theory, which suggests that the origin of the universe from an extremely hot and dense point and has been expanding ever since. The observations of increasing shear and scalar expansion obey the idea of an expanding universe.
2. In presence of a magnetic field for the case $\rho - \lambda = 0$ scenario suggested that the universe in the presence of a magnetic field influences gravitational dynamics, leading to a contraction despite the presence of constant pressure and density (fig 2). The balance between matter energy density and the cosmological constant further shapes the overall evolution of the universe.
3. When the magnetic field is not present for both cases (figures 3 & 4) all the physical quantities follow a straight-line path except at $T = 0$. The universe evolves predictably according to a straight-line path for physical quantities, with a notable deviation at the beginning of time. This suggests that while the absence of a magnetic field may not directly influence the overall evolution of the cosmos, there are still some initial conditions that shape its development.
4. In the absence magnetic field, the equation of state parameter ω_q for $n > 2$ for both the cases comes out to be negative and it is independent of T. This indicates that there is other than baryonic matter and the universe is completely occupied by dark energy and dark matter when the magnetic field is not present. The equation of state parameter ω_q is negative and remains constant over time. This indicates the presence of a dominant energy component with repulsive gravitational effects, similar to dark energy, which influences the overall evolution of the universe.

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