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The Use of Mathematical Working Space Strategies to Improve the Student Proof Process

Isnani[1], S.B. Waluya[2*], Dwijanto[3], T.S.N. Asih[4]

Department Mathematics Education Universitas Negeri Semarang Central Java Indonesia

Correspondence: *s.b.waluya@mail.unnes.ac.id

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Abstract:
The proofing process in Real Analysis involves proofing skills as part of reasoning skills. Currently, the main problem is that students who are prospective mathematics teachers have difficulty in compiling mathematical evidence and considering the abstract of this course. This study aims to investigate the extent to which students master the concept of mathematical proof. Adopt a mixed method with embedded strategies and concurrent research design, using experiments. Participants are students at private universities in the northern coastal area of western Central Java, Indonesia who are taking the Real Analysis course. Precisely in Tegal City and Brebes Regency in the 2022/2023 academic year. Initial ability tests were conducted on the participants, and the results were homogeneous. During the research, students of Tegal city used the Guided Discovery Learning model with the help of worksheets discussion materials with the Mathematics Workspace strategy. Based on the results of several assignments, post-tests, and interviews show a very satisfactory overall implementation rate in the experimental class with high adoption. In the future, this research can be focused on the development of evidentiary problems to improve the quality of evidentiary capabilities and investigate obstacles in studying evidentiary problems. Finally, this research contributes to the effective teaching of Real Analysis by providing insights into evidentiary problems easy to learn.

Keywords: Proof ability, Model Guided Discovery Learning, Mathematical Working Space Strategies, Real Analysis.

1. Introduction

The ability to process proof is one of the abilities that must be possessed by mathematics students and Mathematics Education. Students are at a high level that requires involvement in proof [1]. Constructing and understanding evidence and abstract problems in the proof are the difficulties experienced by students so far [2], [3], [4], [5], [6], [7]. The study program stipulates that students must be involved with mathematical proof to develop students abilities in the proof process [8]. Students must be proficient in developing their working memory and cognitive. This will affect the ability of students in the proof process [9]. Good understanding and basic mathematical concepts are needed in the ability of the proof process. It allows mathematicians to convince others of the correctness of their ideas and assertions. A good understanding of mathematical concepts is essential for constructing rigorous and convincing proofs [1], [2], [10], [11].

The role of proof is to verify, explain, systematize, discover, and communicate. Proofs can demonstrate the truth of mathematical statements, as well as can provide reasonable justifications. The process of proof can lead to new insights, discoveries, and developments [12], [13], [14]. The proof process in the Department of Mathematics and Mathematics Education is found a lot, one of which is in the Real Analysis course. Real analysis is a subject that requires higher-order thinking. In these courses, a strong understanding and concepts is needed to build and understand evidence. Before taking courses on real Analysis students must be prepared to understand the evidence. Furthermore, students must understand proof and improve problem-solving skills, one way is to practice often solving problems related to proof [15], [16].

Students do not understand the problem of proof and are not familiar with the problem of proof. Students need to be confident about evidence to influence their ability to understand and build evidence [6]. Most students have an irreversible mindset in learning mathematical proofs, which leads to a lack of confidence in understanding and building proofs [13], [17]. Student etymology in evidentiary problems in the Real Analysis course. Some research results found difficulties for students in learning proof, namely difficulty understanding concepts (definitions), not knowing how to start proof construction, not knowing the use of definitions and known principles, and not knowing what to prove [18], [19].

Students cannot understand the role and function of evidence in Mathematics for the short and long term. The role of evidence, in general, is to verify the truth of statements, provide logical arguments, construct new mathematics, communicate mathematics, and find the systemization of statements in axiomatic systems [20], [21], [22]. Evidence is very important to apply to mathematics education students because it can improve problem-solving skills and develop critical thinking [23]. Evidence plays a very important role as a fundamental aspect in establishing the truth of mathematical statements. Likewise, others will justify their ideas, and the development and communication of knowledge greatly contribute [24].

These student activities are a need for a model to help students control their thoughts. Learning models that have characteristics that foster students to explore their ideas include *guided discovery learning*. Students make discoveries and lecturers play a role in guiding by analyzing difficulties in solving problems faced by students. The role of the lecturer in this model is as a facilitator to direct students in developing their ideas to formulate answers to conclude the mathematical concepts being studied [25], [26]. The main properties of *guided discovery learning* are: (1) exploring and solving problems to create, integrate, and generalize knowledge. (2) Students determine the stages and frequency of interest in activities, and (3) student activities that encourage the integration of new knowledge [27], [28].

One strategy to help students control their minds in proof activities is the *Mathematical Working Space* (MWS) strategy. *Mathematical Working Space* can be used to analyze and improve the mathematics learning process, including in the development of proof skills. MWS provides a framework that integrates cognitive, instrumental, and social aspects in students' mathematical activities [29], [30] By implementing the MWS strategy, students can develop a better understanding of reality and build arguments and evidence systematically [29], [31], [32]

Mathematical Working Space is a strategy that includes semiotic, instrumental, and discursive genesis. *Semiotic genesis* is the linking of activities that are representations to obtaining visualizations of mathematical problems. Instrumental genesis is the phase in which the knowledge gained can be used in the process of constructing evidence. Next is *discursive genesis*, which is the phase to validate evidence. [33], [34], [35]. Some mathematical research related to proof and proof learning using the Mathematical Working Space strategy has succeeded well, namely research from [36], [37], [38].

Supporting learning facilities in this study are the existence of Discussion Material Worksheets. This facility was created with the Mathematical Working Space strategy. The content of this Discussion Material Worksheet contains semiotic, instrumental, and discursive genesis. Student learning using the *guided discovery learning* model is then also facilitated by student discussion material worksheets containing Sequence (Bartle R.G., 2000). This material contains the concepts of Sequences and Their Limits, Limit Theorems, Monotone Sequences, Subsequence Bolzano Weierstrass's Theorem, and the Cauchy Criterion. Each definition, theorem, lemma, or proof problem in the Discussion Material Worksheets contains semiotic, instrumental, and discursive genesis. The Discussion Material Worksheets were created to make it easier for students to learn Real Analysis.

Referring to the reasons above, to optimize to develop proof skills in Real Analysis lectures. So to find out the condition of students in the problem of proof, a pilot study is carried out and also involves interviewing students to evaluate their understanding. Trends in mathematics education that focus on success and effectiveness in learning. Furthermore, to measure the extent of the success of the learning, interviews were also conducted. The results of the pilot study also contribute to the development of effective teaching practices, then used to assess the application of learning practices in the classroom environment based on the principles of constructivism.

1.1 Research Questions

This study seeks to answer the following two questions: (1) with a definition level of 0.05 in statistics, learning can be said to be successful satisfactorily and more effectively, and (2) the extent of students' success in mastering the problem of proof.

1.2 Study Contribution

Some of the significant contributions of this research both theoretical and practical are as follows: (1) Providing effective learning practices, (2) developing mathematics evaluation materials as lecturers' performance in teaching and learning in proof problems, (3) making it easier for students to learn mathematical proof problems.

1.3 Scope of Study

The formation of effective learning in the learning of Real Analysis in Sequences material in the Mathematics Education study program of private universities in the city of Tegal for the 2022/2023 Academic Year.

2. Research Method

2.1Research Design

This study uses a sequential research design, quantitative data by qualitative data (Creswell & Creswell, 2017). Quantitative research was applied to a quasi-experimental with a single sample: Davison & Smith, 2018 O X1 O X2 Where O is the treatment, X1 is the first test and X2 is the second test.

This research begins with an initial evidentiary ability test for each student. Then Real Analysis learning was carried out for seven meetings where each student meeting was given assignments in groups. After seven meetings of teaching and learning activities, a test was carried out to see the achievement of students' mathematical proof skills.

Methodologically qualitatively used using a case study approach. Of the three prospective mathematics teachers who completed the test, it can be an indication of another response with a similar process. Semi-structured interviews with student responses. And identify the student proofing process where the discussion material worksheet is implemented to help with their mathematical proof.

2.2 Participants

Participants were students who took the Real Analysis program at private universities in the northern coastal region of western Central Java Indonesia. Precisely in Tegal City and Brebes Regency in the 2022/2023 academic year. Participants from Tegal City were 32 people and Brebes Regency was 30 people. With a purposive sampling of engineering students from the city of Tegal as an Experiment class. In the Real Analysis learning experiment class, the Guided Discovery Learning model was used with the help of worksheets, discussion materials with mathematical working space strategies. Meanwhile, students from Brebes district as a control class using conventional learning. The two classes during the research on learning Real Analysis in the same material, namely Sequence, for seven meetings. The sub-materials that will be studied include Sequences and Their Limits, Limit Theorems, Monotone Sequences, Subsequences of Bolzano Weierstrass's Theorem, and The Cauchy Criterion.

2.3 Instruments

The test is a quantitative instrument used in this study, while the interview is a qualitative instrument. The initial evidentiary ability test is carried out before learning. Furthermore, learning was carried out for seven meetings with discussion worksheets in the eighth meeting given a final test.

The Evidentiary Ability and aspects measured are included in the following Table 1:

Table 1 Proof Ability and Measured Aspects

Dimension	Aspects Measured
Reading the Evidence	Ability to apply the stages of proof of a statement to other similar statements The ability to use definitions as a basis in giving reasons for the correct proof step or the improvement of symbols, narratives, and premises, at inappropriate stages/steps of proof Comparing two definitions, then choosing one to use in proving a statement The ability to examine a mathematical statement to determine the truth or to show the error of the statement using examples of denial Making a hypothesis (conjecture) based on the pattern and properties of several statements and proving the conjecture obtained by deduction
Constructing the Evidence	1. The ability to organize and manipulate facts, as well as order the steps of the evidence provided to obtain a valid item construction. 2. The ability to make connections between the facts known in the statement and the elements to be proven. 3. Ability to use premises, definitions, or theorems related to a statement to build proof.

As for qualitative data collection, the data collection is carried out by the researcher himself. At the same time, the data source is experimental class students who are chosen appropriately for the collection and analysis of qualitative data, which is then concluded qualitatively. Things that are concluded qualitatively include the achievement of aspects of evidentiary ability, students and their causative factors, advantages, and weaknesses when applying informal arguments through a *guided discovery learning model with a mathematical working space strategy.*

2.4. Data Analysis

Data analysis used for quantitative research is by the research objectives. The goal is to grow the proof ability of mathematics teachers through a guided discovery learning model with a mathematical working space strategy. It is said to be successful if it meets: 1) the experimental class experiences learning completeness, namely a) 75% of students obtain scores from the evidentiary ability test above completeness, which is 65, the test used with the Proportion test b) the average score above the completeness value that is 65 then use the one-sample t-test, c). Results of student assignments for each sub-material 2). Description Pre-test and post-test statistics in experimental and control classes 3) The results of the evidentiary ability test in the experimental class are better than the control class, then use the independent t-test.

Based on the responses of 32 students, 3 representative answers were chosen. Then the representative responses are analyzed to find the proof process. The completeness of mathematical proof skills was analyzed through answer sheets and interview files. The results of this analysis function as data on the student learning process during the research. The process data shows the credibility of qualitative research.

3. Results

Before the research, the initial ability to prove students who took the Real Analysis course in the Mathematics Education study program of Pancasaki Tegal University and Brebes Civilization University were given a test. The test contains proof of the material before the row, namely the Real Number System material. Then the test results are tested with an independent Samples test Assisted IBM SPSS Statistic 27 with the results in the following Table 2

Table 2 Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Awal_Ps_Pr	Equal variances assumed	,142	,709	4,817	60	,000	5,16200	1,07158	2,99271	7,33129
	Equal variances are not assumed.			4,399	22,133	,000	5,16200	1,17355	2,72905	7,59495

Based on the test results, the sig obtained was 0.709 > 0.05, so the initial ability of students from the two study programs was homogeneous. Then with the purposive sampling technique, Pancasakti Tegal University students were used as an experimental class.

Furthermore, a Proportion test and a one-sample test were carried out to determine the student's ability to prove during learning using the Guided Discovery learning model with the Mathematical Working Space strategy on the row material. Before being tested in the 2 tests above, the data must meet the normal distribution, then a normality test. The test uses the Kolmogorov-Smirnov test with the help of IBM SPSS Statistics 27 with the results in Table 3.

Table 3 Results of the Normality Test

	Kolmogorov-Smirnov[a]		
	Statistic	df	Sig.
Data kemampuan pembuktian Akhir kelas Eksperimen	,141	26	,194

Based on Table 3 above aAcquired Sig = 0.194 > 0.05, which means that the data is normally distributed. Next, a proportion test was carried out with the following results $t_{calculated}$ = 4.48 > t_{table} = 1.710882 means that more than 75% of students who learned Analysis in the experimental class obtained a score above the completeness value of 65. Then a test of one sample using IBM SPSS Statistics 27 was obtained in the following Table 4.

Table 4 One-Sample Test

	Test Value = 65.00					
	t	df	Sig. (2-tailed)	Mean Difference	,95% Confidence Interval of the Difference	
					Lower	Upper
VAR00001	1,742	31	,094	,86692	,8609	,8729

With the test one sample t-test pada Table 4 obtained sig = 0.094 > 0.05 means the average results of the evidentiary ability test in the experimental class above the completeness criterion of 65. From the results of the two tests above, it means that students who study Real Analysis with the help of a Guided Discovery Learning model with the help of a worksheet of rocky discussion material Mathematical working space strategy meet the completeness criteria. The condition of the experimental classroom is seen from the final results of the learning that meets the requirements of the proof. Likewise, the results of student assignments in each sub-material with descriptive statistics are contained in the following Table 5.

Table 5 Descriptive statistics of student assignments in each sub-material

	Sequences and Their Limits	Limit Theorems	Monotone Sequences	Subsequences and Bolzano Weierstrass's Theorem	The Cauchy Criterion
Mean	72.77	77.67	70.45	89.54	80.56
Standard. Deviation	5.75	6.12	5.98	4,65	6.09
Minimum	36.78	45.65	39.24	55.67	46.24
Maximum	78.56	80.25	75.67	90.78	89.99

The results of student assignments for each sub-material in Table 5 can be seen to vary depending on the difficulty level of the questions. Based on the answer sheet, students increasingly understand the problem of the proof process from the dimension of reading the butt and the construction of evidence.

Based on the results of the proportion test, the test of one sample, and the results of student assignments for each sub-material, the experimental class can be said to have experienced completeness. This completeness means that the seven meetings can be said to have been successful. This success is thanks to the model used and the tools used so that the learning has met the credibility.

Furthermore, based on the results of the Pre-test, namely the initial evidentiary ability test, the post-test which is the final test of evidentiary ability, and the results of the Gain test in the experimental class and control class. The results of the three tests are included in Table 6 with descriptive statistics as follows:

Table 6 Descriptive statistics of evidentiary ability

	Eksperimen			Control		
	Pre Tes	Pos Tes	Gain	Pre Tes	Pos Tes	Gain
Standard Deviation	4.78	4.13	0.11	5.09	9.89	0.14
Mean	68.79	78.95	0.71	65.94	66.95	0.45

Table 6 shows a very significant difference between the experimental class and the control class. Where the score in the Experment class is better than in the control class. Although previously at the pre-test time, namely during the initial ability test, the condition of the experimental class and the control class was homogeneous. The difference between the pre-test of the experimental class and the control class was very slim but had no effect. However, when viewing the results of the post-test and the Gain test, it shows a very significant difference. The Experimental Class seems to have improved the ability to prove. With the conditions of the experimental class that uses the right learning model and strategy, it turns out that it can increase the ability to prove when compared to the previous and compared to the experimental class.

Furthermore, whether there is a difference in the ability to prove the end of the experimental class compared to the control class. This will be tested by an independent test with the results contained in Table 7 below:

Table 7 Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Akhir_Ps_pr	Equal variances assumed	,865	,358	2,704	60	,010	2,06493	,76375	,51881	3,61106
	Equal variances are not assumed.			2,855	34,601	,007	2,06493	,72317	,59622	3,53364

Furthermore, to compare the results of the evidentiary ability test in the experimental class with the control class with the t-test for Equality of Means obtained sig = 0.01 < 0.05, meaning that the results of the evidentiary ability in the experimental class are better than the control class. Because the four tests are met, the use of the Guided Discovery Learning learning model with the Mathematical working Space strategy can be said to be complete. This means that some students have improved their proof skills. This can be seen from the results of the final test of evidentiary ability, namely, students can understand the evidentiary questions given, prove the proof questions with the right steps, can trace the wrong evidentiary steps, and provide conclusions correctly.

Students of the Mathematics Education study program at Tegal University, after attending lectures using a guided discovery learning model assisted by worksheets of discussion materials with mathematics workspace strategies. Furthermore, at the final meeting, the seventh was given a final test on the problem of proving Sequence material.

The test can be used to determine the student's ability in the problem of proof. The results of the evidentiary ability test are sorted from the highest to the lowest score. The order of results is divided into three categories, namely high, medium, and low categories. Each category is taken by one student.

Based on the results of observations and interviews in each category, the following were obtained: High category (L) based on observations and interviews high category students feel happy using the newly felt learning, and by using the Discussion Material worksheet, find it easy to learn proof. In this worksheet, students discuss groups and easily learn the concept of proof. Students can read the evidence correctly. Each definition, theorem, lemma, and student proof problem on the worksheet students can interpret in their own language. Furthermore, students can construct evidence using the strategies on the worksheet. Students already know the concepts of definitions and theorems so that students know the steps of proof correctly.

Medium category students (M), referring to the results, namely students in the medium category feel happy learning with the newly used learning model but still ask to be accompanied by lecturers continuously. Students' understanding of the steps of proof and the use of definitions to prove increases. As for examining a mathematical statement to determine truth, it is still confused. Also, have been able to master the evidence steps provided to obtain valid grain construction. However, it is not yet confident to use premise, definitions, or statement-related theorems to build proof.

Students of the lower category (S) obtained the results Feeling happy with the newly used learning, but understanding the worksheet takes longer than in the two categories. In this category, understanding must be done repeatedly.

4. Discussion

This research was conducted on the subject of Real Analysis for Sequences material. In Sequences material, the goal is to prove convergent sequences, but to prove convergent Sequences in Bartle can use several theorems and definitions. Each meeting discusses a theorem to prove the convergent sequence students can understand and can prove problems using the theorem being studied. However, after all the theorems for proving convergent sequences have been studied, a repetition is held. This test is held at the end of the material. Then, when working on the problem, students experience confusion in determining which theorem will be used to solve the problem. Students must be able to use correctly the theorems and definitions related to the problem they are working on. To overcome this, a worksheet of discussion material was made with a mathematical working space strategy then the learning process used the Guided Discovery Learning Model.

Learning with the Guided Discovery learning model makes the student environment more active. This learning model encourages students to collaborate in solving problems in Real Analysis learning. This is in line with several studies that use the Guided Discovery learning model in learning exact sciences to make students active and improve their learning achievement [25], [28], [40], [41]. Likewise, in learning Real Analysis with the discovery method, encouraging students to be more active in proof learning [42]. Likewise, as has been done [43], Real Analysis learning with the Guided Discovery Learning model obtained by students successfully understand the concepts in Real Analysis. So far, students studying real analysis feel confused, but with the worksheet discussion material prepared with the mathematical working space strategy, the burden is lost. True also the statement [44] to build ease in learning concepts and fun learning needs a guide. This discussion material worksheet prepared with a mathematical working space strategy makes it easier for students to learn Real Analysis. Because in it, every definition, theorem, lemma, and related problem helps students to interpret it in their language. The worksheet also helps students construct evidence and validate evidence.

The use of the Guided Discovery learning learning model is very appropriate in Real Analysis learning. This learning model strongly encourages students to complete solutions in proof. The findings of the discussion material worksheet with this mathematical working space strategy can also overcome students' difficulties in learning proof. This is because the worksheet is facilitated with visualization, construction, and validation.

The mathematical working space strategy facilitates representation-visualization activities and construction activities to prove or validate evidence. This visualization makes it easier for students to understand concepts/definitions. Mastery of concepts is very important in real analysis. The ability to use definitions as a basis for providing reasons for the correct evidentiary steps or the improvement of symbols, narratives, and premises at inappropriate stages/steps of proof. If students can understand the definition well, they can analyze a mathematical statement to determine its truth or to show the error of the statement using examples of denial. This constructing activity is the ability to organize and manipulate facts, as well as sequence the steps of evidence provided to obtain valid item construction. Students can make connections between the facts known in the statement and the elements to be proven. These facts can be contained in definitions, theorems, or proven problems related to statements to build proof.

Every student must understand the concept of definition. Because in proving theorems and proof problems, an understanding of evidence is also needed. [45] Have used several strategies to improve student thinking in writing valid evidence. In this study, students can easily understand the definition concepts in the discussion material worksheet with mathematical working space strategies. In the worksheet, each student's definition must know the background, the essence of the definition, and its attributes. This attribute is very helpful for students in the evidentiary steps. These attributes will make it easier for students to prove theorems and proof problems.

During lectures, students often do proof questions that are done in groups. Solving proof problems in groups is very helpful for students in finding solutions. The Guided Discovery learning learning model is very appropriate in Proof learning, especially in Real Analysis courses [43], [45]. Students individually work on questions related to proof at the end of each meeting. The doing of proof questions encourages students to improve their proof skills. Moreover, learning has been facilitated with worksheets of discussion material to overcome student learning difficulties in learning Real Analysis. Worksheets with mathematical working space strategies are perfect for learning proofs [35], [46], [47], [48], [49] The use of discussion material worksheets in this study is very helpful for students in learning Real Analysis. Previously, students found it difficult to prove, but with flexibility in the worksheet, they can overcome their difficulties. These worksheets are very helpful for students in learning to analyze math problems. After learning real analysis using worksheets discussion material with mathematical working space strategies, most students can understand a general statement and successfully test it with an example, can understand the evidence provided and know the errors of the evidence provided, and then correct it, Can give conclusions appropriately If students have mastered the basics of analysis and proof, it helps in learning analysis in mathematical problems [50], [51], [52].

5. Conclusion

Underlining the results of this study the need for comprehensive learning is by the Real Analysis course in the mathematics education study program which aims to improve students' understanding of proof problems. In addition, it provides the necessary support to educators in implementing appropriate learning. This study can be aimed at the curriculum section to show the extent to which effective learning can be applied.

This study uses a quantitative approach using data on the results of the initial test and the final test obtained Real Analysis Learning using the Guided Discovery Learning model with discussion material worksheets with the Mathematical Working Space Strategy experienced completeness, there was an improvement compared to before, and was more effective. Scientific efforts in this area for the next time were centred on designing and implementing learning devoted to improving the quality of mathematical proof problems. Meanwhile, the results of qualitative research can explore the obstacles faced by educators and explore student perceptions in learning integration. Efforts to be made in the future are to improve the development of sustainable professional learning and relationships that are in line with the target. This kind of learning promises to foster a conducive learning environment that can ultimately improve students' ability to prove in learning Real Analysis.

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