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Decomposition of (Gζ, Ξ) -Continuity

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Abstract: In this work, we introduce and investigate a new type of open sets. We also present a decomposition of both (gζ, ξ) - c and decomposition of (ζ, ξ)-c.

Keywords: hereditary generalized topology, α-Hg-O, σ-Hg-O and π-Hg-O sets, β-Hg-O sets.

1. Introduction

In 2002, generalized topology and generalized continuity introduced by Csaszar in [1]. In 2005, Csaszar introduced and studied generalized open sets (ζ-α-O, ζ-σ-O, ζ-π-O, ζ-β-O)[2]. The notion ζ-b-O introduced by Sarsak in [11]. A space Z is called a C0 -space [12], if C0=Z, where C0 is the set of all representative elements of sets of ζ and x is called a represent element of u ∈ ζ if u ⊂ v for each v ∈ ζ(X). A subset A of generalized topological space (X, ζ) is said to be gζ -closed [4] (resp. ωζ -closed [7]), if c(A)⊆M whenever A⊆M and M is ζ-O (resp. ζ-σ-O) in X. The complement of ωζ -closed (resp. gζ-O) is ωζ-O[7] (resp. gζ-O [4]). The gζ-interior (resp. ωζ-interior) is the largest gζ-O (resp. ωζ-O) set contained in A and is denoted by $i_g(A)$ (resp. $i_\omega(A)$). In 2005, Csaszar introduced hereditary class in [3]. In this work hereditary generalized topological space (Z, ζ, H) is denoted by HGTS.

Definition 1.1. [3] The set ψ is said to be α-H-O (resp. σ-H-O, π-H-O, β-H-O, β∗-H-O, ζ∗ -closed), if ψ⊆ic∗(ψ) (resp. ψ⊆c∗i(ψ), ψ⊆ic∗(ψ), ψ⊆cic*(ψ), ψ⊆c*ic*(ψ), c*(ψ)⊂ψ).

Definition 1.2. A set ψ is said to be b-H-O [8], if ψ ⊆ ic∗(ψ)∪c∗i (ψ).

Definition 1.3. [10] A set ψ is said to be

1. α-Hg-O, if ψ ⊆ $i_g c{*} i_g(\psi)$.
2. σ-Hg-O, if ψ ⊆ $c{*} i_g(\psi)$.
3. π-Hg-O, if ψ ⊆ $i_g c{*}(\psi)$.
4. β-Hg-O, if ψ ⊆ $c i_g c{*}(\psi)$.
5. S-β-Hg- O, if ψ ⊆ $c{*} i_g c{*}(\psi)$.

2. b-Hg-open set

Definition 2.1. The set $\psi\subset(Z,\zeta,H)$ is called as b-Hg-open (b-Hg-O), if $\psi\subseteq i_g c*(\psi)\cup c*i_g(\psi)$.

Proposition 2.2. In HGT S, every ζ-O set is b-Hg-O.

Proof. A subset $\psi\subset Z$ is ζ-O. Then $\psi = i(\psi)$. Now $\psi \subseteq i(\psi) \subseteq i_g(\psi) \subseteq i_g c*(\psi) \cup c*i_g(\psi)$. Hence ψ is b-Hg-O.

Remark 2.3. The converse of Proposition 2.2 need not be true from the following example.

Example 2.4. Assume Z = {1, 2, 3, 4}, ζ={∅, {1}, {2}, {1, 2}, {2, 3, 4}, Z}, H = {∅, {1}, {3}}. Then ψ = {1, 2, 3} is b-Hg-O but not ζ-O.

Proposition 2.5. In HGTS (Z, ζ, H), every gζ-O set is b-Hg-O but not conversely.

Proof. Assume a subset ψ of HGTS (Z, ζ, H) is gζ- O. Then $\psi = i_g(\psi)$. Now $\psi \subseteq i_g(\psi) \subseteq i_g c*(\psi) \cup c*i_g(\psi)$. Hence ψ is b-Hg-O.

Proposition 2.6. In HGTS, every ωζ-O set is b-Hg-O but not conversely.

Proof. Assume a subset ψ of HGT S (Z, ζ, H) is ωζ-O. Then $\psi = i_\omega(\psi)$. Now $\psi\subseteq i_\omega(\psi) \subseteq ig(\psi) \subseteq i_g c*(\psi) \cup c*i_g(\psi)$. Hence ψ is b-Hg-O.

Example 2.7. Assume Z = {1, 2, 3, 4}, ζ={∅, {1}, {2}, {1, 2}, {2, 3, 4}, Z}, H = {∅, {1}, {3}}. Then ψ = {1, 2, 3} is b - Hg - O but not gζ-O.

Remark 2.8. The notions of b-Hg-O and ζ-b-O are independent.

Example 2.9. Assume Z = {1, 2, 3, 4}, ζ={∅, {1}, {2}, {1, 2}, {2, 3, 4}, Z}, H = {∅, {1}, {3}}. Then ψ = {4} is b-Hg-O but not ζ-b-O.

Example 2.10. Assume Z = {1, 2, 3, 4}, ζ = {∅, {1}, {1, 2, 3}, {3, 4}, Z}, H ={∅, {1}, {3}}. Then M = {1, 4} is ζ- b- O but not b- Hg-O.

Proposition 2.11. In HGTS (Z, ζ, H) every b-H-O is b-Hg-O.

Proof. Assume ψ be a b-H-O $\psi\subseteq ic*(\psi)\cup c*i(\psi)\subseteq i_g c*(\psi)\cup c*i_g(\psi)$. Hence ψ is b-Hg-O.Remark 2.12. The converse of Proposition 2.11 need not be correct from the following examples.

Example 2.13. Assume Z = {1, 2, 3, 4}, ζ = {∅, {1, 3}, {2, 3}, {1, 2, 3}, {1, 4}, {1, 3, 4}, Z}, H = {∅, {1, 2}}. Then ψ = {1} is b-Hg-O but not b-H- O .

Proposition 2.14. Every α-Hg-O (resp. σ-Hg-O, π-Hg-O) is b-Hg-O but not conversely.

Proof.

1. Assume ψ be α-Hg-O. Then $\psi\subseteq i_g c*i_g(\psi)\subseteq c*i_g(\psi)\cup i_g c*(\psi)$. Which implies ψ is b-Hg-O.
2. Assume ψ is σ- Hg-O. Then $\psi \subseteq c*i_g(\psi) \subseteq c*i_g(\psi)\cup i_g c*(\psi)$. Which implies ψ is b-Hg-O.
3. Assume ψ be π-Hg-O. Then $\psi\subseteq i_g c*(\psi) \subseteq c*i_g(\psi)\cup i_g c*(\psi)$. Which implies ψ is b- Hg-O.

Example 2.15. Assume Z = {1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3}, {1, 4}, {1, 3, 4}, Z}, H = {∅, {1}, {2}}. Then ψ={2} is b-Hg-O but not α-Hg-O (resp. σ-Hg- O, π-Hg-O).

Theorem 2.16. If ψ is b-Hg-O and ζ-σ-O, then it is β-H- O.

Proof. Let ψ is b-Hg-O and ζ-σ- O. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $\psi \subseteq ci(\psi)$. Now $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) \subseteq c*(\psi)$, which implies $ci(\psi) \subseteq cic*(\psi)$. So $\psi \subseteq ci(\psi) \subseteq ci*(\psi)$. Hence ψ is β-H-O.

Theorem 2.17. If ψ is b - Hg - O and ζ∗ -closed, then it is σ-Hg-O.

Proof. Let ψ is b-Hg-O and ζ∗-closed. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $c*(\psi) \subseteq \psi$. Now $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) \subseteq c*i_g(\psi) \cup i_g(\psi) = c*i_g(\psi)$. Hence ψ is σ-Hg-O.

Theorem 2.18. If ψ is b-Hg-O and ζ-closed, then it is σ-Hg-O.

Proof. Let ψ is b-Hg-O and ζ-closed. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $c*(\psi) \subseteq \psi$ by Proposition 2.9 of [6]. Which implies $i_g c*(\psi) \subseteq i_g(\psi)$. Now $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) \subseteq c*i_g(\psi) \cup i_g(\psi) = c*i_g(\psi)$. Hence σ-Hg-O.

Theorem 2.19. If ψ is b-Hg-O such that $ig(\psi) = \emptyset$, then it is π-Hg-O.

Proof. Let ψ be a b-Hg-O and $i_g(\psi)=\emptyset$. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) = i_g c*(\psi)$. Hence ψ is π-Hg-O.

Theorem 2.20. If ψ⊂Z is b-Hg-O and ψ∈H, then it is σ-Hg-O.

Proof. Let ψ is b-Hg-O and ψ∈ H. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $c*(\psi) = \psi$ by Remark 2.10 of [6]. Now $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) = i_g(\psi) \cup c*i_g(\psi) = c*i_g(\psi)$. Hence ψ is σ-Hg-O.

Theorem 2.21. If ψ⊂Z is b-Hg-O and H=P (Z) then it is σ-Hg-O.

Proof. Let ψ is b-Hg-O and H = P (Z) Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $c*(\psi)=\psi$ by Remark 2.10 of [6]. Now $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) = i_g(\psi) \cup c*i_g(\psi) = c*i_g(\psi)$. Hence ψ is σ-Hg-O.

Definition 2.22. For ψ⊂Z, $i_b H_g(\psi)$ is the largest b-Hg-O set contained in ψ.

Definition 2.23. A subset ψ of HGT S (Z, ζ, H) is called D_b(c, Hg)-s, if $ig(\psi)=i_b Hg(\psi)$.

Theorem 2.24. For a subset ψ of HGT S (Z, ζ, H), the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is b-Hg-O and D_b(c, Hg)-s

Proof. (1)⇒(2) Let ψ is gζ-O. Then ψ is b-Hg-O. So $\psi = i_g(\psi)$ and $\psi = i_b H(\psi)$. Therefore $i_g(\psi) = i_b H(\psi)$. Hence ψ is D_b(c, Hg)-s

(2) ⇒ (1) Let ψ is b-Hg-O and D_b(c, Hg)-s Then $\psi = i_b H(\psi)$ and $i_g(\psi) = i_b H(\psi)$. Therefore $i_g(\psi)=\psi$. Hence ψ is gζ-O.

Remark 2.25. The notions of ψ is b-Hg-O and D_b(c, Hg)-s are independent.

Example 2.26. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3}, {1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ = {2} is b-Hg-O but not D_b(c, Hg)-s and M = {4} is D_b(c, Hg)-s but not b-Hg-O set.

3. New types of Sets

Definition 3.1. A subset ψ⊂Z is called

1. α∗-Hg-s, if $i_g c*i_g(\psi) = i(\psi)$.
2. σ∗-Hg-s, if $c*i_g(\psi)=i(\psi)$.

3. $\pi*$-Hg-s, if $i_g c*(\psi)=i(\psi)$.
4. $b*$-Hg-s, if $c*i_g(\psi)\cup i_g c*(\psi)=i(\psi)$.
5. $\beta*$-Hg-s ($\beta*$-Hg-s), if $c i_g c*(\psi) = i(\psi)$.

Remark 3.2. The notions of $\alpha*$-Hg-s (resp. $\sigma*$-Hg-s, $\pi*$-Hg-s, $b*$-Hg-s, $\beta*$-Hg-s) and α-g-O (resp. σ-Hg-O, π-Hg-O, b-Hg-O, β-Hg-O) are independent.

Example 3.3. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H = {∅, {1}, {2}}. Then ψ={1} is α-Hg-O (resp. σ-Hg-O, π-Hg-O, b-Hg-O, β-Hg-O) but not $\alpha*$-Hg-s (resp. $\sigma*$-Hg-s , $\pi*$-Hg-s, $b*$-Hg-s , $\beta*$-Hg-s) and M={2} is $\alpha*$-Hg-s (resp. $\sigma*$- Hg-s , $\pi*$-Hg-s, $b*$-Hg-s, $\beta*$-Hg-s) but not α-Hg-O (resp. σ-Hg O, π-Hg-O, b-Hg-O, β-Hg-O).

Definition 3.4. The subset $\psi \subset Z$ of is called

1. $\alpha*$-B-Hg-s ($\alpha*$-B-Hg-s), if $\psi=U\cap V$, where U is ζ-O and V is $\alpha*$-Hg-s.
2. $\sigma*$-B-Hg-s ($\sigma*$-B-Hg-s), if $\psi=U\cap V$, where U is ζ-O and V is $\sigma*$-Hg-s.
3. $\pi*$-B-Hg-s ($\pi*$-B-Hg-s), if $\psi=U\cap V$, where U is ζ-O and V is $\pi*$-Hg-s.
4. $b*$-B-Hg-s ($b*$-B-Hg-s), if $\psi=U\cap V$, where U is ζ-O and V is $b*$-Hg-s.
5. $\beta*$-B-Hg-s ($\beta*$-B-Hg-s), if $\psi=U\cap V$, where U is ζ-O and V is $\beta*$-Hg-s.

Theorem 3.5. If $\psi\subset Z$ is b-Hg-O and $\pi*$-Hg-s, then it is σ-Hg-O.

Proof. Let ψ is b-Hg-O and $\pi*$-Hg-s. Then $\psi\subseteq i_g c*(\psi)\cup c*i_g(\psi)$ and $i_g c*(\psi)=i_g(\psi)$. Now $\psi \subseteq i_g c*(\psi)\cup c*i_g(\psi)\subseteq c*i_g(\psi)\cup i_g(\psi) = c*i_g(\psi)$. Hence ψ is σ-Hg-O.

Theorem 3.6. If $\psi\subset Z$ is b-Hg-O and $\sigma*$-Hg-s, then it is π-Hg-O.

Proof. Let ψ is b-Hg-O and $\sigma*$-Hg-s.Then $\psi\subseteq i_g c*(\psi) \cup c*i_g(\psi)$ and $c*i_g(\psi) = i_g(\psi)$. Now $\psi\subseteq i_g c*(\psi)\cup c*i_g(\psi)\subseteq i_g c*(\psi)\cup i_g(\psi)=i_g c*(\psi)$. Hence ψ is π-Hg-O.

Proposition 3.7. Let (Z, ζ, H) be a strong HGTS and $\psi\subset Z$. Then the following holds:

1. If ψ is $\alpha*$-Hg-s, then ψ is $\alpha*$-B-Hg-s,
2. If ψ is $\sigma*$-Hg-s, then ψ is $\sigma*$-B-Hg-s.
3. If ψ is $\pi*$-Hg-s, then ψ is $\pi*$-B-Hg-s,
4. If ψ is $b*$-Hg-s, then ψ is $b*$-B-Hg-s,
5. If ψ is $\beta*$-Hg-s, then ψ is $\beta*$-B-Hg-s.

Proof. Let ψ be a $\pi*$-Hg-s. If we take $M=Z\in\zeta$, then $\psi=M\cap\psi$ and hence ψ is a $\pi*$-B-Hg-s.

Proof of (2), (3), (4), (5) are similar of Proof of (1).

Proposition 3.8. For a subset ψ a HGTS (Z, ζ, H), the following properties are hold:

1. If ψ is an $\sigma*$-Hg-s and $g\zeta$-O, then ψ is $\alpha*$-Hg-s.
2. If ψ is an $\pi*$-Hg-s and $g\zeta$-O, then ψ is $\alpha*$-Hg-s.
3. If ψ is an $b*$-Hg-s, then ψ is $\pi*$-Hg-s.
4. If ψ is an $b*$-Hg-s, then ψ is $\sigma*$-Hg-s.

Proof. (1). Let ψ is $\sigma*$-Hg-s and $g\zeta$-O. Then $i_g c*i_g(\psi)\subset c*i_g(\psi) = i(A)$.

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Therefore $i_g c*i_g(\psi)$=i(A). Hence ψ is α∗-Hg-s.

(2). Let ψ is π∗-Hg-s and gζ-O. Then $i_g c*i_g(\psi)\subset i_g c*(\psi)$=i (A). Therefore $i_g c*i_g(\psi)$=i(A). Hence ψ is α∗-Hg-s.

(3). Let ψ b∗-Hg-s. Then $i_g c*(\psi)\subset i_g c*(\psi)\cup c*i_g(\psi)$=i(ψ). Therefore $i_g c*(\psi)$= i(ψ). Hence ψ is π∗-Hg-s.

(4). Let ψ b∗-Hg-s. Then $c*i_g(\psi)\subset i_g c*(\psi)\cup c*i_g(\psi)$=i(ψ). Therefore $c*i_g(\psi)$=i(ψ). Hence ψ is σ∗-H-s.

Theorem 3.9. Let (Z, ζ, H) be a strong HGT S where Z is C0 -space and ψ ⊂ Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is α-Hg-O and α∗-B-Hg-s,
3. ψ is σ-Hg-O and σ∗-B-Hg-s.
4. ψ is π-Hg-O and π∗-B-Hg-s,
5. ψ is β-Hg-O and β∗-B-Hg-s.

Proof. (1) ⇒ (2), (1) ⇒ (3), (1) ⇒ (4), are obvious.

(2)⇒(1). Let ψ is both α-Hg-O and α∗-B-Hg-s. Then $\psi\subseteq i_g c*i_g(\psi)=i_g c*i_g(M\cap N)$, where M∈ζ and N is α∗-Hg-s. Hence $\psi\subseteq i_g c*i_g(M)\cap i_g c*i_g(N)$. Now $\psi\subseteq M\cap\psi\subseteq M\cap[i_g c*i_g(M)\cap i(N)]=M\cap i(N)=i(\psi)$. Hence ψ is ζ-O.

(3)⇒(1). Let ψ is both σ-Hg-O and σ∗-B-Hg-s. Then $\psi\subseteq c*i_g(\psi) = c*i_g(M\cap N)$, where M∈ζ and N is σ∗-Hg-s. Hence $\psi\subseteq c*i_g(M)\cap c*i_g(N)$. Now $\psi\subseteq M\cap\psi\subseteq M\cap[c*i_g(M)\cap i(N)]=M\cap i(N)=i(\psi)$. Hence ψ is ζ-O.

(4)⇒ (1). Let ψ is both π-Hg-O and π∗-B-Hg-s. Then $\psi\subseteq i_g c*(\psi)=i_g c*(M\cap N)$, where M∈ζ and N is π∗-Hg-s. Hence $\psi\subseteq i_g c*(M)\cap i_g c*(N)$. Now $\psi\subseteq M\cap\psi\subseteq M\cap[i_g c*(M)\cap i(N)]=M\cap i(N) = i(\psi)$. Hence ψ is ζ-O.

(5)⇒ (1). Let ψ is both β-Hg-O and β∗-B-Hg-s. Then $\psi\subseteq ci_g c*(\psi)=ci_g c*(M\cap N)$, where M∈ζ and N is β∗-Hg-s. Hence $\psi\subseteq ci_g c*(M)\cap ci_g c*(N)$. Now $\psi\subseteq M\cap\psi\subseteq M\cap[ci_g c*(M)\cap i(N)]=M\cap i(N) = i(\psi)$. Hence ψ is ζ-O.

Remark 3.10. The notions of α-Hg-O (resp. σ-Hg-O, π-Hg-O, β-Hg-O) and α∗-B-Hg-s (resp. σ∗-B -Hg-s, π∗-B-Hg-s, β∗-B-Hg -s) are independent.

Example 3.11. Assume Z = {1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is α-Hg-O (resp. σ-Hg-O, π-Hg-O, β-Hg-O) but not α∗- B-Hg-s (rep. σ∗-B-Hg-s, π∗-B-Hg-s, β∗-B-Hg-s) and M={2} is α∗-B-Hg-s (rep. σ∗-B-Hg -s, π∗-B-Hg-s, β∗-B-Hg-s) but not α-Hg-O (resp. σ-Hg-O, π-Hg-O, β-Hg-O).

Theorem 3.12. Let (Z, ζ, H) be a strong HGT S where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is σ-Hg-O and b∗-B-Hg-s

3. ψ is π-Hg-O and b∗-B-Hg-s
4. ψ is b-Hg-O and b∗-B-Hg-s

Proof. (1) ⇒ (2) ⇒ (4) and (1) ⇒ (3) ⇒ (4) are obvious, since Z is b∗-B-Hg-s

(4) ⇒ (1). Let ψ is b-Hg-O and b∗-B-Hg-s. Then $\psi \subseteq i_g c*(\psi) \cup c*i_g(\psi) = i_g c*(M \cap N) \cup c*i_g(M \cap N)$, where ψ=M∩N, M∈ζ and V is b∗-Hg-s Hence $\psi \subseteq M \cap \psi \subseteq M \cap [i_g c*(M \cap N) \cup c*i_g(M \cap N)] \subseteq [M \cap i_g c*(M) \cap i_g c*(N)] \cup [M \cap c*i_g(M) \cap c*ig(N)] \subseteq [M \cap i_g c*(N)] \cup [M \cap c*i_g(N)] = M \cap [i_g c*(N) \cup c*i_g(N)] = M \cap i(V) = i(\psi)$.

Remark 3.13. The notions of σ-Hg-O (resp. π-Hg-O, b-Hg-O) and b∗-B-Hg-s are independent.

Example 3.14. Assume Z = {1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is σ-Hg-O (resp. π-Hg-O, b-Hg-O) but not b∗-B-Hg-s and M={2} is b∗-B-Hg-s but not σ-Hg-O (resp. π-Hg-O, b-Hg-O).

Theorem 3.15. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is α-Hg-O and σ∗-B-Hg- s,
3. ψ is σ-Hg-O and σ∗-B-Hg-s .

Proof. (1) ⇒ (2). Let a subset ψ of Z is ζ-O. Then it is α-Hg-O and σ∗-B-Hg-s.

(2)⇒(3). Let a subset ψ of Z is both α-Hg-O and σ∗-B-Hg-s. Then it is both σ-Hg-O and σ∗-B-Hg-s.

(3)⇒(1). This is from Theorem 3.9.

Remark 3.16. The notions of α-Hg-O and σ∗-B-Hg-s are independent.

Example 3.17. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is α-Hg-O but not σ∗-B-Hg-s and M={2} is σ∗-B-Hg-s but not α-Hg-O .

Theorem 3.18. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is α-Hg-O and π∗-B-Hg-s,
3. ψ is π-Hg-O and π∗-B-Hg-s.

Proof. (1)⇒(2). Let a subset ψ of Z is ζ-O. Then it is α-Hg-O and π∗-B-Hg-s.

(2)⇒(3). Let a subset ψ of Z is both α-Hg-O and π∗-B-Hg-s. Then it is both π-Hg-O and π∗-B-Hg-s.

(3)⇒(1). This is from Theorem 3.9.

Theorem 3.19. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is α-Hg-O and β∗-B-Hg-s,
3. ψ is β-Hg-O and β∗-B-Hg-s.

Proof. (1)⇒(2). Let a subset ψ of Z is ζ-O. Then it is α-Hg-O and β∗-B-Hg-s.

(2)⇒(3). Let a subset ψ of Z is both α-Hg-O and β∗-B-Hg-s. Then it is both β-Hg-O and β∗-B-Hg-s.

(3)⇒(1). This is from Theorem 3.9.

Remark 3.20. The notions of α-Hg-O and π∗-B-Hg-s are independent.

Example 3.21. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is α-Hg-O but not π∗-B-Hg-s (resp. β∗-B-Hg-s) and M={2} is π∗-B-Hg-s (resp. β∗-B-Hg-s) but not α-Hg-O.

Theorem 3.22. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is σ-Hg-O and β∗-B-Hg-s,
3. ψ is β-Hg-O and β∗-B-Hg-s .

Proof. (1) ⇒ (2). Let a subset ψ of Z is ζ-O. Then it is σ-Hg-O and β∗-B-Hg-s.

(2)⇒ (3). Let a subset ψ of Z is both σ-Hg-O and β∗-B-Hg-s. Then it is both β-Hg-O and β∗-B-Hg-s.

(3)⇒ (1). This is from Theorem 3.9.

Remark 3.23. The notions of σ-Hg-O and β∗-B-Hg-s are independent.

Example 3.24. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3}, {1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is σ-Hg-O but not β∗-B-Hg-s and M={2} is β∗-B-Hg-s but not σ-Hg-O.

Theorem 3.25. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is ζ-O,
2. ψ is π-Hg-O and β∗-B-Hg-s,
3. ψ is β-Hg-O and β∗-B-Hg-s.

Proof. (1) ⇒ (2). Let a subset ψ of Z is ζ-O. Then it is π-Hg-O and β∗-B-Hg-s.

(2)⇒(3). Let a subset ψ of Z is both π-Hg-O and β∗-B-Hg-s. Then it is both β-Hg-O and β∗-B-Hg-s.

(3)⇒(1). This is from Theorem 3.9.

Remark 3.26. The notions of π-Hg-O and β∗-B-Hg-s are independent.

Example 3.27. Assume Z = {1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={1} is π-Hg-O but not β∗-B-Hg-s and M={2} is β∗-B-Hg-s but not π-Hg-O.

Definition 3.28. A subset ψ of a HGTS (Z, ζ, H) is called

1. ξ∗-Hg-s, $i_g c{*} i_g(\psi)=i_g(\psi)$.
2. σ∗-Hg-s, if $c{*} i_g(\psi)=i_g(\psi)$.
3. π∗-Hg-s, if $i_g c{*}(\psi)=i_g(\psi)$.
4. Φ∗-Hg-s, if $c{*} i_g(\psi) \cup i_g c{*}(\psi)=i_g(\psi)$.

5. Δ∗-Hg-s, if $ci_g c*(\psi)=i_g(\psi)$.

Definition 3.29. A subset ψ of HGTS (Z, ζ, H) is called

1. ξ∗-B-Hg-s, if ψ=U∩V, where U is gζ-O and V is ξ∗-Hg-s
2. σ∗-B-Hg-s, if ψ=U∩V, where U is gζ-O and V is σ∗-Hg-s.
3. π∗-B-Hg-s, if ψ=U∩V, where U is gζ-O and V is π∗-Hg-s.
4. Φ∗-B-Hg-s, if ψ=U∩V, where U is gζ-O and V is Φ∗-Hg-s
5. Δ∗-B-Hg-s , if ψ=U∩V, where U is gζ-O and V is Δ∗-Hg-s

Theorem 3.30. If ψ⊂Z is both π-Hg-O and ζ∗-closed, then it is π∗-Hg-s.

Proof. Let ψ is both π-Hg-O and ζ∗-closed. Then $\psi\subseteq i_g c*(\psi)$ and $c*(\psi)\subset\psi$. Now $i_g c*(\psi)\subset c*(\psi)\subset\psi$. So, $\psi=i_g c*(\psi)$. Thus $i_g(\psi) = i_g c*(\psi)$. Hence ψ is π∗-Hg-s.

Theorem 3.31. If ψ⊂Z is both σ-Hg-O and ζ∗-closed, then it is ξ∗-Hg-s

Proof. Let ψ is both σ-Hg-O and ζ∗-closed. Then $\psi\subseteq c*i_g(\psi)$ and $c*(\psi)\subset\psi$. Now $c*i_g(\psi)\subset c*(\psi)\subset\psi$. So, $\psi=c*i_g(\psi)$. Thus $i_g(\psi) = i_g c*i_g(\psi)$. Hence ψ is ξ∗-Hg-s

Theorem 3.32. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and L⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is α-Hg-O and ξ∗-B-Hg-s,
3. ψ is σ-Hg-O and σ∗-B-Hg-s,
4. ψ is π-Hg-O and π∗-B-Hg-s.
5. ψ is β-Hg-O and Δ∗-B-Hg-s

Proof. (1) ⇒ (2), (1) ⇒ (3), (1) ⇒ (4), are obvious.

(2)⇒ (1). Let ψ is both α-Hg-O and ξ∗-B-Hg-s Then $\psi\subseteq i_g c*i_g(\psi)=i_g c*i_g(U\cap V)$, where U is gζ-O and V is ξ∗-Hg-s Hence $\psi\subseteq i_g c*i_g(U)\cap i_g c*i_g(V)$.Now $\psi\subseteq U\cap\psi\subseteq U\cap[i_g c*i_g(U)\cap i_g(V)] = i_g(U)\cap i_g(V)=ig(\psi)$. Hence ψ is gζ-O.

(3) ⇒ (1). Let ψ is both σ-Hg-O and σ∗-B-Hg-s. Then $\psi\subseteq c*i_g(\psi)=c*i_g(U\cap V)$, where U is gζ-O and V is σ∗-Hg-s. Hence $\psi\subseteq c*i_g(U)\cap c*i_g(V)$. Now $\psi\subseteq U\cap\psi\subseteq U\cap[c*i_g(U)\cap i_g(V)] = i_g(U)\cap i_g(V)=i_g(\psi)$. Hence ψ is gζ-O.

(4)⇒ (1). Let ψ is both π-Hg-O and π∗-B-Hg-s. Then $\psi\subseteq igc*(\psi) =i_g c*(U\cap V)$, where U is gζ-O and V is π∗-Hg-s. Hence $\psi\subseteq i_g c*(U)\cap i_g c*(V)$. Now $\psi\subseteq U\cap\psi \subseteq U\cap[i_g c*(U)\cap i_g(V)] = i_g(U)\cap i_g(V)=i_g(\psi)$. Hence ψ is gζ-O.

(5)⇒ (1). Let ψ is both β-Hg-O and Δ∗-B-Hg-s Then $\psi\subseteq ci_g c*(\psi) =ci_g c*(U\cap V)$, where U is gζ-O and V is Δ∗-Hg-s Hence $\psi \subseteq ci_g c*(U)\cap ci_g c*(V)$. Now $\psi\subseteq U\cap\psi\subseteq U\cap[ci_g c*(U)\cap i_g(V)]=i_g(U)\cap i_g(V)=i_g(\psi)$. Hence ψ is gζ-O.

Remark 3.33. The notions of α-Hg-O (resp. σ-Hg-O, π-Hg-O, β-Hg-O) and ξ∗-B-Hg-s (resp. σ∗-B-Hg-s, π∗-B-Hg-s, Δ∗-B-Hg-s) are independent.

Example 3.34. Assume Z={1, 2, 3, 4}, ζ={∅, {1, 3}, {2, 3}, {1, 2, 3},{1, 4}, {1, 3, 4}, Z}, H={∅, {1}, {2}}. Then ψ={3, 4} is σ-Hg-O (resp. π-Hg-O, bπ-Hg-O) but not Φ∗-B-Hg-s and M={2} is Φ∗-B-Hg-s but not σ-Hg-O (resp. π-Hg-O, b-Hg-O).

Theorem 3.35. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is σ-Hg-O and Φ∗-B-Hg-s
3. ψ is π-Hg-O and Φ∗-B-Hg-s
4. ψ is b-Hg-O and Φ∗-B-Hg-s.

Proof. (1)⇒(2)⇒(4) and (1)⇒(3)⇒(4) are obvious, since Z is Φ∗-B-Hg-s

(4)⇒(1). Let ψ is b-Hg-O and Φ∗-B-Hg-s Then $\psi\subseteq i_g c*(\psi)\cup c*i_g(\psi)=i_g c*(M\cap N)\cup c*i_g(M\cap N)$, where ψ=M∩N, M is gζ-O and V is Φ∗-Hg-s. Hence $\psi\subseteq M\cap\psi\subseteq M\cap[i_g c*(M\cap N)\cup c*i_g(M\cap N)]\subseteq[M\cap i_g c*(M)\cap i_g c*(N)]\cup[M\cap c*i_g(M)\cap c*i_g(N)]$ $\subseteq[M\cap i_g c*(N)]\cup[M\cap c*i_g(N)]=M\cap[i_g c*(N)\cup c*i_g(N)]=M\cap i_g(V)=i_g(\psi)$.

Remark 3.36. The notions of σ-Hg-O (resp. π-Hg-O, b-Hg-O) and Φ∗-B-Hg-s are independent.

Theorem 3.37. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O.
2. ψ is α-Hg-O and π∗-B-Hg-s.
3. ψ is π-Hg-O and π∗-B-Hg-s.

Proof. (1)⇒(2). Let a subset ψ of Z is gζ-O. Then it is α-Hg-O and π∗-B-Hg-s.

(2)⇒(3). Let a subset ψ of Z is both α-Hg-O and π∗-B-Hg-s. Then it is both ψ is π-Hg-O and π∗-B-Hg-s.

(3)⇒(1). This is from Theorem 3.32.

Theorem 3.38. Let (Z, ζ, H) be a strong HGTS where Z is C0 -space and L⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is α-Hg-O and σ∗-B-Hg-s,
3. ψ is σ-Hg-O and σ∗-B-Hg-s.

Proof. (1)⇒(2). Let a subset ψ of Z is gζ-O. Then it is α-Hg-O and σ∗-B-Hg-s.

(2) ⇒ (3). Let a subset ψ of Z is both α-Hg-O and σ∗-B-Hg-s. Then it is both σ-Hg-O and σ∗-B-Hg-s.

(3) ⇒ (1). This is from Theorem 3.32.

Theorem 3.39. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and L⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is α-Hg-O and Δ∗-B-Hg-s,
3. ψ is β-Hg-O and Δ∗-B-Hg-s.

Proof. (1) ⇒ (2). Let a subset ψ of Z is gζ-O. Then it is α-Hg-O and Δ∗-B-Hg-s.

(2) ⇒ (3). Let a subset ψ of Z is both α-Hg-O and Δ∗-B-Hg-s Then it is both β-Hg-O and Δ∗-B-Hg-s.

(3) ⇒ (1). This is from Theorem 3.32.

Theorem 3.40. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and L⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is σ-Hg-O and Δ∗-B-Hg-s,
3. ψ is β-Hg-O and Δ∗-B-Hg- s.

Proof. (1) ⇒ (2). Let a subset ψ of Z is gζ-O. Then it is σ-Hg-O and Δ∗-B-Hg-s

(2) ⇒ (3). Let a subset ψ of Z is both σ-Hg-O and Δ∗-B-Hg-s. Then it is both β-Hg-O and Δ∗-B-Hg-s.

(3) ⇒ (1). This is from Theorem 3.32.

Theorem 3.41. Let (Z, ζ, H) be a strong HGTS, where Z is C0 -space and ψ⊂Z. Then the following conditions are equivalent.

1. ψ is gζ-O,
2. ψ is π-Hg-O and Δ∗-B-Hg-s,
3. ψ is β-Hg-O and Δ∗-B-Hg-s.

Proof. (1) ⇒ (2). Let a subset ψ of Z is gζ-O. Then it is π-Hg-O and Δ∗-B-Hg-s

(2)⇒(3). Let a subset ψ of Z is both π-Hg-O and Δ∗-B-Hg- s. Then it is both β-Hg-O and Δ∗-B-Hg-s.

(3) ⇒ (1). This is from Theorem 3.32.

4. Decomposition of (gζ, ξ) -Continuity

Definition 4.1. A map ν:(Z, ζ, H)→ (W, ξ) is (b- Hg, ξ)-c, if $j^{-1}(V)$ is b-Hg-O for each ξ-O set V in (W, ξ).

Definition 4.2. A map ν:(Z, ζ, H)→(W, ξ) is (R*g, ξ)-c ((R*g, ξ)-c), (resp. ((Db(c, Hg), ξ)- c), if $v^{-1}(V)$ is R*g set (resp. (Db(c, Hg)-s for each ξ-O set V in (W, ξ).

Definition 4.3. A function ν:(Z, ζ, H)→(W, ξ) is said to be (α∗-B-Hg, ξ)-c (resp. (π∗-B-Hg, ξ)-c, (σ∗-B-Hg, ξ)-c, (b∗-B-Hg, ξ)-c, (β∗-B-Hg, ξ)-c), if $v^{-1}(V)$ is α∗-B-Hg-s (resp. π∗-B-Hg-s, σ∗-B-Hg-s, b∗-B-Hg-s, β∗-B-Hg-s) for each ξ-O set V in (W, ξ).

Definition 4.4. A function ν:(Z, ζ, H)→(W, ξ) is said to be (ξ∗-B-Hg, ξ)-c (resp. (Π∗-B-Hg, ξ)-c, (Σ∗-B-Hg, ξ)-c, (Φ∗-B-Hg, ξ)-c, (Δ∗-B-Hg, ξ)-c), if $v^{-1}(V)$ is ξ∗-B-Hg-s (resp. π∗-B-Hg-s, Δ∗-B-Hg-s) for each ξ - O set V in (W, ξ).

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Theorem 4.5. For a map ν:(Z, ζ, H)→(W, ξ) where Z is C0 -space, the following results are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (b-Hg, ξ)- c and (Db(c, Hg), ξ)-c.

Proof. The proof is clear by Theorem 2.53.

Theorem 4.6. For a map ν:(Z, ζ, H)→(W, ξ) where Z is C0 -space, the following results are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (α∗-B-Hg, ξ)-c,
3. ν is (σ-Hg, ξ)-c and (σ∗-B-Hg, ξ)-c,
4. ν is (π-Hg, ξ)-c and (π∗-B-Hg, ξ)-c,
5. ν is (β-Hg, ξ)-c and (β∗-B-Hg, ξ)- c.

Proof. The proof is clear by Theorem 3.9.

Theorem 4.7. For a map ν:(Z, ζ, H)→(W, ξ) where Z is C0 -space, the following results are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (σ-Hg, ξ)-c and (b∗ - B - Hg, ξ) - c,
3. ν is (π-Hg, ξ)-c and (b∗ - B - Hg, ξ) - c,
4. ν is (b - Hg, ξ) - c and (b∗ - B - Hg, ξ) - c.

Proof. The proof is clear by Theorem 3.12.

Theorem 4.8. Let (Z, ζ, H) be a strong HGTS for a function ν:(Z, ζ, H) →(W, ξ), Z is C0 -space. Then the following conditions are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (σ∗-B-Hg, ξ)- c,
3. ν is (σ- Hg, ξ)-c and (σ∗-B- Hg, ξ)-c .

Proof. The proof is clear by Theorem 3.15.

Theorem 4.9. Let (Z, ζ, H) be a strong HGTS for a function ν:(Z, ζ, H)→(W, ξ), Z is C0 -space. Then the following conditions are equivalent.

1. ν is (ζ, ξ)-c ,
2. ν is (α - Hg, ξ)-c and (π∗-B-Hg, ξ)-c,
3. ν is (π-Hg, ξ)-c and (π∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.18.

Theorem 4.10. Let (Z, ζ, H) be a strong HGTS for a function ν:(Z, ζ, H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (β∗-B-Hg, ξ)-c,
3. ν is (β-Hg, ξ)-c and (β∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.19.

Theorem 4.11. Let (Z, ζ, H) be a strong HGTS for a function ν:(Z, ζ, H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (σ-Hg, ξ)-c and (β∗-B-Hg, ξ)-c,
3. ν is (β-Hg, ξ)-c and (β∗-B- Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.22.

Theorem 4.12. Let (Z, ζ, H) be a strong HGTS for a function ν:(Z, ζ, H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (ζ, ξ)-c,
2. ν is (π-Hg, ξ)-c and (β∗-B- Hg, ξ)- c,
3. ν is (β-Hg, ξ)-c and (β∗-B- Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.25.

Theorem 4.13. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ,H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (ξ∗-B-Hg, ξ)-c,
3. ν is (σ-Hg, ξ)-c and (Σ∗-B-Hg, ξ)-c,
4. ν is (π- Hg, ξ)-c and (Π∗-B-Hg, ξ)-c,
5. ν is (β-Hg, ξ)-c and (Δ∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.32.

Theorem 4.14. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (σ-Hg, ξ)-c and (Φ∗-B-Hg, ξ)-c,
3. ν is (π-Hg, ξ)-c and (Φ∗-B- Hg, ξ)-c,
4. ν is (b-Hg, ξ)-c and (Φ∗-B- Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.35.

Theorem 4.15. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ), where Z is C0 -space. Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (Π∗-B-Hg, ξ)-c,
3. ν is (π- Hg, ξ)-c and (Π∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.37.

Theorem 4.16. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ). Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,

2. ν is (α- Hg, ξ)-c and (Σ∗-B-Hg, ξ)-c,
3. ν is (σ-Hg, ξ)-c and (Σ∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.38.

Theorem 4.17. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ). Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (α-Hg, ξ)-c and (Δ∗-B-Hg, ξ)-c,
3. ν is (β-Hg, ξ)-c and (Δ∗-B- Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.39.

Theorem 4.18. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ). Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (σ-Hg, ξ)-c and (Δ∗-B-Hg, ξ)-c,
3. ν is (β-Hg, ξ)-c and (Δ∗-B-Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.40.

Theorem 4.19. Let (Z, ζ, H) be a strong HGTS, where Z is c0 space for a function ν:(Z, ζ, H)→(W, ξ). Then the following conditions are equivalent.

1. ν is (gζ, ξ)-c,
2. ν is (π-Hg, ξ)-c and (Δ∗-B-Hg, ξ)-c,
3. ν is (β-Hg, ξ)-c and (Δ∗-B- Hg, ξ)-c.

Proof. The proof is clear by Theorem 3.41.

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